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Soft Generalized αi-Closed Sets in Soft Topological Spaces

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Abstract. In the present paper, we introduced a new kind of soft generalized closed sets and soft generalized open sets are called soft generalized αi-closed sets and soft generalized αi-open sets. The relations among these two families and some other kinds of soft sets as like as soft generalized closed, soft generalized i-closed, soft generalized α-closed and soft semi-generalized closed sets are investigated and clarify by proofs and evidences.

Keywords: Soft generalized αi-closed sets, Soft generalized αi-open sets, Soft i-closed sets.

1. Introduction and Preliminaries

In 2019 (see [10]) Mohammed A.A and Abdullah B.S inserted types of inter-open and ii-open sets. In 1999, 2011 and 2015, Molodtsov and numerous different specialists introduced the concept of soft sets and their properties (see [11], [14], [4]). In (2012, 2013) (see [3] and [6]) the definition of "soft semi-open sets" and "soft α -open sets" was introduced individually in the soft topological spaces by Chen, B. and Kannan, K. In 2020 (see [2]), the idea of soft i-open sets, was introduced by Askandar, S. W. and Mohammed, A. A., which will use in this work.

In this article, soft topological space (X,τ,E) denotes (sTs). What's more, we denote the soft set by (*sS*), $int(K,E)$ and denote the *sS's* (K,E) $Int(K,E)$, $Cl(K,E)$, the soft interior and soft closure respectively. The τ elements are called soft open sets, (sOs), what's more, their complements are called closed soft sets, (sCs). ϕ_E, X_E Separately denote the soft null and soft absolute sets.

In the fragment 1, we give established basic hypothesis of the soft sets and soft topological spaces. In addition, we give fundamental meanings of some soft generalized closed, soft i-open, soft i-closed sets. In the segment 2, we characterize new ideas of soft generalized closed sets as soft generalized αi-closed sets and explore its points of interest. Soft generalized αi-open sets are described in the third section, and numerous significant results are determined.

Proposition 1.1: Consider (K,A), (L,A) belongs to $SS(X_A)$ and there is
$((K,A)\,\tilde{\cup}\,(L,A))^c=(K,A)^c\,\tilde{\cup}\,(L,A)^c$.
$((K,A)\,\tilde{\cap}\,(L,A))^c=(K,A)^c\tilde{\cap}(L,A)^c$. ([14]).

Theorem 1.1: Pick (K,E) to be a soft set in (X,τ,E) and there is

$Int(K,E)^c = (Cl(K,E))^c$.

$Cl(K,E)^c = (Int(K,E))^c$

$Int(K,E) = (Cl(K,E)^c)^c$. ([5]).

Definition 1.1: Let (F,E) be a soft set (*sS*) in (X,τ,E), (F,E) is named:

"Soft i-open set", $(sIOs)$ if there exists a sOs $(O,E) \neq \phi, X$ wherein, $(F,E) \tilde{\subseteq} Cl((F,E) \tilde{\bigcap} (O,E))$ ([2]).

"Soft semi-open set", $(sSOs)$ if: a. "$(F,E) \tilde{\subseteq} Cl(Int(F,E))$". *b.* If there exists a sOs $(O,E) \neq \phi, X$ wherein "$(O,E) \tilde{\subseteq} (F,E) \tilde{\subseteq} Cl(O,E)$" ([3]).

"Soft α-open set", $(s\alpha Os)$ if " $(F,E) \tilde{\subseteq} Int(Cl(Int(F,E)))$ "([6]).

The union of all $sIOs$ (individually, $s\alpha Os$ and $sSOs$) over X contained in (F,E) is named a soft I-interior (individually, soft α-interior and soft Semi-interior) of a soft set (F,E) and designated by $IInt(F,E)$ (individually, $\alpha Int(F,E)$ and $SInt(F,E)$).

The complement of $sIOs$ (individually, $s\alpha Os$ and $sSOs$) is named soft i-closed ($sICs$) (individually, soft α-closed ($s\alpha Cs$) and soft semi-closed ($sSCs$)). The intersection of all $sICs$ (individually, $s\alpha Cs$ and $sSCs$) over X containing (F,E) is called the soft i-closure (individually, soft α-closure and soft semi-closure) of (F,E) and designated by $ICl(F,E)$ (individually, $\alpha Cl(F,E)$ and $SCl(F,E)$

Definition1.2: A (*sS*), (F,E) in (X,τ,E) considers:

"Soft generalized closed set", ($sGCs$) if " $Cl((F,E) \tilde{\subseteq} (O,E)$ "wherein $(F,E) \tilde{\subseteq} (O,E)$ and (O,E) is a sOs in (X,τ,E). The complement of $sGCs$ is named "soft generalized open set", ($sGOs$)([8]).

Definition 1.3: A (*sS*), (F,E) in (X,τ,E) considers:

Soft generalized α-closed set, ($sG\alpha Cs$) if " $Cl((F,E) \tilde{\subseteq} (O,E)$ " wherein $(F,E) \tilde{\subseteq} (O,E)$ and (O,E) is an $s\alpha Os$ in (X,τ,E) ([12]).

Soft α -generalized closed set, ($s\alpha GCs$) if " $\alpha Cl((F,E) \tilde{\subseteq} (O,E)$ " wherein $(F,E) \tilde{\subseteq} (O,E)$ and (O,E) is an sOs in (X,τ,E) ([1]).

"Soft s*g-closed set", ($sS*GCs$) if " $Cl((F,E) \tilde{\subseteq} (O,E)$ " wherein $(F,E) \tilde{\subseteq} (O,E)$ and (O,E) is an $sSOs$ in (X,τ,E) ([9]).

Soft semi generalized closed set, ($sSGCs$) if " $SCl((F,E) \tilde{\subseteq} (O,E)$ "wherein $(F,E) \tilde{\subseteq} (O,E)$ and (O,E) is an $sSOs$ in (X,τ,E) ([7]).

The complement of $sG\alpha Cs$ (resp., $s\alpha GCs$, $sS*GCs$ and $sSGCs$) is named soft generalized α-open ($sG\alpha Os$) (resp., soft α generalized open $s\alpha GOs$, soft s*g open $sS*GOs$ and soft semi generalized open $sSGOs$. All $sG\alpha Cs$ (resp., $s\alpha GCs$, $sS*GCs$, $sSGCs$ and $sGSCs$ in (X,τ,E) obtained $sG\alpha Cs(X_E)$)(resp., $s\alpha GCs(X_E)$ $sS*GCs(X_E)$, $sSGCs(X_E)$ and $sGSCs(X_E)$).

Definition 1.4: Consider (K,E) be an sS in (X,τ,E). Pick " $\tau_{(K,E)}=\{(G,E)\tilde{\sqcap}(K,E):(G,E)\in\tau\}$" the thought of "soft topology" on (K,E). The soft topology is called relative soft topology of τ on (K,E) and $\{(K,E),\tau_{(K,E)}\}$ is defined as (X,τ,E) soft subspace ([13]).

Theorem 1.2: Each sOs is a $sIOs$ ([2])

Corollary 1.1: Each sCs is a $sICs$ ([2])

Theorem 1.3: Each $sSOs$ is a $sIOs$ ([2]).

Corollary 1.2: Each $sSCs$ is a $sICs$.

Proof: Assume that (K,E) be an $sSCs$ in (X,τ,E), we get, $(K,E)^C$ is a $sSOs$, we have, $(K,E)^C$ is a $sIOs$"(Theorem 1.3)". Henceforth, (K,E) is a $sICs$.

Theorem 1.4: Each $s\alpha Os$ is a $sSOs$([2]).

Corollary 1.3: Each $s\alpha Cs$ is a $sSCs$.

Proof: Consider (K,E) as an $s\alpha Cs$ in (X,τ,E), we get, $(K,E)^C$ is a $s\alpha Os$, which implies to $(K,E)^C$ is a $sSOs$"(Theorem 1.4)". Henceforth, (K,E) is a $sSCs$.

Theorem 1.5: Each $s\alpha Os$ is a $sIOs$([2]).

Corollary 1.4: Each $s\alpha Cs$ is a $sICs$.

Proof: Assume that (K,E) be an $s\alpha Cs$ in (X,τ,E), we get, $(K,E)^C$ is a $s\alpha Os$, which implies to $(K,E)^C$ is a $sIOs$"(Theorem 1.5)". Henceforth, (K,E) is a $sICs$.

2. Soft Generalized αi-Closed Sets

Definition 2.1: Let (K,E) be a (*sS*) in (X,τ,E), then (K,E) considers:

Soft generalized αi-closed set, ($sG\alpha ICs$) if " $ICl((F,E)\tilde{\subseteq}(O,E)$ " wherein" $(K,E)\tilde{\subseteq}(O,E)$ and (O,E) " is a $s\alpha Os$ in (X,τ,E). The group of all $sG\alpha ICs$ is denoted by $sG\alpha ICs(X_E)$.

Soft generalized i-closed set, ($sGICs$) if " $ICl((K,E)\tilde{\subseteq}(O,E)$ "wherein" $(K,E)\tilde{\subseteq}(O,E)$ and (O,E) " is a sOs in (X,τ,E). The group of all $sGICs$ is designated by $sGICs(X_E)$.

Theorem 2.1: Each $sICs$ is a $sG\alpha ICs$.

Proof: Assume that a $sICs$, (F,E) in (X,τ,E) and (O,E) is a $s\alpha Os$ wherein, $(F,E)\tilde{\subseteq}(O,E)$, we get, " $ICl(F,E)=(F,E)\tilde{\subseteq}(O,E)$ ". Henceforth, (F,E) is a $sG\alpha ICs$.

Theorem 2.2: If $(K,E)\tilde{\subseteq}(L,E)\tilde{\subseteq}ICl(K,E)$, wherein, (K,E) is a $sG\alpha ICs$, then so is (L,E).

Proof: Consider $(L,E)\tilde{\subseteq}(O,E)$ and (O,E) be an $s\alpha Os$ in (X,τ,E), then $(K,E)\tilde{\subseteq}(O,E)$. Since (K,E) is a $sG\alpha ICs$, $(L,E)\tilde{\subseteq}ICl(K,E)$, we conclude that " $ICl(L,E)\tilde{\subseteq}IC(K,E)$ ". We get" $ICl(L,E)\tilde{\subseteq}IC(K,E)\tilde{\subseteq}(O,E)$ ". We obtained that" $ICl(L,E)\tilde{\subseteq}(O,E)$ ". Henceforth, (L,E) is a $sG\alpha ICs$.

Theorem 2.3: If $(K,E)\tilde{\subseteq}(W,E)\tilde{\subseteq}(X,\tau,E)$, wherein, (K,E) is an $sG\alpha ICs$ in (X,τ,E), then (K,E) is $sG\alpha ICs$ relative to (W,τ_W,E).

Proof: Consider $(K,E)\tilde{\subseteq}(O,E)$ and (O,E) is a $s\alpha Os$ in (W,E), since $(K,E)\tilde{\subseteq}(W,E)$, we get $(K,E)\tilde{\subseteq}(O,E)\tilde{\bigcap}(W,E)$, (O,E) is a $s\alpha Os$ in (W,τ_W,E), then, there exists a $s\alpha Os$ (M,E) in X wherein, $(O,E)=(M,E)\tilde{\bigcap}(W,E)$, then $(K,E)\tilde{\subseteq}(O,E)\tilde{\subseteq}(M,E)$ with (K,E) is an $sG\alpha ICs$ in (X,τ,E), we get, $ICl(K,E)\tilde{\subseteq}(M,E)$ by "$ICl(K,E)\tilde{\bigcap}(W,E)$"is a soft i-closure of (K,E) in (W,τ_W,E), we get, "$ICl(K,E)\tilde{\bigcap}(W,E)\tilde{\subseteq}(O,E)$". Henceforth, (K,E) is a $sG\alpha ICs$ relative to (W,τ_W,E).

Theorem 2.4: If $(K_1,E),(K_2,E)$ are $sG\alpha ICs$, then so is their intersection.

Proof: Consider $(K_1,E),(K_2,E)$ as a $sG\alpha ICs$ in (X,τ,E) and $(O_1,E),(O_2,E)$ are any $s\alpha Os$ wherein $(K_1,E)\tilde{\subseteq}(O_1,E)$ and $(K_2,E)\tilde{\subseteq}(O_2,E)$, then $Cl(K_1,E)\tilde{\subseteq}(O_1,E)$ and $ICl(K_2,E)\tilde{\subseteq}(O_2,E)$ (by suppose). Since,"$ICl((K_1,E)\tilde{\bigcap}(K_2,E))\tilde{\subseteq}ICl(K_1,E)\tilde{\bigcap}ICl(K_2,E)$", we get, $Cl((K_1,E)\tilde{\bigcap}(K_2,E))\tilde{\subseteq}((O_1,E)\tilde{\bigcap}(O_2,E))$ and since the intersection between any two $s\alpha Os$ is a $s\alpha Os$, put "$((O_1,E)\tilde{\bigcap}(O_2,E))=(O,E)$"with (O,E) is a $s\alpha Os$, $ICl((K_1,E)\tilde{\bigcap}(K_2,E))\tilde{\subseteq}(O,E)$. Henceforth, $(K_1,E)\tilde{\bigcap}(K_2,E)$ is a $sG\alpha ICs$

Theorem 2.5: Each $sG\alpha ICs$ is a $sGICs$.

Proof: Consider (K,E) is a $sG\alpha ICs$ in (X,τ,E) and (O,E) is any sOs wherein $(K,E)\tilde{\subseteq}(O,E)$, then $ICl(K,E)\tilde{\subseteq}(O,E)$ since each sOs is $s\alpha Os$ with (O,E) is a sOs . Henceforth, (K,E) is a $sGICs$.

Theorem 2.6: Each $sG\alpha Cs$ is a $sG\alpha ICs$.

Proof: Assume that (K,E) is a $sG\alpha Cs$ in (X,τ,E) and (O,E) is any $s\alpha Os$ wherein "$(K,E)\tilde{\subseteq}(O,E)$", "$ICl(K,E)\tilde{\subseteq}Cl(K,E)\tilde{\subseteq}(O,E)$"will that mean $ICl(K,E)\tilde{\subseteq}(O,E)$ with (O,E) is a $s\alpha Os$. Henceforth, (K,E) is a $sG\alpha ICs$.

Theorem 2.7: Each $sSGCs$ is a $sG\alpha ICs$.

Proof: Consider (K,E) is an $sSGCs$ in (X,τ,E) and $(K,E)\tilde{\subseteq}(O,E)$ where (O,E) is any $s\alpha Os$ and $sSOs$, since, each $sSCs$ is a $sICs$ "(Corollary 1.2)", then $ICl(K,E)\tilde{\subseteq}SCl(K,E)\tilde{\subseteq}(O,E)$. Henceforth, (K,E) is a $sG\alpha ICs$.

3. Soft Generalized αi-Open Sets

Definition 3.1: Let (K,E) be a (*sS*) in (X,τ,E), then (K,E) considers as a Soft generalized αi-open set, ($sG\alpha IOs$) when its complement $(K,E)^C$ is a $sG\alpha ICs$. The group of every $sG\alpha IOs$ is denoted by $sG\alpha IOs(X_E)$.

Theorem 3.1: A soft set (K,E) in (X,τ,E) is $sG\alpha IOs$ when and if only $(U,E)\tilde{\subseteq}IInt(K,E)$ where (U,E) is a $s\alpha Cs$ within $(U,E)\tilde{\subseteq}(K,E)$.

Proof: Assume that (K,E) be $sG\alpha IOs$ with " $(U,E)\tilde{\subseteq}(K,E)$" and (U,E) is a $s\alpha Cs$. Then $(K,E)^C$ is $sG\alpha ICs$ and $(U,E)^C$ is $s\alpha Os$ with " $(K,E)^C\tilde{\subseteq}(U,E)$ $ICl(K,E)^C\tilde{\subseteq}(U,E)^C$". Henceforth, $(U,E)\tilde{\subseteq}IInt(K,E)$.

Inverse, assume that $(U,E)\tilde{\subseteq} IInt(K,E)$ with $(U,E)\tilde{\subseteq}(K,E)$ and (U,E) is a $s\alpha Cs$. Consider (M,E) as a $s\alpha Os$ containing $(K,E)^C$, thus, "$(M,E)^C \tilde{\subseteq} IInt(K,E)$ ", then " $ICl(K,E)^C \tilde{\subseteq} (M,E)$". Henceforth, $(K,E)^C$ is $sG\alpha ICs$. Which implies (K,E) be $sG\alpha IOs$.

Theorem 3.2: If $IInt(K,E)\tilde{\subseteq}(L,E)\tilde{\subseteq}(K,E)$, wherein, (K,E) is a $sG\alpha IOs$, then so is (L,E).

Proof: Consider "$IInt(K,E)\tilde{\subseteq}(L,E)\tilde{\subseteq}(K,E)\,(L,E)\tilde{\subseteq}(O,E)$" then " $(K,E)^C \tilde{\subseteq}(L,E)^C \tilde{\subseteq} ICl((K,E)^C)$ " with $(K,E)^C$ is $sG\alpha ICs$. Thus, $(L,E)^C$ is $sG\alpha ICs$ "(Theorem 2.2)". Henceforth, (L,E) is $sG\alpha IOs$.

Corollary 3.1: Each $sG\alpha IOs$ is a $sGIOs$.

Proof: Consider (K,E) as an $sG\alpha IOs$ in (X,τ,E), we get, $(K,E)^C$ is a $sG\alpha ICs$, this implies $(K,E)^C$ is $sGICs$"(Theorem 2.5)". Henceforth, $(K,E)^C$ is a $sGIOs$.

Corollary 3.2: Each $sG\alpha Os$ is a $sG\alpha IOs$.

Proof: Assume that (K,E) is an $sG\alpha Os$ in (X,τ,E), we have, $(K,E)^C$ is a $sG\alpha Cs$, thus $(K,E)^C$ is $sG\alpha ICs$ "(Theorem 2.6)". Henceforth, (K,E) is a $sG\alpha IOs$.

Corollary 3.3: Each $sSGOs$ is a $sG\alpha IOs$.

Proof: Consider (K,E) is an $sSGOs$ in (X,τ,E), we get, $(K,E)^C$ is $sSGCs$, this implies $(K,E)^C$ is $sG\alpha ICs$ "(Theorem 2.7)". Henceforth, (K,E) is a $sG\alpha IOs$.

Theorem 3.3: If $(K_1,E),(K_2,E)$ are $sG\alpha IOs$, then so is their union.

Proof: Consider $(K_1,E),(K_2,E)$ as a $sG\alpha IOs$ in (X,τ,E). We get, $(K_1,E)^C,(K_2,E)^C$ are $sG\alpha ICs$. By "(Theorem 2.4)", we get, "$(K_1,E)^C \tilde{\cap} (K_2,E)^C = ((K_1,E)\tilde{\cup}(K_2,E))^C$", is a $sG\alpha ICs$. Henceforth, $(K_1,E)\tilde{\cup}(K_2,E)$ is a $sG\alpha IOs$.

4. Conclusions

The relation among $sG\alpha ICs$, $sG\alpha Cs$, $sGCs$, $sGICs$ and $sSGCs$, depends on the relation among $s\alpha Os$, $sIOs$, $sSOs$ and sOs.

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