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Pre-service Mathematics Teachers' Knowledge of History of Mathematics and Their Attitudes and Beliefs Towards Using History of Mathematics in Mathematics Education

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Abstract This study examined pre-service mathematics teachers' knowledge of history of mathematics and their attitudes and beliefs towards using history of mathematics in mathematics education based on year level in teacher education program and gender. The sample included 1,593 freshman, sophomore, junior, and senior pre-service middle school (grades 4–8) mathematics teachers from nine universities in Turkey. Data were collected through Knowledge of History of Mathematics Test and Attitudes and Beliefs towards the Use of History of Mathematics in Mathematics Education Questionnaire. Results indicate that pre-service teachers have moderate knowledge of history of mathematics and positive attitudes and beliefs towards using history of mathematics. Their knowledge scores increase as the year level in teacher education program advanced. Males' knowledge scores are significantly higher than females' scores in the first 2 years. This situation reverses in the last 2 years, but it is not statistically significant. Pre-service teachers have more positive attitudes and availing beliefs towards using history of mathematics as they progress in their teacher education program. Females have greater attitudes and beliefs mean scores than males in each of the years. The results indicate that the teacher education program may have enhanced the pre-service teachers' knowledge of history of mathematics by related courses. However, the moderate knowledge scores indicate that there is a need for revision of these courses. The pre-service teachers' positive attitudes and beliefs towards using history of mathematics stress the importance of teacher education program in order to prepare them for implementing this alternative strategy in the future.

1 Introduction

Using history has been a strategy for teaching mathematics since the beginning of the twentieth century, when history of mathematics was introduced as a new discipline of

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study (Fried 2001; Furinghetti and Radford 2002). The initial aim for using history was to improve teachers' own mathematical education and also to provide a different pedagogical orientation for classroom mathematics (Furinghetti 2004). A considerable number of studies investigated the effects of specially designed practices of using history within pre-service teacher education (e.g., Clark 2012; Furinghetti 2007; Philippou and Christou 1998), particularly in recent decades. However, a focus on the contribution of history of mathematics to pre-service teachers' related knowledge and their attitudes and beliefs towards using history in mathematics courses is rare. It may not be possible for teachers to employ history in their teaching without the relevant knowledge and favorable dispositions towards using it in their classrooms. Teacher education is an appropriate place for training mathematics teachers for the potential uses of history and promoting the necessary positive attitudes and availing beliefs. In this sense, the present study explores Turkish pre-service middle school mathematics teachers' knowledge of history of mathematics and their attitudes and beliefs towards using history in mathematics education. It is our hope that the results of this study will provide teacher education programs with the initial information about the state of pre-service teachers' aforementioned knowledge, attitudes and beliefs.

2 History of Mathematics in Mathematics Education

Jankvist (2009) classified the reasons for using history of mathematics within two themes: *history as a tool* and *history as a goal*. *History as a tool* deals with using history from the aspect of the didactics of mathematics for three reasons: (a) assisting learners to gain motivation, (b) strengthening their mathematical cognition, and (c) being an inherent component of their conceptual development in mathematics (i.e., the genetic principle). These three reasons also seem to be valid grounds for employing history in middle school mathematics education, as a way to help students in a variety of ways when learning mathematics.

History can be used *as a tool* in middle grades mathematics in order to develop favorable dispositions and strong mathematical understanding. Students have certain dispositions such as attitudes towards mathematics and motivations to study mathematics in middle grades that might be negatively influenced as they progress to higher grades (Aiken 1976; Callahan 1971; Caraisco-Alloggiamento 2008; McLeod 1992; Middleton and Spanias 1999). Middle grades students (grades 6–8) also experience difficulties in learning mathematics such as transition from relatively concrete arithmetic thinking to abstract algebraic thinking that requires studying with symbols (Filloy and Rojano 1989). Hence, they are in need of multiple mathematical perspectives that apply to multiple learning objectives (Hiebert and Grouws 2007). Students' learning difficulties and obstacles in this process may be parallel with those encountered in the evolution of mathematics. For instance, it may not be easy for them to comprehend operations with negative numbers on the number line before having a symbolic mathematical sense (Heeffer 2011).

History can also be used *as a goal* by itself. This class of reasons for using history is about revealing issues in the development and evolution of mathematics as a discipline such as mathematics being a cultural heritage of mankind (Tzanakis and Arcavi 2000). Using history in this sense can also be helpful for middle grades students because research indicates that students hold misconceptions about what mathematics is and how it has developed throughout history (Furinghetti 2000).

The joint origin of the idea of using history of mathematics for teacher education and student learning (Furinghetti 2004) brings out certain commonalities. However, as educational needs have evolved, specificities have also emerged in the reasons for and the ways of using history for teachers and students. Previous studies propose that history can be used *as a cognitive tool* (Jankvist 2009) in order to enhance the learning of mathematics for both students and pre-service teachers. Herein, this use for pre-service teachers should be differentiated with the purpose of deep understanding of mathematical concepts for improving mathematical knowledge for teaching. For example, Clark (2012) asserted that pre-service mathematics teachers' understandings of solving quadratic equations could be enhanced through study of Al-Khwarizmi's approach of completing the square. In a similar study, Furinghetti (2007) enabled pre-service teachers to track the historical traces of school algebra through tapping into original texts of algebra masters. History can serve *as a motivational tool* (Jankvist 2009) so as to enhance affective dispositions such as attitudes towards mathematics and mathematical activity, both for students (Lit et al. 2001) and pre-service teachers (Philippou and Christou 1998). It is also possible to learn history of mathematics for its own sake *as a goal* to uncover the "developmental and evolutionary aspects of mathematics" (Jankvist 2009, p. 239) in school mathematics (e.g., Jankvist 2010) and teacher education programs.

Despite commonalities in the reasons for using history of mathematics with students and pre-service teachers, the practices in these groups exhibit variation. Pre-service teachers engage in history of mathematics through special courses or activities integrated into other courses in teacher education programs. On the other hand, students engage in history within the ordinary course of mathematics courses in which there are various objectives to be achieved. Pre-service teachers have more time to comprehensively examine primary or secondary historical material and think about pedagogical implications for future teaching. As for students, they need a compact integration of history into the pedagogical objectives in classroom mathematics. Otherwise, the use of history is avoided for relevant reasons which may decrease its value for their teachers (for reasons in detail, see Siu 2007).

Considering the reasons for using the history of mathematics along with the corresponding needs of middle grades students, it can be asserted that the use of history might be utilized as an alternative by pre-service teachers to meet students' needs while learning mathematics. In order to effectively use history, teachers should initially master the essential historical knowledge behind the mathematics concepts taught in middle school (Fried 2001). Yet, the mastery of history of mathematics may not be sufficient to encourage teachers to employ it in teaching mathematics if they do not hold positive attitudes and availing beliefs towards this teaching practice (Ajzen 2001; Thompson 1992). The two substantial factors of knowledge of history of mathematics and attitudes and beliefs towards using history in mathematics education should be restructured and enriched in pre-service teacher education programs as a relatively long formal education process before the mathematics teachers enter their profession (Freudenthal 1981; Heiede 1996). In order to meet this goal, pre-service teachers' related knowledge and attitudes and beliefs should be initially identified in order to inform teacher educators.

3 Pre-service Mathematics Teachers' Knowledge of History of Mathematics

School mathematics is based on mathematical notations, terms, concepts, and procedures coming from history (Bagni 2008) and hence cannot be seen as completely separate from its own origins (Siu 2000). In this respect, history of mathematics is able to provide "solid knowledge, dependable knowledge, theorems one can prove, definitions with consequences, proofs one can understand" (Freudenthal 1981, p. 31) for pre-service and in-service middle school mathematics teachers. Pre-service teachers who intend to use history in their future instruction must have knowledge of the history of mathematics regarding the mathematics topics they will teach (Fried 2001) prior to being trained about how to use history and having any formal experience in it, especially if they learnt mathematics in an ahistorical manner before their pre-service teacher education program (Heiede 1996). Being knowledgeable about history of mathematics will enable pre-service teachers to make critical judgments about historical information and to do the related further research before using it in mathematics teaching (Heiede 1996).

In the literature, studies from several countries address pre-service teachers' inadequate insights about the nature of mathematics and mathematical activity. Though this is not generally true across all countries, attention should be paid to the findings of the relevant studies. For example, Furinghetti (2000) gave voice to this conception, claiming that in Italy, "mathematics exists because it is taught" (p. 44). In the US, pre-service teachers reportedly believe that mathematics is composed of clusters of unrelated formulas and rules (Ball 1990) and it is based on computations that are used to solve problems (Foss and Kleinsasser 1996). Nyaumwe (2004) detected a procedural approach to mathematics by pre-service teachers in Zimbabwe. Similarly, Turkish pre-service middle school mathematics teachers tended to address mathematics as a collection of rules determined by an outside authority and knowing mathematics is equivalent to performing calculations to solve problems (Haser 2006). Pre-service teachers may also hold negative attitudes regarding mathematics (Nisbett 1991; Philippou 1994; Philippou and Christou 1998).

Introducing history of mathematics to pre-service teachers could be an effective tool for enhancing affect towards mathematics and its teaching (Philippou and Christou 1998). History has the potential to provide pre-service teachers with the answers behind the foundations and development of mathematics, which they can then address for students with an active curiosity in middle grades (Freudenthal 1981). Pre-service teachers should give importance to the fact that history has formed the basis of the subject that they will teach in the future (Heiede 1996). Such insights are likely to reflect on their students' parallel conceptions about mathematics (Bagni 2008).

Pre-service teachers' familiarity with the history of the mathematical concepts they will teach may help them develop pedagogical approaches in teaching mathematics including history (Furinghetti 1997). However, it is reported that many mathematics teachers are trained unsatisfactorily for the history of mathematics (Fauvel 1991), and they generally learn history through books and popular publications (Furinghetti 1997). Turkish pre-service middle school mathematics teachers are also trained in history of mathematics with poor attention to pedagogical links and by consulting secondary sources (Alpaslan and Haser 2012).

Empirical studies investigating pre- and in-service mathematics teachers' knowledge of history of mathematics are scarce in the mathematics education literature. Goodwin's (2007) study revealed that in the US there was a relationship between high school teachers' knowledge of history of mathematics and their views about the discipline of mathematics: teachers with stronger knowledge considered the development of mathematics as an ongoing process, which could be done by anyone and that it included contributions from several cultures. On the other hand, teachers with less knowledge considered mathematics as a collection of rigid rules and static knowledge formed by gifted individuals. However, these findings are somewhat limited in understanding pre-service mathematics teachers' knowledge of history of mathematics during their training in teacher education programs.

4 Pre-service Teachers' Attitudes and Beliefs Towards Using History of Mathematics in Mathematics Education

Changing pre-service teachers' attitudes and beliefs in teacher education programs in order to help them perform reform-oriented practices has been a major concern for teacher educators (Richardson 1996) on the grounds that attitudes and beliefs are keys to teacher adoption and implementation of a reformed curriculum (Handal and Herrington 2003). Although researchers investigated this complex relationship and how attitudes and beliefs have changed or remained over time (Philipp 2007), they do not agree on definitions of these constructs in the field of mathematics education (Furinghetti and Pehkonen 2002; Zan and Di Martino 2007).

Attitudes and beliefs can be considered as a part of the affective domain in mathematics education including beliefs, attitudes, emotional states, and values (DeBellis and Goldin 2006), where attitudes address a more affective construct of "orientations of dispositions toward certain sets of emotional feelings (positive or negative) in particular (mathematical) contexts" (p. 135) and beliefs refer to a highly cognitive construct of "attribution of some sort of external truth or validity to systems of propositions or other cognitive configurations" (p. 135). The common point of these constructs, which is also the driving force for investigating them in this study, is that teachers mirror their attitudes and beliefs towards an object, a situation, or another person in their related feelings and behaviors in the classroom such as interest, demand for learning, and utilization (Philipp 2007).

Studies focusing on the affective aspects of using history in mathematics education are generally employed with only one experimental group. For instance, Gönülateş (2008) integrated history of mathematics with a methods of teaching mathematics course for senior pre-service mathematics teachers. She found that the courses improved the pre-service teachers' attitudes towards the integration of history in mathematics teaching. However, the improvement was not statistically significant. She suggested that the pre-service teachers already had considerably positive attitudes towards the integration before the implementation. Gürsoy (2010) also designed a course to give pre-service mathematics teachers insight into how history of mathematics could be integrated within the Turkish middle school mathematics curriculum. In this study, senior-level pre-service teachers' attitudes and beliefs about using history positively changed as a result of the course, and the result was statistically significant. These few studies provide contradictory evidence for how pre-service teachers' attitudes and beliefs towards the use of history can be influenced with intervention on the issue. Inconsistent results emphasize the need for further studies regarding attitudes and beliefs towards the use of history.

Investigating the role of gender has been an internationally increasing trend in the learning and teaching of mathematics in recent years (Fennema 2002). Additionally, the interest in gender might be due to its complex nature, which often also yields inconsistent results (Gallagher and Kaufman 2005). Educators are concerned with the gender-imbalanced representations of history in schools that emphasize male figures predominantly in politics, science, and war throughout history (Osler 1994). The omission of women in these contexts becomes a barrier for equal access to the historical content to be taught (Adams 1983). Losing the gender balance may also create a gender gap in learning from the issues in history of mathematics. For example, Picker and Berry (2000) found that a majority of male students aged 12–13 chosen from the US, the UK, Finland, Sweden, and Romania drew a male as the typical image for a mathematician. In the same study, the drawing of the mathematician in respect of gender equity was more desirable but still inadequate for females. Gender phenomenon "as a dynamic of history" (Osler 1994, p. 231) is important because it may affect pre-service teachers' interest and motivation to learn history and how they may employ history in their teaching. In addition, their attitudes and beliefs towards using history in mathematics education may also be influenced by gender on the grounds that it may have a role in the instructional beliefs and preferences employed in mathematics classroom, such as how they perceive mathematics curriculum, teaching mathematics, and mathematics as a subject (Fennema 1990; Li 1999). Yet there is a need in investigating such presumptions that are based on somewhat limited literature on the issue (Li 1999).

Studies investigating pre-service teachers' knowledge of history of mathematics in the accessible literature are scarce, and most of the existing studies have been conducted with in-service teachers. Pre-service teachers' attitudes and beliefs towards using history in mathematics education have been investigated through specific cases and are mainly focused on the effects of training on the integration of history of mathematics (e.g., Gönülateş 2008; Gürsoy 2010). The instruments in these studies were mostly focused on the specific cases of using history of mathematics. There is a need for a more general picture of pre-service teachers' knowledge of history of mathematics and their attitudes and beliefs towards using history in order to understand the contributions of teacher education programs on these constructs for the future use of history in mathematics classrooms. Additionally, gender as a possible factor affecting these constructs should be investigated extensively to explore the role of this variable and to provide information for further research. Hence, the study described here aims to investigate pre-service teachers' knowledge of history of mathematics and their attitudes and beliefs towards using history in mathematics education with respect to year level in teacher education program and gender. Through this aim, the following four research questions are proposed:

1. What is pre-service middle school mathematics teachers' knowledge of the history of mathematics?
2. How positive and availing are pre-service middle school mathematics teachers' attitudes and beliefs towards using history in mathematics education?
3. Is there a significant mean difference in pre-service teachers' knowledge of the history of mathematics with respect to year level in the teacher education program and gender?
4. Is there a significant mean difference in pre-service teachers' attitudes and beliefs towards using history in mathematics education with respect to year level in the teacher education program and gender?

5 Methods

The aim of this study is to reveal pre-service teachers' existing knowledge of history of mathematics and their attitudes and beliefs towards using history in mathematics education with respect to year level in their teacher education program and gender in a generalizable sense. Therefore, a cross-sectional survey research design was employed in which surveys or tests are implemented in a fixed time (Cohen et al. 2000).

5.1 Participants and Context

In order to generalize the results to the target population of all Turkish pre-service mathematics teachers for middle grades, the participants were selected by means of cluster random sampling from all 47 four-year middle grades mathematics teacher education programs in Turkey. The clusters were randomly chosen as a 20 percent sample of these teacher education programs in each of the seven official geographical regions in Turkey. Volunteering pre-service teachers from all four of the year levels participated in the study in their regular class hours in the fall semester of 2010–2011 academic year. The participants consisted of a total of 1,593 pre-service mathematics teachers including 478 freshmen (30 %), 432 sophomores (27.1 %), 409 juniors (25.7 %), and 274 seniors (17.2 %). The gender distribution was 66.8 % (1,064) female and 33.2 % (529) male. The reason for choosing pre-service teachers from all year levels was because the Knowledge of History of Mathematics Test (to be explained later) was appropriate even for the freshmen, since the questions were limited to historical factual information in the middle school mathematics curriculum (Ministry of National Education [MoNE] 2009) and official textbooks (MoNE 2010) for middle grades students in Turkey. Furthermore, pre-service teachers may have developed the origins of attitudes and beliefs towards using history in mathematics education through observations in their school years (Lortie 1975). In this way, it is possible to seek for a change in pre-service teachers' knowledge, attitudes and beliefs of interest within teacher education programs.

Teacher education programs in Turkey are centralized to a certain extent through the governance of the Council of Higher Education (CHE). Therefore, middle school mathematics teacher education programs in Turkey can be considered to offer the same courses for pre-service teachers, to a great extent. The study was conducted in a four-year program training future mathematics teachers of grades 6–8. Grade 4 and 5 mathematics was also part of these programs. The program included courses on mathematics, statistics, physics, English, history, technology, mathematics education, educational sciences, and teaching practice (CHE 2007a). History of mathematical concepts was addressed in the program through elective courses titled *History of Science* and *History of Mathematics* in order to improve pre-service teachers' knowledge about the discipline (CHE 2007a). These two were offered as courses for *general knowledge* in the fall semesters of the third and fourth years, in that order. They were scheduled for two class hours (theoretical) per week during a 14-week semester. The contents of *History of Science* and *History of Mathematics* are presented in Table 1 as expressed in the documents guiding middle grades mathematics teacher education programs (CHE 2007b).

Middle grades mathematics teacher education programs consider the course content given in Table 1 as a base in designing and implementing *History of Science* and *History of Mathematics* courses due to their centralized nature. The structure of *History of Mathematics* courses in the program does not emphasize links between history and pedagogy of

Table 1 Content of courses titled History of Science and History of Mathematics

Course title	Course content
History of Science	Science in Ionian-Hellene and Turkish-Islamic (Arab, Khorasan, Seljuk, Andalusia, Ottoman) periods. Evolution of the branches of science such as astronomy, mathematics, physics, medicine, biology in these periods and dated from the western Renaissance. Scientific and technologic revolutions within the twentieth century
History of Mathematics	Studies on mathematics together with information about mathematicians (e.g., biographies) in the topics of numbers, sets, arithmetic, operations, algebra, equations, binomial theorem, analytic and modern geometry, area, solids, geometric tools, metric systems, vectors, graphs, logarithms, trigonometry, integrals, and computers

mathematics (CHE 2007b). Alpaslan and Haser (2012) analyzed such a typical *History of Mathematics* course in Turkey according to Jankvist's (2009) *whys* and *hows* of using history in mathematics education. They found that although the course could essentially develop pre-service teachers' ideas about using *history as a goal*, it failed in terms of *history as a tool* or at least narrowed it down to a motivational tool. As regards the *hows*, pre-service teachers formed conceptions of only *the illumination approaches* that limited the use of history to isolated historical information (e.g., names, dates, biographies of mathematicians). No emphasis was given to using history in middle school mathematics teaching. The instructor of the course guided the pre-service teachers to secondary sources due to the limited resources in the Turkish language, which made sources in other languages a problem. Pre-service teachers enrolled in this course claimed that history was more dominant than mathematics and pedagogy in the course.

Another course that is predicted to possibly influence pre-service teachers' enrollment in history of mathematics and its use in mathematics education is the *Methods of Teaching Mathematics* course. This course has four class hours per week and includes theoretical issues about *knowledge for teaching* together with their practices. Pre-service teachers are introduced to the goals of Turkish middle school mathematics education and examine and evaluate middle school mathematics curriculum and the related techniques, materials, and textbooks (CHE 2007b). Furthermore, it is expected that they understand "the historical development of mathematics and correspondingly its role and value in the development of human thought" (MoNE 2009, p. 9) through educational materials such as tangrams and clinometers (MoNE 2010).

5.2 Instruments

Two instruments, the Knowledge of History of Mathematics Test and the Attitudes and Beliefs Questionnaire towards Using History of Mathematics in Mathematics Education (see "Appendix 1 and 2") were employed for this study (Alpaslan et al. 2011a, b).

Mathematics teachers' training on history of mathematics is not similar across countries (Furinghetti 1997). Given that the history of mathematics course context in Turkey is somewhat narrow (Alpaslan and Haser 2012), Turkish pre-service mathematics teachers can be categorized into what Furinghetti (1997) calls those who "use manuals of history of mathematics, popularization books, readers, etc." rather than "original sources and research papers" (p. 55). Therefore, the content of the Knowledge of History of Mathematics Test was determined considering the content of the History of Mathematics courses in teacher education programs (CHE 2007b), the place of history in middle school mathematics education curriculum and textbooks (MoNE 2009, 2010), and the mathematics teacher competencies identified by MoNE (2008). It is based on historical facts representing the place of history of mathematics in these formal documents in order to understand to what extent pre-service teachers have the knowledge about the history of mathematics as addressed in the Turkish middle school mathematics education context. Validity of the knowledge test was outlined by the formal documents that pre-service teachers were responsible for and the table of specifications for the test, which was examined by three mathematics education researchers. The English versions of the items were reviewed for their correctness by a researcher working on the history of mathematics. The test consisted of a total of 11 multiple choice, true or false, and short answer items including drawings, photographs, and visuals of items of an historical nature. Two items had two sub-questions. Item 11 was based on the questions asked in Goodwin's (2007) survey instrument. The remainder of the questions was originally constructed by the researchers. The test items address ancient civilizations, historical teaching materials (such as tangrams), historical evolution of mathematics concepts, and the people who contributed to mathematics in rational numbers, addition and subtraction, polygons, and measurement within the middle school mathematics education curriculum (MoNE 2009) and textbooks (MoNE 2010) in Turkey. Participants were given 10 min to respond to the test items. The responses for the test were dichotomously coded as *true* (1) or *false* (0). Hence, pre-service teachers' scores on this test ranged between 0 and 13. As for the reliability, the internal consistency among the items of the test was determined by KR-20 coefficient as .56. This value is acceptable considering the limited number of items and the diagnostic characteristic of the test. In such tests, participants' scores might cluster close to a certain value which lowers the standard deviation and hence the reliability (Peşman and Eryılmaz 2010).

The Attitudes and Beliefs Questionnaire towards Using History of Mathematics in Mathematics Education used in the study aimed to investigate pre-service teachers' attitudes and beliefs towards using history in mathematics education. The attitudes and beliefs were investigated as a whole in this instrument (i.e., the items were not separated as 'attitudes items' and 'beliefs items') on the grounds that the definitions of attitude have overlaps with those of belief and vice versa, and that there is not a certain definition for both of these affective constructs in the literature (Leder and Forgasz 2002; McLeod 1992). Three researchers who studied affective variables in mathematics education provided insight on the format of the items and the adequacy of the items of the questionnaire. The first part of the questionnaire includes questions about demographics (e.g., gender, year level in teacher education program) and two descriptive questions asking about participants' familiarity with the history of mathematics and its usage in mathematics education. For the descriptive items, pre-service teachers were asked whether they had been instructed through history of mathematics during their entire school experience or not, and for the extent to which they followed history of mathematics-related publications (e.g., books, documentaries). The second (main) part of the questionnaire has 35 items on a five-point Likert type scale. These items are partially adapted from former questionnaires implemented in the related research studies of history in mathematics education (Gönülateş 2008; Marshall 2000; Sullivan 2000). The items in this part address possible contributions to uncover the nature and structure of mathematics accompanied by its interrelations with other branches of science; its usefulness, advantages and disadvantages; and the value of history of mathematics in pre-service education. The five response categories are *strongly agree* (5), *agree* (4), *undecided* (3), *disagree* (2), and *strongly disagree* (1) and thus total scores varied between 35 and 175. The participants were given 20 min to respond to the questionnaire. The internal consistency among the items was calculated by Cronbach's Alpha as .90 and indicated a high reliability (Cronbach 1951).

In the analysis of the data gathered by the questionnaire, positive attitudes refer to more enjoyable emotions and negative attitudes to less enjoyable ones (McLeod 1992). Zan and Di Martino (2007) stated that identifying beliefs as positive and negative might be problematic. Instead, an availing versus non-availing dichotomy for beliefs is alternatively employed in this study. According to Muis (2004), availing beliefs refer to beliefs that have a potential to facilitate the learning and teaching of mathematics, and hence to produce the desired learning outcomes. As for non-availing beliefs, they "have no influence on learning outcomes or negatively influence learning outcomes" (Muis 2004, p. 323). For instance, if a pre-service teacher believes that learning history of mathematics will enrich his or her professional repertoire, this belief can be labeled as *availing* because it is to the students' advantage.

According to Leder and Forgasz (2002), measuring existing attitudes and beliefs is a demanding issue since there is no consensus on the best way to draw inferences about these hidden constructs. Approaches to measure them have both disadvantages and advantages. Leder and Forgasz alert researchers about the use of subject-completed questionnaires because participants might be alerted to the constructs of interest and the possible effect of their responses. For example, participants may give their responses to impress society (or, the researchers) through the desired outcome (Edwards 1957). In order to minimize this possibility, pre-service teachers in this study were not required to give any personal information during the data collection. It was also announced that they could leave at any time they wished. Still, the findings are limited by the pre-service teachers' self-reported responses for the items in the questionnaire.

6 Results

Responses for the descriptive questions in the first part of the Attitudes and Beliefs Questionnaire towards Using History of Mathematics in Mathematics Education revealed that more than half of the pre-service teachers (59.8 %) were not introduced to history in mathematics courses during their school life before they enrolled in teacher education programs. While most pre-service teachers *sometimes* (65.2 %) followed publications related to history of mathematics such as books, magazines and documentaries, few said *often* (3.1 %) and some *never* (31.3 %). When the instruments were implemented, 226 seniors (14.2 %) were taking the *History of Mathematics* course; 611 (38.4 %) juniors and seniors successfully completed the *History of Science* course; and all of the 409 juniors (25.7 %) and 274 seniors (17.2 %) were taking or had taken *Methods of Teaching Mathematics* course.

6.1 Knowledge of History of Mathematics

Pre-service teachers' mean (M) and standard deviation (SD) scores of the Knowledge of History of Mathematics Test are presented with regard to year level in teacher education program and gender in Table 2.

According to Table 2, it can be asserted that pre-service teachers' scores in the knowledge test are nearly moderate (M = .44) and females' related knowledge (M = .43) is close to that of males (M = .46) in general. The mean knowledge scores increased with year level in teacher education programs (freshmen, M = .36; sophomores, M = .39; juniors, M = .50; and seniors, M = .56). This increase was also seen both for the females (freshmen, M = .34; sophomores, M = .38; juniors, M = .50; and seniors, M = .56) and

Table 2 Descriptive statistics regarding knowledge of history of mathematics

Year level in teacher education program	Knowledge of history of mathematics M (SD)			Sample size (female/male)
	Female	Male	Total	
Freshman	.34 (.12)	.39 (.14)	.36 (.13)	477 (328/149)
Sophomore	.38 (.15)	.42 (.14)	.39 (.15)	428 (300/128)
Junior	.50 (.16)	.49 (.16)	.50 (.16)	410 (260/150)
Senior	.56 (.15)	.56 (.14)	.56 (.15)	278 (176/102)
Total	.43 (.17)	.46 (.16)	.44 (.16)	1,593 (1,064/529)

males (freshmen, M = .39; sophomores, M = .42; juniors, M = .49; and seniors, M = .56).

Two-way Analysis of Variance was conducted to explore the potential influence of year level in the program and gender on pre-service teachers' knowledge of the history of mathematics. An interaction effect between year level in the program and gender on pre-service teachers' knowledge of history of mathematics mean scores is statistically significant [$F(3, 1585) = 4.02$, $p < .05$]. In other words, the influence of year level in the program on pre-service teachers' knowledge of history of mathematics varies significantly with respect to gender, which is shown in Fig. 1.

Since there was a significant interaction effect between year level in teacher education program and gender on pre-service teachers' knowledge of history of mathematics, main effects of these two variables on knowledge of history of mathematics were not analyzed. Instead, simple main effects of these two variables on knowledge of history of mathematics were examined by year level in teacher education program across gender and gender across the year level through multiple comparisons of the related knowledge mean scores. Results of the comparisons are given in Table 3.

Considering Table 3, it is asserted that female pre-service teachers' knowledge of history of mathematics increased significantly on a regular trend as they progressed in the teacher education program. More precisely, females' knowledge of history of mathematics in each of the years was significantly higher than those in the former year(s). The similar case was also valid for male pre-service teachers except for the pair between freshmen and sophomores, in which the increase is not statistically significant.

The multiple comparisons of knowledge of history of mathematics mean scores by gender across year level in teacher education program are given in Table 4. The results revealed that male pre-service teachers' knowledge of history of mathematics mean scores were significantly higher than those of females in the initial two years in teacher education program. In the

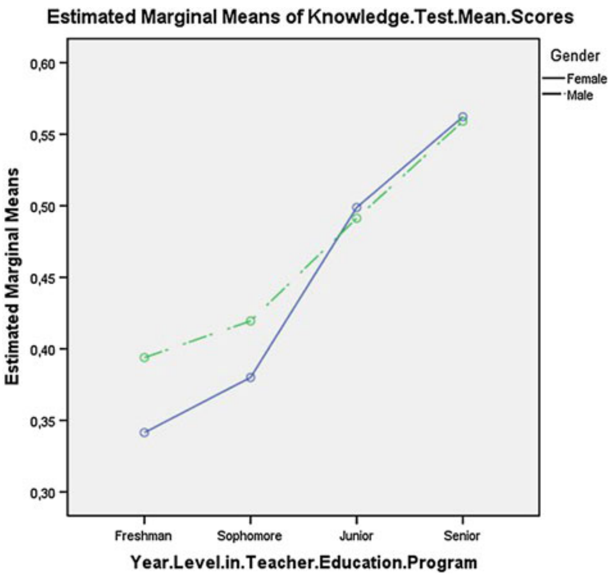

Fig. 1 The interaction between year level in teacher education program and gender on pre-service teachers Knowledge of History of Mathematics Test mean scores

Table 3 Multiple comparisons of knowledge of history of mathematics mean scores by year level in teacher education program across gender

Comparison		Mean difference (J − I)	Standard error	95 % confidence interval	
Gender	Year level in teacher education program (I vs. J)			Lower bound	Upper bound
Female	Freshman vs. sophomore	.039*	.011	−.069	−.008
	Freshman vs. junior	.157*	.012	−.189	−.126
	Freshman vs. senior	.220*	.013	−.255	−.184
	Sophomore vs. junior	.119*	.012	−.151	−.087
	Sophomore vs. senior	.181*	.014	−.217	−.145
	Junior vs. senior	.062*	.014	−.099	−.025
Male	Freshman vs. sophomore	.026	.017	−.071	.020
	Freshman vs. junior	.097*	.017	−.141	−.054
	Freshman vs. senior	.163*	.018	−.212	−.115
	Sophomore vs. junior	.072*	.017	−.117	−.026
	Sophomore vs. senior	.138*	.019	−.188	−.088
	Junior vs. senior	.066*	.018	−.017	−.115

* Indicates the mean difference is significant at the .05 level

Table 4 Multiple comparisons of knowledge of history of mathematics mean scores by gender across year level in teacher education program

Comparison		Mean difference (J − I)	Standard error	95 % confidence interval	
Year level in teacher education program	Gender (I vs. J)			Lower bound	Upper bound
Freshman	Female vs. male	.052*	.014	−.080	−.025
Sophomore	Female vs. male	.039*	.015	−.069	−.010
Junior	Female vs. male	−.008	.015	−.021	.036
Senior	Female vs. male	−.004	.018	−.031	.039

last two years, the females' knowledge of history of mathematics was higher than that of the males, but this time the difference is not statistically significant.

6.2 Attitudes and Beliefs Towards Using History of Mathematics in Mathematics Education

Mean (M) and standard deviation (SD) values for pre-service teachers' Attitudes and Beliefs Questionnaire towards Using History of Mathematics in Mathematics Education scores are given with regard to year level in teacher education program and gender in Table 5.

Pre-service teachers' attitudes and beliefs towards using history in mathematics education were considered as positive and availing (M = 3.67). Female pre-service teachers' related attitudes and beliefs (M = 3.71) are more positive and availing than those males have (M = 3.61). Attitudes and beliefs mean scores improve with year level in teacher education program (freshmen, M = 3.58; sophomores, M = 3.66; juniors, M = 3.74; and seniors, M = 3.76). The improvement was also noticed for females across the years (freshmen, M = 3.61; sophomores, M = 3.70; juniors, M = 3.77; and seniors, M = 3.80), but was valid for the males only up to the third year (freshmen, M = 3.51; sophomores, M = 3.56; juniors, M = 3.69; and seniors M = 3.69).

In order to investigate possible effects of year level in teacher education program and gender on pre-service teachers' attitudes and beliefs towards using history in mathematics education, two-way Analysis of Variance was conducted. The interaction effect between year level in the program and gender is not statistically significant [$F(3, 1585) = .24$, $p > .05$], which means that there was no significant difference in the effect of year level in the program on pre-service teachers' attitudes and beliefs towards using history mean scores for females and males (see Fig. 2).

Since the interaction effect is not significant, main effects of year level in the program and gender were investigated separately. The main effect of year level in the program on

Table 5 Descriptive statistics regarding attitudes and beliefs towards using history of mathematics in mathematics education

Year level in teacher education program	Attitudes and beliefs towards using history of mathematics in mathematics education M (SD)			Sample size (female/male)
	Female	Male	Total	
Freshman	3.61 (.52)	3.51 (.54)	3.58 (.53)	477 (328/149)
Sophomore	3.70 (.36)	3.56 (.49)	3.66 (.41)	428 (300/128)
Junior	3.77 (.46)	3.69 (.53)	3.74 (.49)	410 (260/150)
Senior	3.80 (.46)	3.69 (.58)	3.76 (.51)	278 (176/102)
Total	3.71 (.46)	3.61 (.54)	3.67 (.49)	1,593 (1,064/529)

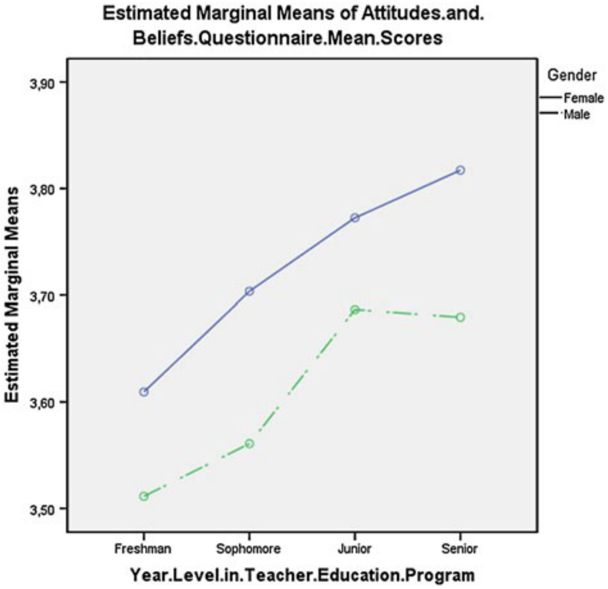

Fig. 2 The interaction between year level in teacher education program and gender on pre-service teachers' Attitudes and Beliefs Questionnaire towards Using History of Mathematics in Mathematics Education mean scores

the attitudes and beliefs towards using history mean scores is statistically significant [F(3, 1585) = 11.61, $p < .05$] and post hoc test and Tukey's Honestly Significant Difference Test results are given in Table 6.

According to Table 6, pre-service teachers' attitudes and beliefs towards using history in mathematics education regularly increased from the first to the last years in the program. Moreover, the attitudes and beliefs mean score differences between the pairs of freshmen and junior, freshmen and senior, and sophomore and senior are statistically significant. The differences have small effect size (r = .021) pointing to small practical significance (Cohen 1988).

The main effect of pre-service teachers' gender on the attitudes and beliefs towards using history is statistically significant [F(1, 1585) = 18.06, $p < .05$] in favor of female pre-service teachers, which means that their relevant mean scores (M = 3.71) were significantly higher than that of males (M = 3.61). The difference has small effect size (r = .01) indicating that the practical significance of the result is small (Cohen 1988).

7 Discussion and Implications

The survey results initially indicate that pre-service teachers have been inadequately introduced to history of mathematics during their entire school life. This might be due to the late introduction of history in Turkish mathematics education curricula (MoNE 2005a, b) and in the teacher education programs (CHE 2007a). The low level of interest in the history of mathematics might be another consequence of this late introduction in the last two years of the teacher education programs (CHE 2007a). The in-depth study by Alpaslan and Haser (2012) about a history of mathematics-related course in a middle grades mathematics teacher education program revealed that such courses have the potential to raise an early interest in pre-service teachers; however, the insufficiency and inaccessibility of history of mathematics-related publications in Turkish students' native language might result in a low level of interest.

Taking a course on the history of mathematics might have considerable effects on teachers' knowledge of history of mathematics. Pre-service teachers in this study did not take a history of mathematics-related course in their first two years in the teacher education programs, which might have resulted in lower scores on the Knowledge of History of Mathematics Test for those in the first two years. The study's results show that male pre-service teachers scored higher on the knowledge of history of mathematics than the females in the first two years, but the differences between the scores of these groups become slighter within the last two years of the programs. It might be speculated that this

Table 6 Multiple comparisons of attitudes and beliefs towards using history of mathematics in mathematics education scores among different year levels in teacher education program

Year level in teacher education program	Year level in teacher education program	Mean difference	Significance
Freshman	Sophomore	−.08	.053
	Junior	−.16	.000*
	Senior	−.18	.000*
Sophomore	Junior	−.08	.073
	Senior	−.10	.033*
Junior	Senior	−.02	.949

finding may be due to males' early personal interests in the field, resulting from male-dominated treatment of history in school years (Osler 1994) and its possible influence on preferences for future course selection (Linn and Hyde 1989). Careful integration of women's perspectives and thinking into general history and history of mathematics courses may have the potential to increase female students' interests (Pengelley 2012). What is more important in this finding is that teacher education programs may contribute to pre-service teachers' knowledge of history of mathematics through history-related courses and courses on methods of teaching mathematics. However, the nearly moderate level of this knowledge for all years in a teacher education program addresses the insufficiency of the history of mathematics-related courses, which is also consistent with the findings in a middle grades mathematics teacher education program in Turkey (Alpaslan and Haser 2012). A revision of the content and pedagogy of the history of mathematics-related courses in the teacher education programs might provide pre-service teachers with more knowledge of history of mathematics that they are likely to use (Schubring et al. 2000) in middle school mathematics classrooms.

The result regarding pre-service teachers' positive attitudes and availing beliefs towards using history is parallel with the studies in Turkey which indicate that senior pre-service mathematics teachers generally have positive feelings and desirable beliefs about this alternative teaching strategy (Gürsoy 2010; Gönülateş 2008). This study also shows that pre-service teachers begin the program with considerably positive attitudes and desirable beliefs. The favorable attitudes and beliefs may be a result of seeking for new orientations in mathematics education while preparing for teaching mathematics in the future. Respectively higher attitudes and beliefs in senior pre-service teachers may indicate that they favor using history for their future teaching after they begin taking more teaching-related courses and after they are introduced to alternative understandings of teaching mathematics. Hence, teacher educators should take advantage of these positive attitudes and availing beliefs to educate pre-service teachers on the history of mathematics and its use in middle school mathematics teaching through courses in teacher education programs. Courses related to history of mathematics might also provide pre-service teachers with certain ideas about the use of history. Senior pre-service teachers may notice certain problems in middle school mathematics classes in their classroom practice courses as well as in their pre-college mathematics education, and they may consider history of mathematics as an alternative solution for these problems and hence develop positive attitudes and desirable beliefs for the use of history in the mathematics classrooms.

In his review of mathematics teachers' beliefs and gender differences, Li (1999) asserted that female teachers tend to adopt more student-centered and collaborative learning environments in their teaching. The result that females hold significantly more positive attitudes and availing beliefs than males in each year level may be connected to Li's assertion and thus, female pre-service teachers may consider the use of history as a way of providing such student-centered instruction.

Research targeting knowledge, attitudes, and beliefs in the field of using history in mathematics education has been scarce. This study's findings document pre-service teachers' knowledge of history of mathematics and their attitudes and beliefs towards using history in mathematics education in the Turkish teacher education context through randomly reaching a considerable number of subjects. The knowledge instrument used in this study specifically reflected how history of mathematics is addressed in the Turkish context. Similar studies are needed in other contexts in order to understand how and to what extent history of mathematics is emphasized in teacher education and how this emphasis is reflected in pre-service teachers' dispositions. Case studies exploring possible effects of methods of teaching mathematics courses, history of mathematics-related courses, and classroom practice courses on pre-service teachers' knowledge of history of mathematics and their related attitudes and beliefs are also needed to understand the nature of this field. The field of history in mathematics education should also investigate how pre-service teachers transform their existing knowledge of history of mathematics and the related attitudes and beliefs into actual teaching and what factors influence this transformation. Research on gender differences in the field of using history in mathematics education is relatively new and gender differences are sensitive to cultural and situational contexts (Linn and Hyde 1989). Thus, the results regarding the gender variable should be carefully interpreted and clarified through further research.

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Appendix 1: Attitudes and Beliefs towards the Use of History of Mathematics in Mathematics Education (ABHME) Questionnaire

Demographic Information

1. Gender: Female (1) Male (2)

2. Year of Enrollment in the Pre-service Program: (1) (2) (3) (4)

3. Type of High School Graduated:

(1) Anatolian Teacher Education High School

(2) Anatolian High School

(3) Foreign Language Weighted High School

(4) General High School

(5) Other

4. Have you ever been instructed to history of mathematics during your school life?

No (1) Yes (2)

5. How often do you follow the publications (e.g., magazines, books, documentaries) about history of mathematics?

Never (1) Sometimes (2) Often (3) Always (4)

ABHME Questionnaire

The questionnaire is formed of 35 items. Please mark your choice for each item that presents your attitudes and beliefs towards the use of history of mathematics in mathematics education.

	Totally Disagree	**Disagree**	**Uncertain**	**Agree**	**Totally Agree**
1. It is difficult to integrate history of mathematics in mathematics education.					
2. Having knowledge about history of mathematics gives an idea about why humans felt the need for mathematics.					
3. The use of history of mathematics in mathematics education makes positive contribution to the learning of mathematics by providing a different standpoint and mode of presentation.					
4. Using history of mathematics in mathematics education causes students to lose their enthusiasm for learning mathematics.					
5. Noticing that great mathematicians also made mistakes when dealing with mathematics enhances students' motivation for the learning of mathematics.					
6. Learning history of mathematics enriches pre-service teachers' professional repertoire.					
7. Pre-service teachers must be given courses about how to use history of mathematics in mathematics education.					
8. History of mathematics enables one to link the mathematical concepts and see the close relationships between them.					
9. History of mathematics helps students comprehend mathematics in depth via introducing alternative approaches and various examples.					
10. History of mathematics makes students notice that mathematics is a universal product of various cultures.					

	Totally Disagree	Disagree	Uncertain	Agree	Totally Agree
11. I do not have an idea about how to use history-based didactical materials (e.g., pantograph, tangram).					
12. I do not know how to integrate history of mathematics into mathematics teaching processes.					
13. Mathematics education integrated with history of mathematics displays a more realistic and comprehensive picture about what mathematics is.					
14. Real life problems chosen from history of mathematics should be used in mathematics education.					
15. History of mathematics is a practical tool for the teaching of mathematics.					
16. History of mathematics should be integrated into mathematics education.					
17. I do not have enough information about the historical evolutions of the concepts which I will teach in the future.					
18. Written and visual didactical materials can be developed by using history of mathematics (e.g., worksheets, plays, puzzles, documentaries and cartoons).					
19. History of mathematics helps grasp the role and importance of mathematics in society.					
20. Integrating history of mathematics in mathematics lessons increase students' mathematics anxiety.					
21. Including history of mathematics in mathematics education hinders mathematics teaching.					
22. History of mathematics enables students to rediscover mathematics by tapping into their own talent and experiences.					
23. The integration of history of mathematics in elementary (K6-8) mathematics curriculum increases teachers' and students' course load.					

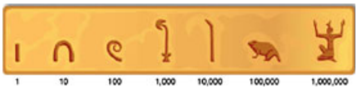
	Totally Disagree	Disagree	Uncertain	Agree	Totally Agree
24. The examination of the original sources of mathematics allows teachers and students to notice the advantages of modern mathematics.					
25. History-based learning activities will not attract students' interest in classroom setting.					
26. History of mathematics assists the change of classroom environment from a place in which knowledge is transferred to a platform in which research is made.					
27. History-based learning activities should be included in the elementary (K6-8) mathematics curriculum.					
28. Pre-service mathematics teachers must have knowledge of and ideas about the historical evolution of mathematical concepts.					
29. I do not know the place of the history of mathematics in the elementary mathematics curriculum.					
30. History of mathematics provides us to notice the contributions of mathematics to the other scientific disciplines (e.g., physics) and their interrelationship.					
31. The comparison between the original and modern versions of the way mathematics concepts are handled helps students understand mathematics.					
32. I do not plan to use the learning activities based on history of mathematics.					
33. Using history-based didactical materials in mathematics lessons causes waste of time.					
34. Knowing the historical development of the mathematical topic being studied enables students to learn that topic better.					
35. It is not important to use history of mathematics in mathematics lessons.					

Appendix 2: Knowledge of History of Mathematics (KHM) Test

This test is formed of 11 multiple choice, true-false and short answer questions. The estimated time for finishing the test is nearly **10 minutes**.

The items from 1 to 4 are multiple choice questions. You are expected to choose your response for each item.

1. - They have one of the oldest number systems.
- They developed a number system up to millions approximately 5000 years ago.
- Numerals in their mathematics are formed by juxtaposing some certain symbols.
- 7 different symbols constituting their numeration system is given below:

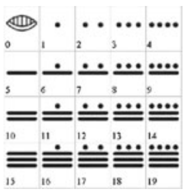


Which antique civilization has the above mentioned characteristics?

A) Mesopotamian Civilization
B) Roman Civilization
C) Egyptian Civilization
D) Babylon Civilization

Right Answer: **C**

2.



As it can be seen in the figure on the left side, which ancient civilization used the numeral system which was just composed of , and symbols?

A) Egyptians
B) Mayans
C) Babylonians
D) Mesopotamians

Right Answer: **B**

3.

Which of the following statements are <u>wrong</u> about π?
i. It has been used for approximately 4000 years in the computations about circles.
ii. Its value was calculated differently in various civilizations during history.
iii. It was found that its decimal component had a certain pattern.

A) Only iii **B)** i & ii **C)** i & iii **D)** All of them

Right Answer: **A**

4.

i. He took the first step for translating mathematical terms into Turkish.
ii. He defined and exemplified the geometrical terms like line, circle, parallel and triangle in his book named *Geometry*, which is for sale at present by The Council of Turkish Language.
iii. He became famous around the world with his studies on Algebra.

Which of the above argument(s) about Atatürk is/are correct?

A) Only ii **B)** i & ii **C)** i & iii **D)** All of them

Right Answer: **B**

The 5th and 6th items are formed of true-false statements. Please mark ‘T’ if the statement is correct or ‘F’ if it is incorrect.

5. T F Human beings who lived in different regions of the world used only natural numbers for many years. In time, rational numbers were introduced when natural numbers and integers failed to satisfy tasks of measurement.

True: **T**

6. T F There were numbers from 1 to 59 of an ancient civilization given below. The symbols for stating the numbers were 𒁹 and 𒌋 This civilization was Babylonians.

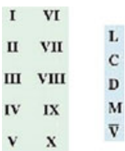
True: **T**

The items from 7 to 11 are short answer and matching questions. Please find the most appropriate answer for the blanks in those questions.

7. What is the name of the historical teaching material which is,
i. an old Chinese puzzle,
ii. formed of 7 regular geometric figures (5 triangles, 1 square, and 1 parallelogram),
iii. used for creating more than 7000 different figures like the ones below:

……………………………………………….

Answer: **Tangram**

8. 

Which ancient civilization has used the numeral symbols given on the left side?

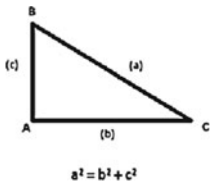
…………………………………………….

Answer: **Roman Civilization**

9. Please write the name of the person associated with the following contributions to mathematics under the figure he/she is well known for.

a. 

……………………………….

b. 

……………………………….

Answers: a. **Escher** b. **Pythagoras**

10. **i.** Hipparchus (170 – 125 B.C.), who was admitted as the first person studying on this field, has set its origins in the 2nd century A.D.
ii. Its name was composed of two words which meant 'triangle' and 'metric' in Greek.
iii. Egyptians and Babylonians utilized in measurement of fields, construction of buildings, astronomy, and sundial.

What is the name of the mathematical field whose historical background was given above?
....

Answer: **Trigonometry**

Please name the mathematician whose biography is given in parts a and b of the 11th item.

11.
a.

He was a French mathematician who lived in 17th century. He laid the foundations of modern probability theory and studied on conics and projective geometry as well. He invented the first digital calculator in order to help his father's tax collection. He spent much time on a triangle formed of numbers, which is named after him. This triangle was actually founded by Omar Khayyam, but it is thought that the Chinese had this knowledge before Khayyam.
Who is this mathematician? ....

Answer: **Pascal**

b.

He was an Italian mathematician who lived in the 12th and 13th centuries A.D. The name of his famous book is 'Liber abaci'. He is well known for a number series. Each element of this number series equals to the sum of the previous two elements (numbers). The series appears in many creatures' formation in the nature. In addition, he introduced Hindu-Arabian numbers to Europe.

Who is this mathematician? ....

Answer:**Fibonacci**

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