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Advanced Fractional Calculus Approach to RC Electrical Circuit Modeling: Analytical Solutions and Comparative Behavioral Analysis

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Abstract

This paper introduces a novel fractional-order model of the classical RC electrical circuit by incorporating the generalized Caputo fractional derivative of order $0 < \psi \leq 1$ and a fractional time constant τ_ψ. Using generalized Laplace and inverse Laplace transform techniques, explicit analytical solutions of the proposed model are derived. The study also conducts a comparative analysis between the new fractional RC circuit model and existing models based on classical integer-order derivatives, Caputo, Caputo–Fabrizio, and conformable fractional operators. The results demonstrate that the proposed model offers improved flexibility and accuracy in capturing the memory-dependent dynamics characteristic of real electrical systems. This work contributes to the growing field of fractional calculus applications in electrical engineering by providing a more comprehensive framework for modeling and analysis of RC circuits with non-integer order behavior.

Keywords Generalized Laplace transform · Wright function · Mittag-Leffler function · Fractional calculus · Electrical circuits · Mathematical models

1 Introduction

Fractional calculus constitutes a generalized framework of classical calculus wherein differentiation and integration are extended to arbitrary (non-integer) orders. This formalism provides a robust mathematical apparatus for accurately modeling anomalous and memory-dependent behaviors observed in complex systems. Due to its inherent ability to capture hereditary and non-local dynamics, fractional-order operators have found extensive utility in diverse domains such as viscoelasticity, electrochemistry, anomalous diffusion, control theory, and biomedical modeling. Unlike classical integer-order models, which often impose restrictive assumptions and fail to encapsulate the intricacies of real-world dynamical phenomena, fractional-order models offer superior fidelity in the representation of systems characterized by power-law memory kernels and spatial heterogeneity. This distinction renders fractional calculus particularly well-suited for modeling real dynamic systems where classical integer-order descriptions are inadequate or overly simplistic. The mathematical rigor and flexibility of fractional differential equations (FDEs) have stimulated significant theoretical and computational advances, resulting in a proliferation of analytical methods and numerical techniques tailored for fractional operators. The literature on fractional derivatives and integrals is extensive and continues to expand rapidly. For foundational treatments and recent developments, the reader is directed to Refs. [1–17].

An electrical circuit is defined as an interconnected network of electrical components that facilitates the flow of electric current. Depending on the nature of the current, electrical circuits are broadly classified into direct current (DC) and alternating current (AC) systems. In DC circuits, current flows unidirectionally with constant polarity, whereas AC circuits are characterized by a periodic reversal of the current direction, typically taking various forms such

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as sinusoidal, square, triangular, or sawtooth. DC circuits are further subdivided into series and parallel configurations. A series circuit provides a single conductive path for current flow, resulting in the same current passing through all components. In contrast, a parallel circuit offers multiple paths, allowing the current to divide among branches according to the impedance. The proper functioning of an electrical circuit depends on the presence of fundamental circuit elements, each performing a specific electrical function; including resistance, measured in ohms Ω and denoted by the symbol R, inductance, measured in henries H and denoted by L, capacitance, measured in farads F and denoted by C, voltage sources represented by V, and conductors. Each of these elements contributes to the dynamic behavior and functionality of the circuit, and their proper integration is essential for accurate modeling and performance analysis in both theoretical and practical applications.

An RC circuit is a fundamental type of linear electrical circuit consisting of a resistor (R) and a capacitor (C), which can be connected either in series or in parallel with a voltage source. Upon application of a DC voltage, the capacitor begins to charge through the resistor, with the voltage across the capacitor asymptotically approaching the supply voltage. In contrast, when the applied voltage is removed or reduced, the capacitor discharges in the reverse direction, releasing the stored electrical energy. Capacitors exhibit energy storage characteristics analogous to those of batteries in that they accumulate and release electrical energy via charge separation on their plates. Here, we consider the analogy to batteries solely in terms of the energy storage function. Although capacitors and batteries serve a similar purpose in storing energy, their storage mechanisms are fundamentally different; batteries store energy chemically, whereas capacitors store energy through an electrostatic field. However, unlike ideal energy storage systems, the charging and discharging processes of a capacitor are not instantaneous. These processes are governed by an exponential time-dependent behavior characterized by the circuit's time constant, denoted as $\tau_0 = RC$, which represents the time required for the capacitor to charge or discharge to approximately 63.2% of its final value. The dynamic behavior of RC circuits is of particular interest in both theoretical and applied contexts, especially in systems exhibiting memory and non-locality. In this regard, numerous studies have investigated the modeling of RC circuits using fractional-order differential equations, which provide a more accurate representation of the temporal evolution of such systems. For further insights and developments in this area, the reader is referred to Refs. [18–28], where RC circuit models involving fractional-order derivatives are extensively analyzed.

We present the following figure for the purpose of visualizing the RC electrical circuit.

The objective of this study is two-fold. First, motivated by the prior works cited above, we propose a novel fractional-order model of the RC electrical circuit and derive its analytical solution using the generalized Laplace transform technique. Second, we perform a comparative analysis between the proposed model and several existing models available in the literature, with a focus on evaluating their dynamic behaviors and modeling capabilities. It is important to emphasize that the proposed model extends the classical integer-order RC circuit to a fractional-order RC circuit by incorporating additional components of physical significance. Specifically, the model introduces a fractional time constant τ_ψ, which generalizes the classical time constant to account for memory effects, and employs the generalized Caputo fractional derivative to more accurately describe the temporal evolution of the system. These enhancements aim to provide a more realistic and flexible framework for modeling electrical circuits with noninteger-order dynamics.

The remainder of the paper is organized as follows. Section 2 presents the preliminary concepts and mathematical tools necessary for the subsequent analysis. In Section 3, we formulate the novel fractional RC electrical circuit model and derive its analytical solutions by employing the generalized Laplace and inverse Laplace transforms. Section 4 is devoted to a comparative study of the approximate dynamic behaviors of the proposed fractional model and several existing RC circuit models reported in the literature. Section 5 provides a discussion of the results, highlighting their significance and implications in the context of fractional-order circuit modeling. Finally, Section 6 concludes the paper with a summary of the main findings and suggestions for potential future research directions.

2 Mathematical background of fractional calculus

In this section, we present the fundamental mathematical preliminaries required for the subsequent analysis, including essential definitions, lemmas, a theorem, and a corollary that form the theoretical basis of the proposed model.

Definition 2.1 (See Ref. [1]) The gamma function is defined by

$$\Gamma(\omega) = \int_0^\infty t^{\omega-1} \exp(-t) dt,$$

where $\Re(\omega) > 0$.

Definition 2.2 (See Ref. [1]) The Mittag-Leffler function is given by

$$E_{\varpi}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\varpi k+1)},$$

where $\varpi, z\in\mathbb{C}$ and $\Re(\varpi)>0$.

Definition 2.3 (See Ref. [1]) The Wright function is introduced by

$$\phi(\varpi,\Delta;z)=\sum_{k=0}^{\infty}\frac{1}{\Gamma(\varpi k+\Delta)}\frac{z^k}{k!},$$

where $\varpi,\Delta,z\in\mathbb{C}$ and $\Re(\varpi)>-1$.

Definition 2.4 (See Ref. [29]) The generalized Caputo fractional derivative of order ψ is defined by

$$\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)}f\right)(t)=\int_0^t (t-q)^{-\psi}\phi\Big(\varpi,1-\psi;-\varpi\Delta\psi(1-\psi)(t-q)^{\varpi}\Big)f'(q)dq,$$

where $0<\psi\leq 1$ and $\varpi>-1$.

Definition 2.5 (See Ref. [30]) The generalized Laplace ($\mathfrak{L}_n$) and the inverse Laplace ($\mathfrak{L}_n^{-1}$) transforms are described by respectively

$$\mathfrak{L}_n\left\{f(t);s\right\}=s^{n-1}\int_0^{\infty}\exp(-s^n t)f(t)dt$$

and

$$\mathfrak{L}_n^{-1}\left\{\mathfrak{L}_n\left\{f(t);s\right\};t\right\}=f(t),$$

where $t\geq 0$, $n\in\mathbb{R}-\{0\}$ and $\Re(s^n)>0$.

Definition 2.6 (See Ref. [30]) The convolution of the functions $f(t)$ and $g(t)$ is given by

$$f(t)*g(t)=s^{n-1}\int_0^t f(t-q)g(q)dq. \tag{1}$$

Theorem 2.1 *(See Ref. [30]) The following formulas holds true*

$$\mathfrak{L}_n\left\{f(t)*g(t);s\right\}=\mathfrak{L}_n\left\{f(t);s\right\}\mathfrak{L}_n\left\{g(t);s\right\} \tag{2}$$

and

$$\mathfrak{L}_n^{-1}\left\{\mathfrak{L}_n\left\{f(t);s\right\}\mathfrak{L}_n\left\{g(t);s\right\};t\right\}=f(t)*g(t). \tag{3}$$

Lemma 2.1 *Let $\varpi>-1$, $0<\psi\leq 1$, $n\in\mathbb{R}-\{0\}$, $s^n>0$, $t\geq 0$. Then*

$$\mathfrak{L}_n\left\{t^{-\psi}\phi\Big(\varpi,1-\psi;\lambda t^{\varpi}\Big);s\right\}=s^{n\psi-1}\exp\left(\frac{\lambda}{s^{\varpi n}}\right). \tag{4}$$

Proof Using the $\mathfrak{L}_n$ transform and the Wright function, we have

$$\begin{aligned}
&\mathfrak{L}_n\left\{t^{-\psi}\phi\Big(\varpi,1-\psi;\lambda t^{\varpi}\Big);s\right\}\\
&=\sum_{k=0}^{\infty}\frac{1}{\Gamma(\varpi k+1-\psi)}\frac{\lambda^k}{k!}\mathfrak{L}_n\left\{t^{\varpi k-\psi}\right\}\\
&=\sum_{k=0}^{\infty}\frac{1}{\Gamma(\varpi k+1-\psi)}\frac{\lambda^k}{k!}\frac{\Gamma(\varpi k+1-\psi)}{s^{(\varpi k-\psi)n+1}}\\
&=s^{n\psi-1}\sum_{k=0}^{\infty}\left(\frac{\lambda}{s^{\varpi n}}\right)^k\frac{1}{k!}\\
&=s^{n\psi-1}\exp\left(\frac{\lambda}{s^{\varpi n}}\right).
\end{aligned}$$

□

Corollary 2.1 *If we apply the $\mathfrak{L}_n^{-1}$ transform to Eq. (4), we obtain*

$$\mathfrak{L}_n^{-1}\left\{s^{n\psi-1}\exp\left(\frac{\lambda}{s^{\varpi n}}\right);t\right\}=t^{-\psi}\phi\Big(\varpi,1-\psi;\lambda t^{\varpi}\Big). \tag{5}$$

Lemma 2.2 *Let $\varpi>-1$, $0<\psi\leq 1$, $n\in\mathbb{R}-\{0\}$, $s^n>0$, $t\geq 0$. Then*

$$\begin{aligned}
&\mathfrak{L}_n\left\{\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)}f\right)(t);s\right\}\\
&=\left(s^{n\psi}\mathfrak{L}_n\left\{f(t);s\right\}-s^{n\psi-1}f(0)\right)\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right).
\end{aligned} \tag{6}$$

Proof Using the $\mathfrak{L}_n$ transform, we have

$$\begin{aligned}
&\mathfrak{L}_n\left\{\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)}f\right)(t);s\right\}\\
&=\frac{\mathfrak{L}_n\left\{s^{n-1}\int_0^t(t-q)^{-\psi}\phi\Big(\varpi,1-\psi;-\varpi\Delta\psi(1-\psi)(t-q)^{\varpi}\Big)f'(q)dq;s\right\}}{s^{n-1}}.
\end{aligned}$$

Considering Eqs. (1) and (2), we find

$$\begin{aligned}
&\mathfrak{L}_n\left\{\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)}f\right)(t);s\right\}\\
&=\frac{\mathfrak{L}_n\left\{f'(t);s\right\}\mathfrak{L}_n\left\{t^{-\psi}\phi\Big(\varpi,1-\psi;-\varpi\Delta\psi(1-\psi)t^{\varpi}\Big);s\right\}}{s^{n-1}}.
\end{aligned}$$

Using Eq. (4), we obtain

$$\mathfrak{L}_n\left\{\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)}f\right)(t);s\right\}$$
$$=\left(s^{n\psi}\mathfrak{L}_n\{f(t);s\}-s^{n\psi-1}f(0)\right)\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right).$$

□

3 Modelling and analytical solutions

Ohm's Law is a formula used to calculate the relationship between voltage, current, and resistance in an electrical circuit and is expressed mathematically as follows:

$$V(t)=RI(t), \tag{7}$$

where t is time, I is current, V is the potential difference between two reference points, and R is resistance. Electrical current is the movement of particles carrying an electrical charge. This charge is usually carried by electrons that move through the cables in electrical circuits. Thus, the charge on the electrical current is mathematically expressed as follows:

$$I_C(t)=C\frac{dV_C(t)}{dt}, \tag{8}$$

which, using this equation in Ohm's Law, we get the following:

$$V(t)=RC\frac{dV_C(t)}{dt}. \tag{9}$$

Our main goal here is to rewrite the relation expressed by (9) in terms of a fractional derivative. For this purpose, consider the following fractional time derivative operator

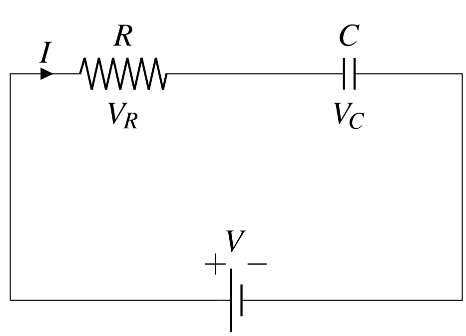


Fig. 1 RC electrical circuit

$$\frac{d^\psi}{dt^\psi},\quad 0<\psi\le 1, \tag{10}$$

where ψ is an arbitrary parameter representing the order of the derivative and becomes an ordinary derivative operator when $\psi=1$. The ordinary time derivative operator has dimensions of the inverse second s^{-1}. However, the expression (10) is of the following form in the $s^{-\psi}$ dimension:

$$\left[\frac{d^\psi}{dt^\psi}\right]=\frac{1}{s^\psi},\quad 0<\psi\le 1. \tag{11}$$

To be consistent in dimensionality, consider the following form

$$\left[\frac{1}{\sigma^{1-\psi}}\frac{d^\psi}{dt^\psi}\right]=\frac{1}{s},\quad 0<\psi\le 1, \tag{12}$$

where σ has the dimension $[\sigma]=s$ seconds and is related to the temporal components of the system [31]. For $\psi=1$ the expression (12) becomes the ordinary time derivative operator. Hence, the ordinary time derivative operator can be written as the fractional time derivative operator as follows:

$$\frac{d}{dt}\to\frac{1}{\sigma^{1-\psi}}\frac{d^\psi}{dt^\psi},\quad 0<\psi\le 1. \tag{13}$$

By applying Kirchhoff's Voltage Law to the RC electrical circuit given in Figure 1, the following is obtained

$$V_R(t)+V_C(t)=V(t), \tag{14}$$

where $V(t)$ is the voltage source, $V_R(t)$ is the voltage across the resistor, and $V_C(t)$ is the voltage across the capacitor. Here, considering that the current in the entire circuit is the same, using Eq. (8) and Ohm's Law $V_R=RI(t)$ in Eq. (14), the RC electrical circuit model is obtained as follows:

$$RC\frac{dV_C(t)}{dt}+V_C(t)=V(t). \tag{15}$$

We use the generalized Caputo fractional derivative instead of the fractional derivative operator used in the expression (13) and then substitute it in Eq. (15), we obtain the new fractional RC electrical circuit model as follows:

$$\frac{RC}{\sigma^{1-\psi}}\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)}V_C\right)(t)+V_C(t)=V(t),\quad 0<\psi\le 1, \tag{16}$$

where

$$\tau_\psi = \frac{RC}{\sigma^{1-\psi}} = \frac{\tau_0}{\sigma^{1-\psi}}, \tag{17}$$

which τ_ψ is the fractional time constant and τ_0 is the time constant in the classical case. Also, $\tau_\psi = \tau_0$ for $\psi = 1$. Hence, we substitute Eq. (17) into Eq. (16) and make the necessary adjustments, then the new fractional RC electrical circuit model has the following form

$$\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)} V_C\right)(t) + \frac{1}{\tau_\psi} V_C(t) = \frac{1}{\tau_\psi} V(t), \quad 0 < \psi \le 1. \tag{18}$$

Applying the $\mathfrak{L}_n$ transform to Eq. (18), we have

$$\mathfrak{L}_n\left\{\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)} V_C\right)(t); s\right\} + \frac{1}{\tau_\psi}\mathfrak{L}_n\{V_C(t); s\} = \frac{1}{\tau_\psi}\mathfrak{L}_n\{V(t); s\}.$$

Then

$$\mathfrak{L}_n\{V_C(t); s\} = \frac{1}{\tau_\psi}\frac{\mathfrak{L}_n\{V(t); s\}}{s^{\psi n}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right) + \frac{1}{s_\psi}} + V_C(0)\frac{s^{\psi n-1}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right)}{s^{\psi n}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right) + \frac{1}{s_\psi}},$$

where

$$\mathfrak{L}_n\{P(t); s\} := \frac{1}{s^{\psi n}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right) + \frac{1}{s_\psi}}$$

and

$$\mathfrak{L}_n\{R(t); s\} := \frac{s^{\psi n-1}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right)}{s^{\psi n}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right) + \frac{1}{s_\psi}},$$

so that

$$\mathfrak{L}_n\{V_C(t); s\} = \frac{1}{\tau_\psi}\mathfrak{L}_n\{V(t); s\}\mathfrak{L}_n\{P(t); s\} + V_C(0)\mathfrak{L}_n\{R(t); s\}.$$

By applying $\mathfrak{L}_n^{-1}$ transform, we get

$$V_C(t) = \frac{s^{n-1}}{\tau_\psi}\int_0^t V(t-q)P(q)dq + V_C(0)R(t),$$

where

$$P(t) = t^{\psi-\frac{1}{n}}\sum_{k=0}^{\infty}\left(-\frac{t^\psi}{\tau_\psi}\right)^k \times \phi\left(\varpi, 1+\psi k+\psi-\frac{1}{n}; \varpi\Delta\psi(1-\psi)(k+1)t^\varpi\right)$$

and

$$R(t) = \sum_{k=0}^{\infty}\left(-\frac{t^\psi}{\tau_\psi}\right)^k \phi\left(\varpi, 1+\psi k; \varpi\Delta\psi(1-\psi)kt^\varpi\right).$$

Thus,

$$V_C(t) = \frac{s^{n-1}}{\tau_\psi}\int_0^t V(t-q)q^{\psi-\frac{1}{n}}\sum_{k=0}^{\infty}\left(-\frac{q^\psi}{\tau_\psi}\right)^k \times \phi\left(\varpi, 1+\psi k+\psi-\frac{1}{n}; \varpi\Delta\psi(1-\psi)(k+1)q^\varpi\right)dq + V_C(0)\sum_{k=0}^{\infty}\left(-\frac{t^\psi}{\tau_\psi}\right)^k \phi\left(\varpi, 1+\psi k; \varpi\Delta\psi(1-\psi)kt^\varpi\right).$$

Now we choose the voltage source as various elementary functions and obtain the analytical solutions via the $\mathfrak{L}_n$ and $\mathfrak{L}_n^{-1}$ transforms.

Case 1. If we take the voltage source $V(t) = V_0$, the fractional RC electrical circuit model has the following form

$$\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)} V_C\right)(t) + \frac{1}{\tau_\psi} V_C(t) = \frac{V_0}{\tau_\psi}, \quad 0 < \psi \le 1, \tag{19}$$

where V_0 is constant. By applying the $\mathfrak{L}_n$ transform to Eq. (19), we have

$$\mathfrak{L}_n\left\{\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)} V_C\right)(t); s\right\} + \frac{1}{\tau_\psi}\mathfrak{L}_n\{V_C(t); s\} = \frac{V_0}{\tau_\psi}\mathfrak{L}_n\{1; s\}.$$

Then,

$$\mathfrak{L}_n\{V_C(t); s\} = (V_C(0) - V_0)\frac{s^{\psi n-1}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right)}{s^{\psi n}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right) + \frac{1}{\tau_\psi}} + \frac{V_0}{s}.$$

By applying the $\mathfrak{L}_n^{-1}$ transform, we get

$$V_C(t) = \left(V_C(0) - V_0\right)\sum_{k=0}^{\infty}\left(-\frac{t^\psi}{\tau_\psi}\right)^k \times \phi\left(\varpi, 1+\psi k; \varpi\Delta\psi(1-\psi)kt^\varpi\right) + V_0. \tag{20}$$

Case 2. If we take the voltage source $V(t) = 0$, the fractional RC electrical circuit model has the following form

$$\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)} V_C\right)(t) + \frac{1}{\tau_\psi} V_C(t) = 0. \tag{21}$$

Applying the $\mathfrak{L}_n$ transform to Eq. (21), we have

$$\mathfrak{L}_n\left\{\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)} V_C\right)(t); s\right\} + \frac{1}{\tau_\psi}\mathfrak{L}_n\left\{V_C(t); s\right\} = 0.$$

Then

$$\mathfrak{L}_n\left\{V_C(t); s\right\} = V_C(0)\frac{s^{\psi n-1}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right)}{s^{\psi n}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right) + \frac{1}{\tau_\psi}}.$$

Applying the $\mathfrak{L}_n^{-1}$ transform, we have

$$V_C(t) = V_C(0)\sum_{k=0}^{\infty}\left(-\frac{t^\psi}{\tau_\psi}\right)^k \phi\left(\varpi, 1+\psi k; \varpi\Delta\psi(1-\psi)kt^{\varpi}\right).$$

Case 3. If we take the voltage source $V(t) = H(t)$, the fractional RC electrical circuit model has the following form

$$\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)} V_C\right)(t) + \frac{1}{\tau_\psi} V_C(t) = \frac{1}{\tau_\psi}H(t), \quad 0 < \psi \le 1, \tag{22}$$

where *H*(*t*) is known as the Heaviside function in [32] and is defined as

$$H(t) = \begin{cases} 1, & \text{for } t \ge 0, \\ 0, & \text{for } t < 0. \end{cases}$$

Applying the $\mathfrak{L}_n$ transform to Eq. (22), we have

$$\mathfrak{L}_n\left\{\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)} V_C\right)(t); s\right\} + \frac{1}{\tau_\psi}\mathfrak{L}_n\left\{V_C(t); s\right\} = \frac{1}{\tau_\psi}\mathfrak{L}_n\left\{H(t); s\right\}.$$

Then

$$\mathfrak{L}_n\left\{V_C(t); s\right\} = \left(V_C(0) - 1\right)\frac{s^{\psi n-1}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right)}{s^{\psi n}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right) + \frac{1}{\tau_\psi}} + \frac{1}{s}.$$

Applying the $\mathfrak{L}_n^{-1}$ transform, we get

$$V_C(t) = \left(V_C(0) - 1\right)\sum_{k=0}^{\infty}\left(-\frac{t^\psi}{\tau_\psi}\right)^k \phi\left(\varpi, 1+\psi k; \varpi\Delta\psi(1-\psi)kt^{\varpi}\right) + 1.$$

Case 4. If we take the voltage source $V(t) = V_0\sin(\omega t)$, the fractional RC electrical circuit model has the following form

$$\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)} V_C\right)(t) + \frac{1}{\tau_\psi} V_C(t) = \frac{V_0}{\tau_\psi}\sin(\omega t), \quad 0 < \psi \le 1, \tag{23}$$

where V_0 is constant. Applying the $\mathfrak{L}_n$ transform to Eq. (23), we have

$$\mathfrak{L}_n\left\{\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)} V_C\right)(t); s\right\} + \frac{1}{\tau_\psi}\mathfrak{L}_n\left\{V_C(t); s\right\} = \frac{V_0}{\tau_\psi}\mathfrak{L}_n\left\{\sin(\omega t); s\right\}.$$

Then

$$\mathfrak{L}_n\left\{V_C(t); s\right\} = \frac{V_0}{\tau_\psi}\frac{\frac{\omega s^{n-1}}{s^{2n}+\omega^2}}{s^{\psi n}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right) + \frac{1}{\tau_\psi}} + V_C(0)\frac{s^{\psi n-1}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right)}{s^{\psi n}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right) + \frac{1}{\tau_\psi}}.$$

Applying the $\mathfrak{L}_n^{-1}$ transform, we get

$$V_C(t) = \frac{V_0}{\tau_\psi}s^{n-1}\int_0^t \sin\left(\omega(t-q)\right)q^{\psi-\frac{1}{n}}\sum_{k=0}^{\infty}\left(-\frac{q^\psi}{\tau_\psi}\right)^k \times \phi\left(\varpi, 1+\psi k+\psi-\frac{1}{n}; \varpi\Delta\psi(1-\psi)(k+1)q^{\varpi}\right)dq + V_C(0)\sum_{k=0}^{\infty}\left(-\frac{t^\psi}{\tau_\psi}\right)^k \phi\left(\varpi, 1+\psi k; \varpi\Delta\psi(1-\psi)kt^{\varpi}\right).$$

Case 5. If we take the voltage source $V(t) = V_0\cos(\omega t)$, the fractional RC electrical circuit model has the following form

$$\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)} V_C\right)(t) + \frac{1}{\tau_\psi} V_C(t) = \frac{V_0}{\tau_\psi}\cos(\omega t), \quad 0 < \psi \le 1, \tag{24}$$

where V_0 is constant. Applying the $\mathfrak{L}_n$ transform to Eq. (24), we have

$$\mathfrak{L}_n\left\{\left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)} V_C\right)(t); s\right\} + \frac{1}{\tau_\psi}\mathfrak{L}_n\left\{V_C(t); s\right\} = \frac{V_0}{\tau_\psi}\mathfrak{L}_n\left\{\cos(\omega t); s\right\}.$$

Then

$$\mathfrak{L}_n\left\{V_C(t);s\right\} = \frac{V_0}{\tau_\psi}\frac{\frac{s^{2n-1}}{s^{2n}+\omega^2}}{s^{\psi n}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right)+\frac{1}{\tau_\psi}} + V_C(0)\frac{s^{\psi n-1}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right)}{s^{\psi n}\exp\left(-\frac{\varpi\Delta\psi(1-\psi)}{s^{\varpi n}}\right)+\frac{1}{\tau_\psi}}.$$

Applying the $\mathfrak{L}_n^{-1}$ transform, we get

$$V_C(t) = \frac{V_0}{\tau_\psi}s^{n-1}\int_0^t \cos\left(\omega(t-q)\right)q^{\psi-\frac{1}{n}}\sum_{k=0}^{\infty}\left(-\frac{q^\psi}{\tau_\psi}\right)^k \times \phi\left(\varpi, 1+\psi k+\psi-\frac{1}{n};\varpi\Delta\psi(1-\psi)(k+1)q^\varpi\right)dq + V_C(0)\sum_{k=0}^{\infty}\left(-\frac{t^\psi}{\tau_\psi}\right)^k\phi\left(\varpi, 1+\psi k;\varpi\Delta\psi(1-\psi)kt^\varpi\right).$$

4 Comparative Behavioral Analysis

In this section, we first present the definitions of the Caputo and Caputo–Fabrizio fractional derivatives, along with the conformable fractional operator, which serve as the mathematical foundations for alternative fractional-order models. Subsequently, we introduce both the classical (integer-order) RC electrical circuit model and the corresponding fractional RC circuit models formulated using the aforementioned derivatives and operator. Finally, we provide a comparative analysis of the numerical simulations for all these models, including the new fractional RC circuit model proposed in this study, using selected parameter values to illustrate their dynamic behaviors.

The Caputo fractional derivative in [1] and the Caputo-Fabrizio fractional derivative in [33] and the conformable operator in [34] are respectively as follows:

$$\text{Caputo Fractional Derivative}: {}^{C}D_{0+}^{\psi}f(t) = \frac{1}{\Gamma(1-\psi)}\int_0^t (t-q)^{-\psi}f'(q)dq,$$
$$\text{Caputo-Fabrizio Fractional Derivative}: {}^{CF}D_{0+}^{\psi}f(t) = \frac{M(\psi)}{1-\psi}\int_0^t \exp\left(-\frac{\psi(t-q)}{1-\psi}\right)f'(q)dq,$$
$$\text{Conformable Operator}: T_\psi f(t) = \lim_{\epsilon\to 0}\frac{f\left(t+\epsilon t^{1-\psi}\right)-f(t)}{\epsilon},$$

where $0<\psi\leq 1$ and $M(\psi)$ is the normalization function and $M(0)=M(1)=1$.

The ordinary RC electrical circuit model, the RC electrical circuit models defined by these fractional derivatives and the operator (see, e.g., [21]), and the fractional RC electrical circuit model introduced in this paper are, respectively, as follows:

$$\text{Ordinary Model}: \frac{dV_C(t)}{dt}+\frac{1}{\tau}V_C(t) = \frac{1}{\tau}V(t),$$
$$\text{Caputo Model}: {}^{C}D_{0+}^{\psi}V_C(t)+\frac{1}{\tau^\psi}V_C(t) = \frac{1}{\tau^\psi}V(t),$$
$$\text{Caputo-Fabrizio Model}: {}^{CF}D_{0+}^{\psi}V_C(t)+\frac{1}{\tau^\psi}V_C(t) = \frac{1}{\tau^\psi}V(t),$$
$$\text{Conformable Model}: T_\psi V_C(t)+\frac{1}{\tau^\psi}V_C(t) = \frac{1}{\tau^\psi}V(t),$$
$$\text{Generalized Caputo Model}: \left(\mathfrak{D}_{0+}^{\varpi,\Delta;(\psi)}V_C\right)(t)+\frac{1}{\tau_\psi}V_C(t) = \frac{1}{\tau_\psi}V(t),$$

where $0<\psi\leq 1$, and $\tau = RC$ denotes the classical time constant. Furthermore, the generalized time constant τ_ψ is defined as $\tau_\psi = \frac{RC}{\sigma^{1-\psi}}$, where σ is a parameter with the dimension of time ($[\sigma] = \mathrm{s}$), representing intrinsic temporal characteristics of the system.

The interpretations of the models considered along with their respective resistance formulations can be succinctly summarized as follows.

In the ordinary model, the time constant τ is defined as the product of resistance *R* and capacitance *C*. Consequently, if the capacitance of a given circuit is known and the time constant is experimentally determined, the resistance can be straightforwardly evaluated. Furthermore, the time constant characterizes the system's response speed to variations in the applied voltage.

In contrast, fractional derivative models describe systems in which historical behavior influences the present response, thus introducing memory effects. In these models, the effective time constant deviates from that of the classical counterpart and explicitly depends on the fractional derivative order parameter ψ. Consequently, resistance in such models is not only a function of the classical time constant and capacitance but is modified by fractional order, reflecting the hereditary properties of the system.

The fractional order ψ serves as a quantifier of the extent of memory retained by the system. As ψ approaches unity, corresponding to the classical first-order derivative, the resistance converges to the classical value. In contrast, a decrease in ψ corresponds to enhanced memory effects, which manifest themselves as pronounced deviations in resistance.

Moreover, the generalized Caputo model introduces an additional parameter σ associated with the system's time scale. This parameter captures time-dependent variations within the system, allowing the resistance to vary not only as a function of the classical resistance and capacitance, but also with respect to σ. This inclusion enables a more accurate representation of phenomena such as time-varying resistance and aging effects commonly observed in practical electrical circuits.

In summary, while the classical model offers a simplified and fixed characterization of resistance, fractional derivative models extend this framework by incorporating memory and temporal variability. This results in a more flexible and realistic description of resistance, making such models particularly advantageous for capturing the complex behavior of real-world electrical systems.

In these models, taking the initial condition $V_C(0) = 0$ and $V(t) = V_0$, the solution functions are, respectively, as follows (see [21]):

$$V_C(t) = V_0 \left(1 - \exp\left(-\frac{t}{\tau}\right)\right), \tag{25}$$

$$V_C(t) = V_0 \left(1 - E_\psi\left(-\left(\frac{t}{\tau}\right)^\psi\right)\right), \tag{26}$$

$$V_C(t) = V_0 \left(1 - \frac{\tau^\psi}{1-\psi+\tau^\psi} \exp\left(-\frac{\psi t}{1-\psi+\tau^\psi}\right)\right), \tag{27}$$

$$V_C(t) = V_0 \left(1 - \exp\left(-\frac{\left(\frac{t}{\tau}\right)^\psi}{\psi}\right)\right), \tag{28}$$

where V_0 is constant. Also by rewriting Eq. (20) with the initial condition $V_C(0) = 0$, we have

$$V_C(t) = V_0\left(1 - \sum_{k=0}^{\infty}\left(-\frac{t^\psi}{\tau_\psi}\right)^k \phi\left(\varpi, 1+\psi k; \varpi\Delta\psi(1-\psi)kt^\varpi\right)\right). \tag{29}$$

We also compare the simulations of solution functions (25), (26), (27), (28) and (29) in Figure (2).

5 Results and Discussion

In this study, a novel fractional-order model was developed for RC electrical circuits based on the generalized Caputo fractional derivative with order $0 < \psi \leq 1$. Unlike traditional integer-order models, the proposed framework inherently captures memory and non-local effects that are often observed in real-world electrical systems. The inclusion of the generalized Caputo operator allows for a richer mathematical representation, accommodating fractional-order dynamics that more accurately reflect the physical behavior of certain circuit components.

Analytical solutions for the proposed model and its specific cases were derived using generalized Laplace and inverse Laplace transform operators, denoted by $\mathfrak{L}_n$ and $\mathfrak{L}_n^{-1}$, respectively. These solutions demonstrate the analytical tractability of the model and establish a foundation for further analytical and numerical investigations. The analytical results were validated and supported by extensive numerical simulations.

A comparative performance analysis was performed between the generalized Caputo-based RC model and several existing formulations in the literature, including the classical integer-order RC model, the conformable derivative-based model and fractional models based on Caputo and Caputo–Fabrizio derivatives. As illustrated in Figure 2, the proposed generalized Caputo model exhibits approximation characteristics superior to the behavior of the classical RC circuit, particularly in terms of the accuracy of the dynamics of the transient and steady state.

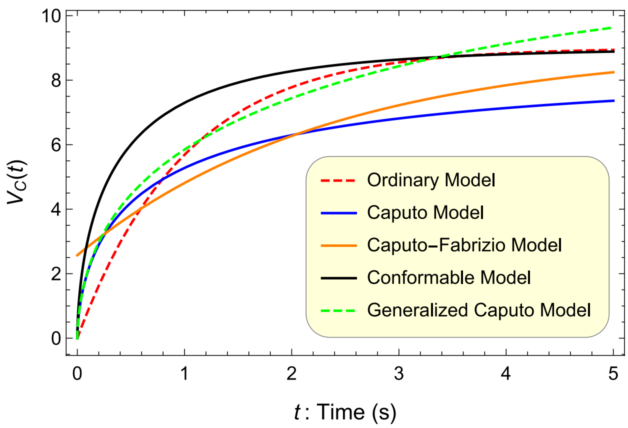
This improved accuracy is attributed to the ability of the generalized Caputo derivative to account for both short-term and long-term memory effects, which are either oversimplified or entirely neglected in other formulations. The simulation results highlight that the generalized Caputo model not only retains the desirable features of the classical model, but also extends its descriptive power, offering a more flexible and comprehensive modeling strategy for systems governed by fractional-order behavior.

Moreover, the proposed framework provides a robust and generalizable basis for modeling other types of electrical circuits. The findings underscore the versatility of the generalized fractional derivative approach, making it a compelling alternative to conventional and existing fractional-order modeling techniques. This approach holds particular promise in applications where capturing complex dynamical characteristics, such as hereditary behavior, energy dissipation, and anomalous diffusion, is essential.

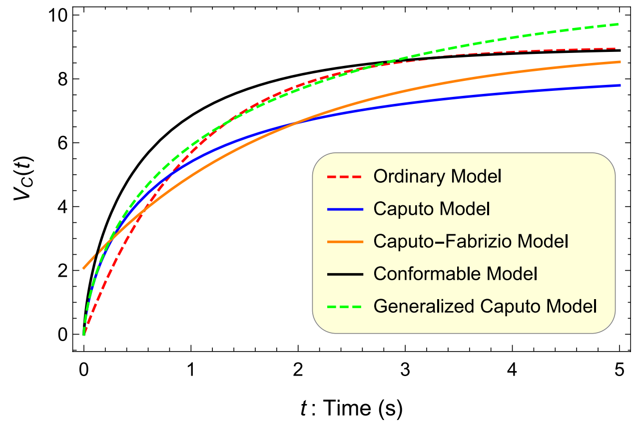
6 Conclusion

This work introduced a new fractional-order modeling framework for RC electrical circuits, grounded in the generalized Caputo fractional derivative of order $0 < \psi \leq 1$. The proposed model enhances the classical integer-order formulation by incorporating non-local and memory-dependent dynamics, which are essential for accurately describing the behavior of real-world electrical systems that exhibit fractional-order characteristics.

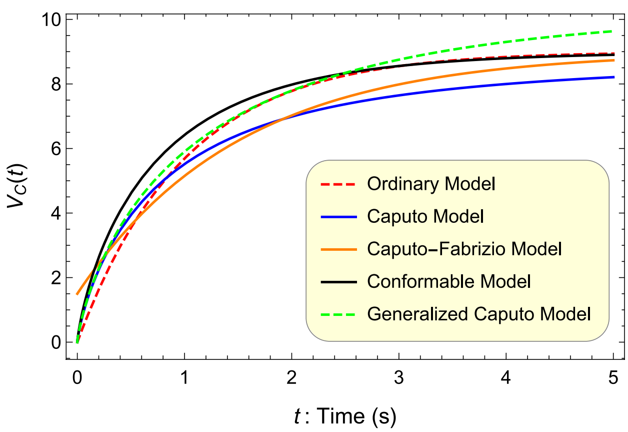
The analytical tractability of the model was demonstrated through the application of generalized Laplace and inverse Laplace transform operators, providing explicit solutions for the system response. Numerical simulations validated the model's accuracy and revealed its strong alignment with classical RC circuit behavior, while outperforming alternative fractional and conformable derivative-based models.



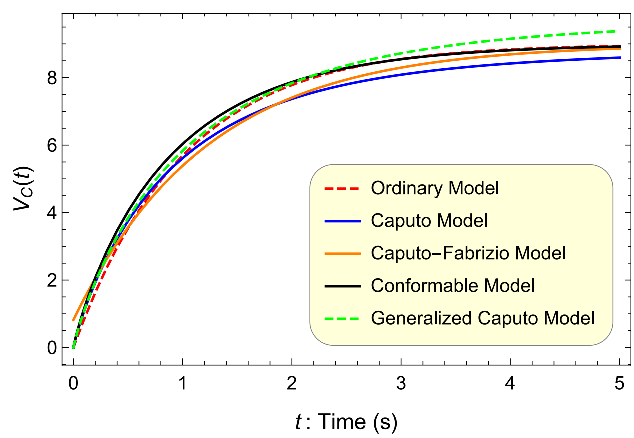
(a) $\tau = \tau_\psi = \varpi = \Delta = 1$, $V_0 = 9$ volt and fractional-order parameter $\psi = 0.60$.



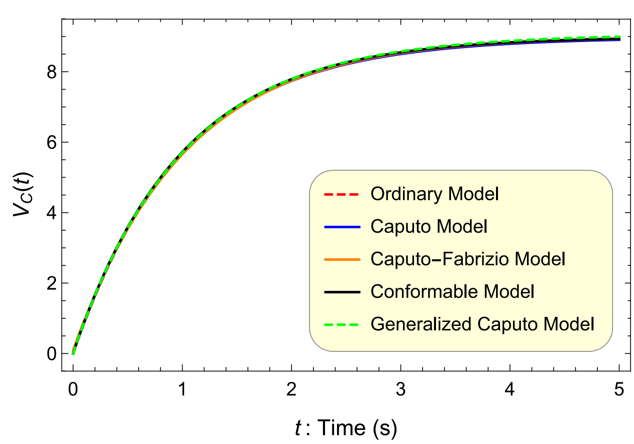
(b) $\tau = \tau_\psi = \varpi = \Delta = 1$, $V_0 = 9$ volt and fractional-order parameter $\psi = 0.70$.



(c) $\tau = \tau_\psi = \varpi = \Delta = 1$, $V_0 = 9$ volt and fractional-order parameter $\psi = 0.80$.



(d) $\tau = \tau_\psi = \varpi = \Delta = 1$, $V_0 = 9$ volt and fractional-order parameter $\psi = 0.90$.



(e) $\tau = \tau_\psi = \varpi = \Delta = 1$, $V_0 = 9$ volt and fractional-order parameter $\psi = 0.99$.

Fig. 2 Comparative evaluation of the RC circuit model responses obtained from integer-order and various fractional-order formulations defined by Eqs. (25)–(29), based on the truncated Mittag-Leffler and Wright function expansions up to $k = 100$

Key contributions of this study include the formulation of a generalized fractional-order RC circuit model and a systematic comparative evaluation against existing models. The results clearly indicate the superior capability of the generalized Caputo derivative to capture essential circuit dynamics while preserving the qualitative and quantitative fidelity of the classical formulation.

The presented findings lay the groundwork for future investigations into fractional-order electrical systems. Extensions of this model to other circuit topologies (e.g., RL, LC, RLC), development of efficient numerical schemes, and experimental validation with physical prototypes represent promising avenues for further research. The generalized Caputo-based framework also has significant potential for interdisciplinary applications, including control systems, signal processing, bioelectrical modeling, and materials science, where memory effects and fractional dynamics play a critical role.

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Declarations

Conflict of interest Not applicable.

Using of AI tools The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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