



# Multi-Objective Optimal Power Flow Using a Modified Weighted Teaching-Learning Based Optimization Algorithm

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**Abstract**—The optimal power flow (OPF) problem is one of the most popular and important issues that researchers need to solve in power systems. In this study, a modified weighted teaching-learning-based optimization (WTLBO) algorithm was designed and implemented for the OPF problem, regarding the teaching-learning-based optimization (TLBO) algorithm, which is one of the meta-heuristic algorithms. To demonstrate the effectiveness of TLBO and developed WTLBO algorithms in OPF solutions, six different single/multi-objective functions consisting of targets, such as fuel cost, total active power losses, and voltage deviation were tested on standard IEEE 30 and 57 bus systems. Multi-objective functions were transformed into a single objective function using the weighted sum method. Analysis results were compared with different optimization algorithms used in the literature. When the developed WTLBO algorithm is compared according to the single/multi-objective functions in OPF solutions, it has been proven that it performs better than the original TLBO algorithm and other algorithms in the literature. As a result of the analysis, according to the TLBO algorithm, in the IEEE 30 bus power system, in the proposed WTLBO algorithm, a decrease of 0.05% in the total fuel cost, 2.20% in active power losses, and 13.37% in voltage deviation are observed.

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## 1. INTRODUCTION

The idea of optimal power flow (OPF) has played an important role in the operation and planning of power systems since the early 1960s [1]. The purpose of OPF is to minimize one or more selected objective functions of a given power system while maintaining a set of distinct equality and inequality constraints. Here, the equality constraints are defined as the load flow equations. Inequality constraints are the limits of independent and dependent variables. The independent variables are expressed as the actual power of the generators other than the slack bus, generator bus voltages, transformer tap settings, and reactive power injection. Dependent variables are slack bus power, load bus voltages, generator reactive powers, and

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line flows [2, 3]. One or more objective functions can be specified in OPF solutions. The most basic purpose function is to minimize the fuel cost of generators. Apart from this, objective functions which are one or a combination of multiples, such as emission, real active power loss, and bus voltage deviation can be determined. Here, it is aimed to minimize the cost of electricity generation, environmental pollution, power losses, and voltage instability. Therefore, OPF solutions are an inevitable field of study in the optimization of power systems.

Many meta-heuristic algorithms have been developed in recent years to solve the OPF problem of power systems [4]. However, OPF solutions in power systems can be divided into two categories as classical methods and meta-heuristic algorithm solutions. In power systems optimization, solutions, such as linear programming [5], Newton-based method [6], nonlinear programming [7, 8], and quadratic programming [9] are expressed as classical methods. Classical methods have disadvantages, such as poor convergence properties [7], the inability of solutions to guarantee accuracy [10], and high computational complexity [11]. Therefore, meta-heuristic methods give more effective results in OPF studies. In OPF solutions, many meta-heuristic optimization algorithms, such as genetic algorithm (GA), evolutionary algorithm (EA), particle swarm optimization (PSO), artificial bee colony algorithm (ABC), gravity search algorithm (GSO), and differential evolution (DE) has been suggested. OPF is a nonlinear programming problem. Meta-heuristic methods can be very effective in solving these nonlinear problems.

Various optimization techniques have been presented by many researchers using metaheuristic algorithms to solve the OPF problem. Kumari *et al.* implemented a multi-purpose OPF solution to minimize fuel cost, loss, and voltage stability index in IEEE 30 bus test system with an improved genetic algorithm (EGA) and decoupled Quadratic Charge Flow (DQLF) algorithm [12]. Chen *et al.* proposed an algorithm (MPIO-PFM) by combining the modified pigeon-inspired optimization algorithm with the penalty function method (PFM) to optimize active power loss, emission, and fuel cost in OPF solutions in power systems [13]. Bhattacharya *et al.* performed the proposed OPF solutions in 26 bus and 118 bus power systems using the multi-objective gravity search algorithm (GSA) to minimize fuel cost, active power loss, and voltage drift [14]. Khorsandi *et al.* He presented a multi-objective fuzzy logic-based artificial bee colony algorithm (MABC) for minimizing fuel cost, emission, power loss, and voltage deviation using IEEE 30 bus and IEEE 118 bus power systems in OPF solution [15]. Elattar *et al.* performed OPF solutions with four different objective functions using a

modified JAYA (MJAYA) algorithm in two different power systems including renewable energy sources [16]. Panda and Tripathy implemented an OPF solution using external reactive power support in a wind-integrated power system using a modified bacterial collection algorithm (MBFO) and genetic algorithm separately [17]. Li *et al.* IEEE has implemented a multi-purpose OPF solution to minimize total fuel cost, active power losses, voltage drift, voltage stability, and emissions by using adaptive restraint differential evolution algorithm in 30 bus power systems [18]. Dao *et al.*, using the chaotic equilibrium optimization (CEO) algorithm, performed an OPF solution for fuel cost optimization depending on the effect of temperature increase in power systems with IEEE 30 and IEEE 118 buses [19]. Mahdad and Srairi developed an adaptive partitioning pollination algorithm (APFPA) for the OPF solution, which includes safety constrained, fuel costs, power losses, and voltage drift, considering failures in the generation unit [20]. Panda *et al.* presented comparative solutions of genetic algorithm, hybrid algorithm, and a modified bacterial aggregation-based (MBFA) algorithm to solve STATCOM-dependent OPF in hydro-thermal-wind power systems [21]. In addition, Naderi *et al.* proposed a multi-purpose OPF solution that includes targets, such as fuel cost, active power losses, and emissions in a new fuzzy adaptive hybrid FAHSPSO-DE algorithm with a self-adaptive particle swarm optimization (SPSO) and differential evolution algorithms in three different test systems (IEEE 30-, 57-, and 118) [22]. On the other hand, Naderi *et al.* have solved the optimal active power distribution problem in IEEE 30 busbar power systems connected to a unified power flow controller (UPFC) from FACT devices by using a hybrid algorithm in fuzzy frame with particle swarm optimization (PSO) and differential evolution algorithm [23].

One of the algorithms developed to solve OPF problems in recent years is teaching-learning based optimization (TLBO) [24]. This algorithm is a new efficient optimization method inspired by its effect on teachers and students in a classroom. The advantage of this method is that it has fewer parameters than other meta-heuristic algorithms [25].

In this article, a teach-learning-based optimization (TLBO) [24, 25] and a modified weighted learning-teach-based optimization (WTLBO) algorithm for reducing fuel cost, active power losses, and voltage deviation are proposed to solve the OPF problem in power systems. With these proposed algorithms, OPF solutions with six different single or multi-purpose functions have been realized on IEEE 30 bus and IEEE 57 bus systems. The results were compared with these two algorithms studied and other algorithms given in the literature.

In OPF problems, multi-objective functions are applied to optimize different objective functions simultaneously. In this study, multi-objective functions were transformed into a single objective function using the weighted sum method with the WTLBO algorithm. In this method, a single objective function is created by assigning a weighting factor to each objective in the multiple objective function.

In this study, a modified weighted teaching–learning based optimization algorithm is proposed to solve the optimal power flow considering critical situations. The advantages and contributions of the algorithm used in the OPF solution of this article can be listed as follows:

- A new variant method based on TLBO is proposed to improve the solution of OPF problems.
- TLBO algorithm has been preferred because of its simplicity of search mechanism. The original TLBO and the proposed WTLBO algorithm do not need additional parameters compared to most algorithms used in the literature.
- The traditional TLBO algorithm randomly selects the teacher and the student, and as the iterations progress, it changes the teacher and student values by using the weight coefficients according to the results of the busbar voltage values it finds. In the WTLBO algorithm, a method in which the weights are determined initially has been created to obtain the best optimization results of the traditional TLBO algorithm with fewer iterations and better values.
- The performances of the proposed WTLBO strategy in terms of solution quality and convergence properties have been tested and verified on two standard test systems, IEEE 30 bus and IEEE 57 bus.
- Constraints in OPF are effectively handled with the predominance of viable solutions.
- The proposed WTLBO algorithm can be an effective alternative for constraint OPF problems.
- The WTLBO algorithm proposed in OPF solutions has better convergence than the traditional TLBO algorithm.

The rest of the article is arranged as follows. In Chapter 2, the problem formulation and objective functions of the OPF are presented. In Chapter 3, TLBO and the developed WTLBO algorithm are introduced. In Section 4, OPF simulation results and other algorithm comparison results in power systems are given. In the last section, the conclusion and recommendations of the study are introduced.

## 2. OPTIMAL POWER FLOW FORMULATION

In general, the OPF problem aims to best determine the power system control parameters to reduce certain targets

that are subject to some inequality and equality constraints. The general mathematical form of the OPF problem is as follows.

$$\text{Minimize: } f(x, u) \quad (1)$$

$$\text{Subject to: } g(x, u) \quad (2)$$

$$h(x, u) \leq 0 \quad (3)$$

Here,  $f$  is the objective function to be minimized,  $x$  and  $u$  are the state and control variable vectors,  $g(x, u)$ , equality constraints, and  $h(x, u)$ , inequality constraints, respectively. The control and state variables of the OPF problem are expressed as:

### 2.1. State Variables

In power systems, state variable vectors ( $x$ ) are expressed by Eq. (4).

$$x = [P_{G_1}, V_{L_1} \dots V_{L_{NL}}, Q_{G_1} \dots Q_{G_{NG}}, S_{l_1} \dots S_{l_m}] \quad (4)$$

Here,  $NL$  is the number of the load bus,  $NG$  is the generator number,  $nl$  is the transmission line number,  $P_{G_1}$  is the slack bus generator's active power output,  $V_L$  is the load bus voltage,  $Q_G$  is the generator's reactive power output, and  $S_l$  is the transmission line load.

### 2.2. Control Variables

In power systems, the control variables ( $u$ ) can be expressed as in Eq. (5).

$$u^T = [P_{G_2} \dots P_{G_{NG}}, V_{G_1} \dots V_{G_{NG}}, Q_{C1} \dots Q_{C_{NC}}, T_1 \dots T_{NT}] \quad (5)$$

Here,  $NT$  is expressed as regulating transformer number,  $NC$  as the parallel reactive power compensator number,  $Q_C$  as parallel reactive power compensator values, and  $T$ , transformer tap settings.

### 2.3. Constraints

In power flow optimization, the active and reactive values of the bus should be kept within certain limits. The limit values given as equality and inequality constraints are important in terms of reflecting the algorithm structure and power system properties.

*2.3.1. Equality Constraints.* The sum of the generated, consumed, and lost power values in the power system should be 0 (zero). This condition required for a balanced power system is given by Eqs. (6) and (7).

$$P_{Gi} - P_{Di} - V_i \sum_{j=1}^{NB} V_j [G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)] = 0, \quad (6)$$

$$Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^{NB} V_j [G_{ij} \sin(\delta_i - \delta_j) + B_{ij} \cos(\delta_i - \delta_j)] = 0 \quad (7)$$

where  $NB$  is the total number of buses.  $P_{Gi}$  is the  $i$ . generator active power.  $Q_{Gi}$  is the  $i$ . reactive power on the generator.  $P_{Di}$  is the  $i$ . load active power in the load bus.  $Q_{Di}$  is the load reactive power in the  $i$ . load bus.  $G_{ij}$  represents the real part of the  $ij_{th}$  element of the Y-bus matrix, and  $B_{ij}$  the imaginary part of the  $ij_{th}$  element of the Y-bus matrix. Violations of the equality constraints mentioned above are checked and detected using load flow calculations.

**2.3.2. Inequality Constraints.** Generator limitations, transformer limitations, and safety limitations are called inequality limitations. Maximum and minimum power generation values are generator limits. Here, the maximum and minimum limit values of the generators should be determined to meet the total load demand.

Generator voltage limits and active and reactive power limits form generator limits. Generator active power limits are defined in Eq. (8).

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}, \quad 1 \leq i \leq N_G \quad (8)$$

Here,  $P_{Gi}^{\min}$  is the minimum active power of the generator and  $P_{Gi}^{\max}$  is the maximum active power of the generator.  $N_G$  is the number of generators.

Generator reactive power limits are given by Eq. (9).

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max}, \quad 1 \leq i \leq N_G \quad (9)$$

Here,  $Q_{Gi}^{\min}$  and  $Q_{Gi}^{\max}$  are the minimum and maximum reactive power values of generator  $i$ , respectively.

Generator voltage limits are given by Eq. (10).

$$V_{Gi}^{\min} \leq V_{Gi} \leq V_{Gi}^{\max}, \quad 1 \leq i \leq N_G \quad (10)$$

Here,  $V_{Gi}^{\min}$  and  $V_{Gi}^{\max}$  are the minimum and maximum voltage values of generator  $i$ , respectively.

Shunt compensator limits are given by Eq. (11).

$$Q_{Ci}^{\min} \leq Q_{Ci} \leq Q_{Ci}^{\max}, \quad 1 \leq i \leq N_C \quad (11)$$

Here,  $Q_{Ci}^{\min}$  and  $Q_{Ci}^{\max}$  are the  $i$  shunt compensator minimum and maximum values, respectively.  $N_C$  is the number of shunt compensators.

Some lines in the interconnected power system may have to regulate transformers for adjusting voltage drops.

Voltage drop adjustment is made by arranging transformer step values. Transformer step values are normally taken as 1. The step value limits are expressed as in Eq. (12).

$$T_i^{\min} \leq T_i \leq T_i^{\max}, \quad 1 \leq i \leq N_T \quad (12)$$

Here,  $T_i^{\min}$  and  $T_i^{\max}$  are the minimum and maximum tapping values of the  $i$  transformer, respectively.  $N_T$  is the number of the tap transformer.

The safety limitations are the voltage amplitudes at the load bus and these amplitudes are defined by Eq. (13).

$$V_{Li}^{\min} \leq V_{Li} \leq V_{Li}^{\max}, \quad 1 \leq i \leq N_L \quad (13)$$

Here,  $V_{Li}^{\min}$  and  $V_{Li}^{\max}$  are the minimum and maximum voltage values of the load bus  $i$ , respectively.  $N_L$  is the load bus number.

The apparent power flow limits are given by Eq. (14).

$$S_{li} \leq S_{li}^{\max}, \quad 1 \leq i \leq N_l \quad (14)$$

Here,  $S_{li}$ ,  $i$  apparent power load flow,  $S_{li}^{\max}$  are the maximum apparent power load flow values.

It should be noted that the control variables are self-limiting. Inequality constraints of state variables containing load bus voltage magnitudes can be included in an objective function as quadratic penalty terms. These inequality constraints are real power generation output, reactive power generation output in Slack Bus. and line loading. In these terms, a penalty factor multiplied by the square of the neglected value of the dependent variable is added to the objective function and any solution obtained is rejected.

Mathematically, the penalty function can be expressed as:

$$\begin{aligned} Penalty = & K_p \left( P_{G1} - P_{G1}^{\text{Lim}} \right)^2 + K_Q \sum_{i=1}^{N_G} \left( Q_{Gi} - Q_{Gi}^{\text{Lim}} \right)^2 \\ & + K_V \sum_{i=1}^{N_L} \left( V_{Bi} - V_{Bi}^{\text{Lim}} \right)^2 + K_S \sum_{i=1}^{N_l} \left( S_{Li} - S_{Li}^{\text{Lim}} \right)^2 \end{aligned} \quad (15)$$

Here,  $K_p$ ,  $K_Q$ ,  $K_V$ , and  $K_S$  are the positive value penalty factors of the load voltage [13].  $x^{\text{Lim}}$ , is the violated limit value of the dependent variable  $x$  and is given as [26]:

$$x^{\text{Lim}} = \begin{cases} x^{\max} & \text{if } x > x^{\max} \\ x^{\min} & \text{if } x < x^{\min} \end{cases} \quad (16)$$

## 2.4. Objective Functions

For the OPF problem in power systems, 3 single-purpose and 3 multi-purpose targets were determined.

**2.4.1. Minimization of Basic Fuel Cost.** The most frequently used objective function in the OPF problem is to minimize the total cost of active power generation. The fuel costs of each generator are represented by quadratic curves. The objective function can be expressed as the sum of fuel costs and is expressed as:

$$f_1 = F_{cost} = \sum_{i=1}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^2, \quad (17)$$

Here,  $a_i$ ,  $b_i$ , and  $c_i$  are the cost coefficients of the generators.

**2.4.2. Minimization of Power Loss.** The aim of minimizing the total power losses in the transmission lines is expressed in Eq. (18):

$$f_2 = P_{loss} = \sum_{i=1}^{nl} \sum_{j \neq i}^{nl} G_{ij} [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)] \quad (18)$$

where  $P_{loss}$ , represents the total active power losses of the power system.  $nl$ , shows the number of transmission lines,  $G_{ij}$ , shows the conductivity of the line connected between  $i$  bus and  $j$  bus,  $\delta_i$  and  $\delta_j$ , respectively, the voltage angle difference between  $i$  bus and  $j$  bus, and  $V_i$  and  $V_j$  show the voltage magnitude of  $i$  and  $j$  bus, respectively.

**2.4.3. Minimization of Voltage Deviation.** In power systems, one of the most important indicators of stability is to optimize the total voltage deviation in the load buses. The total voltage deviation value is given in Eq. (19).

$$f_3 = V_D = \sum_{l=1}^{N_L} |V_L - 1| \quad (19)$$

where  $N_L$  is the number of load buses,  $V_d$  is the load bus total voltage deviation, and  $V_L$  is  $l$  is the load bus voltage.

**2.4.4. Minimization of Fuel Cost and Active Power Losses.** In this case, the objective function aims to minimize generator fuel costs and power transmission losses simultaneously.

$$f_4 = \sum_{i=1}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^2 + \lambda_l \sum_{i=1}^{nl} \sum_{j \neq i}^{nl} G_{ij} [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)] \quad (20)$$

Here,  $\lambda_l$  is the weight factor. It was chosen as 40 [26].

**2.4.5. Minimization of Fuel Cost and Voltage Deviation.** In this case, the objective function aims to minimize generator fuel costs and voltage deviation simultaneously.

$$f_5 = \sum_{i=1}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^2 + \lambda_D \sum_{l=1}^{N_L} |V_L - V_L^{ref}| \quad (21)$$

Here,  $\lambda_D$  is the weight factor. It was chosen as 100 [26].

**2.4.6. Minimization of Cost, Losses, and Voltage Deviation.** In this case, the objective function aims to minimize generator fuel costs, power transmission losses, and voltage deviation values simultaneously.

$$f_4 = \sum_{i=1}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^2 + \lambda_l \sum_{i=1}^{nl} \sum_{j \neq i}^{nl} G_{ij} [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)] + \lambda_D \sum_{l=1}^{N_L} |V_L - V_L^{ref}| \quad (22)$$

### 3. OPTIMIZATION ALGORITHMS

#### 3.1. Teaching-Learning-Based Optimization (TLBO)

The TLBO algorithm was developed by Rao *et al.* between 2011 and 2012 as a new optimization technique according to many optimization algorithms [24, 27]. The TLBO algorithm is social-based. The TLBO algorithm references the interaction between students and teachers in a classroom. The TLBO algorithm, like other meta-heuristic algorithms, does not need any other algorithm-specific parameters and only needs control parameters, such as population size and the number of generations to work [8]. Each step of the algorithm is based on the selection of successful students and best students [27]. TLBO consists of a population of students and teachers who know best. The best solution to date, as the teacher is considered the most learned person in society; is similar to the teacher in TLBO. The TLBO algorithm has three parameters (number of students, number of classes, and number of renewals) and two phases (teacher phase and student phase). In the teacher stage, students learn by imitating the teacher. The teacher is the most knowledgeable person in the class. Therefore, the best student in the class can learn only as much as the teacher [25].

The stages of the TLBO algorithm are given below.

$X_{i,j}$  indicates the result of the  $j$  topic in  $i$  refresh. In Eq. (23), a student in the class is determined during any renewal.

$$X_i^g = [x_{i,1}^g, x_{i,2}^g, x_{i,3}^g, \dots, x_{i,j}^g, \dots, x_{i,D}^g] \quad (23)$$

Here,  $N$  is the number of learners in a class, *i.e.*, “class capacity,”  $D$  is the number of lessons offered to students,  $X$  is the population. The value of  $X_{i,j}$  is randomly determined using Eq. (24).

$$X_{i,j} = X_j^{\min} + ri(X_j^{\max} - X_j^{\min}) \quad (24)$$

Here,  $r_i$  is a random number between 0 and 1,  $X_j^{\max}$  and  $X_j^{\min}$  are the maximum and minimum values of the  $j$  parameter, respectively.

**3.1.1. Teacher Phase.**  $M^g$ , the main parameter of the students in each lesson is given in Eq. (25).

$$M^g = [M_1^g, M_2^g, M_3^g, \dots, M_j^g, \dots, M_D^g] \quad (25)$$

The student with the minimum objective function value,  $X_{teacher}^g$  is considered the best knower for the relevant refresh. In the teacher phase, the algorithm advances the learner’s average by shifting it toward the teacher. To obtain a new set of enhanced learners, a standard weighted differential vector is constructed from the current mean and desired mean parameters and added to the existing learners.

$$X_{new,l}^g = X_l^g + ri(X_{teacher}^g - T_F M^g) \quad (26)$$

$T_F$  is the teaching factor that determines the value to change. The value of  $T_F$  can be 1 or 2. The value of  $T_F$  is determined with a random probability.  $T_F$  is defined as in Eq. (27),

$$T_F = \text{round}[1 + ri(0, 1)\{1, 2\}] \quad (27)$$

If the value of  $X_{new,l}^g$  is better than the current result  $X_l^g$ , it will replace  $X_l^g$  on the next refresh.

**3.1.2. Student Phase.** After the teacher phase, all the best function values are saved for use in the learner phase. At this stage, students learn knowledge by interacting and discussing with each other. If one student is more knowledgeable, the other updates himself/herself through interaction [27]. Random interaction between students enhances their knowledge. For a particular student  $X_i^g$ , another learner  $X_r^g$  is randomly selected ( $i \neq r$ ). In Eq. (28), the  $i$ . parameter of the  $X_{new,i}^g$  matrix at the student stage is given. Figure 1 shows the flow chart of the TLBO algorithm.

$$X_{yeni,i}^g = \begin{cases} X_i^g + ri(X_i^g - X_r^g) \\ e\check{g}er(X_i^g) < f(X_r^g) \\ X_i^g + ri(X_r^g - X_i^g) \end{cases} \quad (28)$$

### 3.2. Modified Weighted Learning-Teach Based Optimization Algorithm (WTLBO)

In this study, the parameter  $w$  known as “weight” is added to Eqs. (26) and (28) in traditional TLBO for this case. Unlike traditional TLBO, when calculating the new student value, our approach takes a portion of the previous value into account and is decided by a weighting factor.

In general, it dictates that individuals sample various regions of the search area in the early stages of regeneration. It is important to fine-tune the results of the solutions in the later stages. Thus, it performs the analysis in the interior of a small area where the intended optimum exists. To achieve this goal, we linearly reduce the value of the predetermined weight factor from a maximum to a minimum value.

$$w = w_{\max} - \left( \frac{w_{\max} - w_{\min}}{MAXIT} \right) \times i \quad (29)$$

Here the maximum and minimum weighting factors are the maximum and minimum values of  $w$ , iterations  $i$  is the number of iterations available, and the maximum number is the number of iterations allowed. The  $w_{\max}$  and  $w_{\min}$  values take values ranging from [0,1].  $w_{\max}$  must always be greater than  $w_{\min}$ . That is, it must satisfy the  $w_{\max} > w_{\min}$  condition. In this context, if Eqs. (30) and (31) are rearranged by adding weighting factors to Eqs. (26) and (28),

In the teacher phase, the learner’s group is written as follows with the new method developed by us.

$$X_{new,l}^g = w \times X_l^g + ri(X_{teacher}^g - T_F M^g) \quad (30)$$

In the student phase,

$$X_{new,i}^g = \begin{cases} w \times X_i^g + ri(X_i^g - X_r^g) \\ e\check{g}er(X_i^g) < f(X_r^g) \\ w \times X_i^g + ri(X_r^g - X_i^g) \end{cases} \quad (31)$$

The traditional TLBO algorithm randomly selects the teacher and student and changes the teacher and student values using weight coefficients according to the results of the busbar voltage values it finds as the iterations progress. In the WTLBO algorithm, a method in which the weights are determined at the beginning was created to achieve the best optimization results of the traditional TLBO algorithm with fewer iterations and better values. On the other hand, the number of students is an important variable in the traditional TLBO algorithm, and if it is more or less, the running time of the algorithm is affected by this value. If the number of students is low, the processing time in each

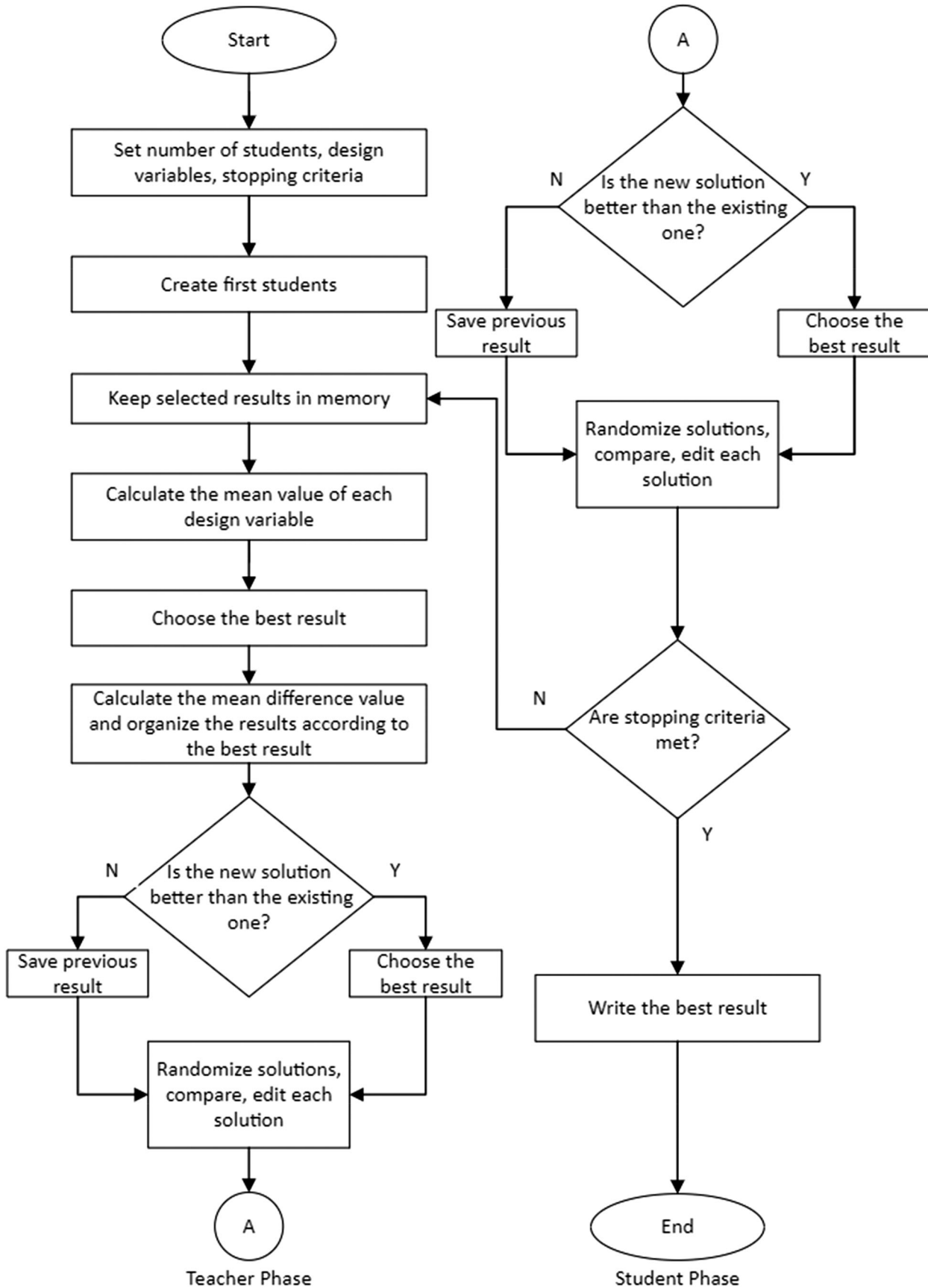


FIGURE 1. Flow chart for TLBO algorithm.

iteration of the algorithm increases a lot, and in the case of an increase in the number of students, the computation time becomes very short and in some iterations, optimum results are avoided. For this value, as a result of many iterations, it has been seen that taking the number of students as 25 gives the best results in power systems. The flow diagram of the WTLBO algorithm is given in Figure 2 [28].

#### 4. SIMULATION AND RESULTS

In this study, TLBO and WTLBO algorithms are used to show the effectiveness of the OPF problem in IEEE 30 bus (in Figure 3) and IEEE 57 bus (in Figure 4) test systems. A total of six optimization targets, three single-objective and three multi-objective, were determined for the OPF problem in the two power systems mentioned. Table 1 shows the optimization targets used in the test systems. In both power systems, limit values of generators' active and reactive power, bus voltage, transformer step value, and shunt compensator inequality restrictions have been determined for OPF solution. In Figure 5, the graph of the limit values against the bus voltage values for the IEEE 30 bus is shown. As can be seen from Figure 5, in the IEEE 30 bus power system, all constraint limits are satisfied in the original TLBO and proposed WTLBO algorithms. In Figure 6, the graph of the limit values against the bus voltage values for the IEEE 57 bus is shown. As can be seen from Figure 6, the original TLBO algorithm for three cases (cases 7–9) did not provide voltage limit values for some buses. However, it has been observed that the proposed WTLBO algorithm provides voltage limit values for all buses. For the proposed algorithms, the population size is 25, the maximum number of iterations is 100, and 30 test runs are performed for each OPF problem. All simulation results are analyzed in Matlab R2017b program using Intel Core(TM) i7-2620 2.7 GHz and 8.00 (64-bit) GB RAM PC.

##### 4.1. IEEE 30-Bus System

The proposed TLBO and WTLBO algorithms are tested on a standard IEEE 30 bus power system with OPF solutions. In the IEEE 30 bus system, there are 41 transmission lines, six generator buses connected to buses 1, 2, 5, 8, 11, and 13, 24 load bus, 4 (6–9; 6–10; 4–12 and 27–28) transformer and nine shunts VAR compensators (10, 12, 15, 17, 20, 21, 23, 24 and 29). Active and reactive power demands are 283.4 MW and 126.2 MVar. Each of the shunt VAR

compensators has 5 MVar capacity, and transformer step setting limits are 0.9–1.1. It is assumed that the maximum and minimum voltages of all load buses are 0.95–1.05 in p.u [29, 30].

To demonstrate the effectiveness of the proposed algorithm, different cases with various purposes are considered for OPF solutions in IEEE 30 bus system as follows:

*4.1.1. Case 1: Minimization of Basic Fuel Cost.* In this case, it is aimed to minimize the total fuel cost of the production system. It is calculated according to Eq. (17). The generator cost coefficients in the IEEE 30 bus system are given in Table 2 [30]. The fuel costs obtained by the original TLBO and proposed WTLBO algorithms are given in Table 3 as \$801,000/hr for TLBO and \$800,570/hr for WTLBO. The convergence graph of the total fuel cost objective function is shown in Figure 5(a). The proposed WTLBO convergence provides better performance than TLBO. To evaluate the effectiveness of the WTLBO algorithm, its comparisons with the total fuel cost results in the literature are given in Table 4. WTLBO achieves better solutions than the original TLBO and other methods.

*4.1.2. Case 2: Minimization of Power Loss.* In this case, it aims to reduce the real power loss in transmission lines. This situation objective function is given in Eq. (18). The results of active power losses obtained by TLBO and WTLBO algorithms are given in Table 3 as 3.179 and 3.109 MW, respectively. TLBO and WTLBO convergence graph is given in Figure 5(b). In Table 5, it is seen that the active power loss performance of WTLBO is better than other algorithms.

*4.1.3. Case 3: Minimization of Voltage Deviation.* In this case, it is to optimize the total deviation in voltage magnitudes at the load bus. The objective function is calculated according to Eq. (19). As can be seen in Table 3, the TLBO and WTLBO voltage deviation values were calculated as 0.1089 and 0.093 p.u, respectively. The purpose here is to get the voltage deviation value closest to 0 (Zero) p.u. As can be seen in Table 6, the proposed algorithm has been proven to give better results than other algorithms in the literature by calculating the closest value to 0 (zero) p.u. According to the convergence graph in Figure 5(c), WTLBO performed even better than TLBO.

*4.1.4. Case 4: Minimization of Fuel Cost and Active Power Losses.* In this case, it is considered the multi-purpose function of OPF to minimize fuel cost and power losses simultaneously. The calculation of this situation according

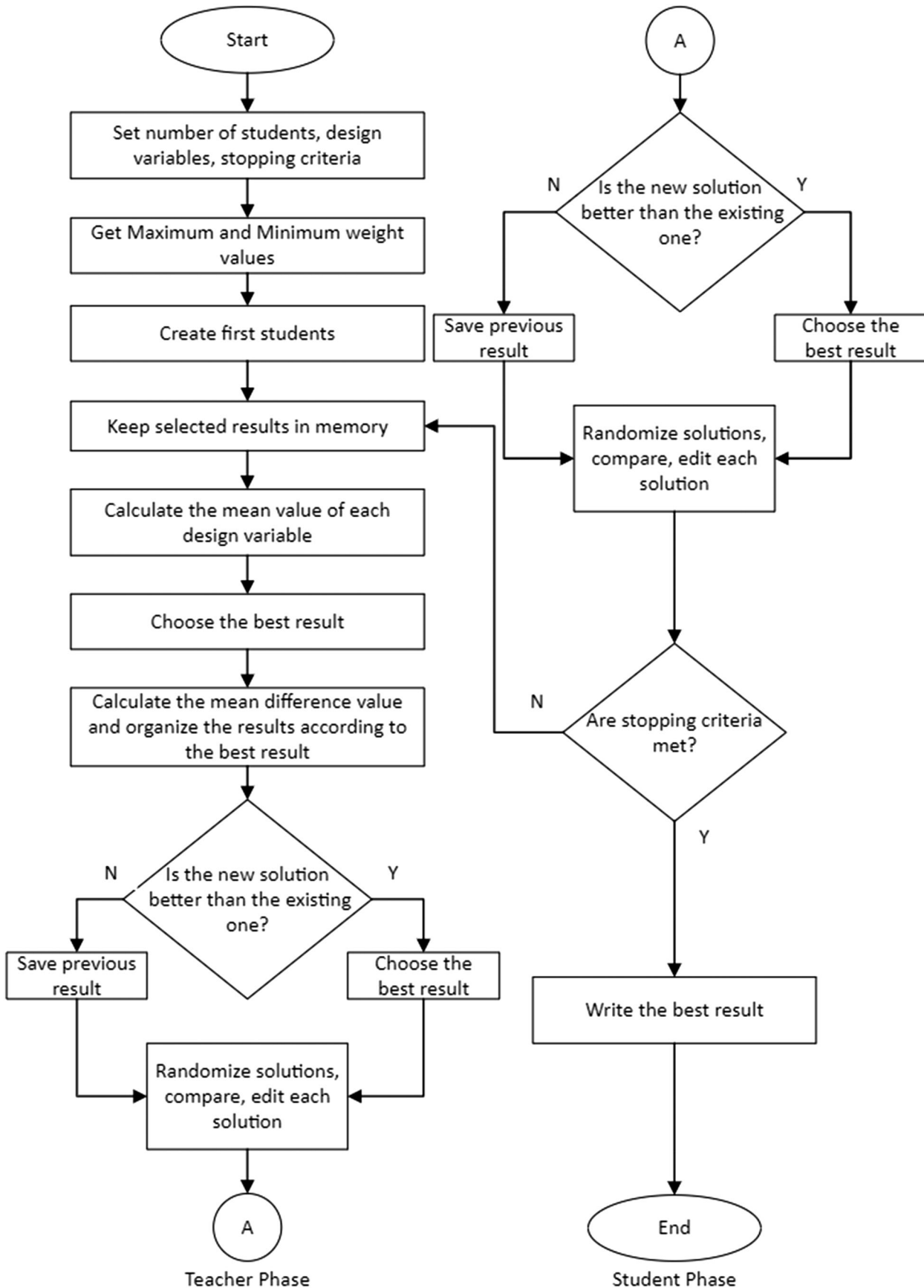


FIGURE 2. Flow chart for the developed WTLBO algorithm.



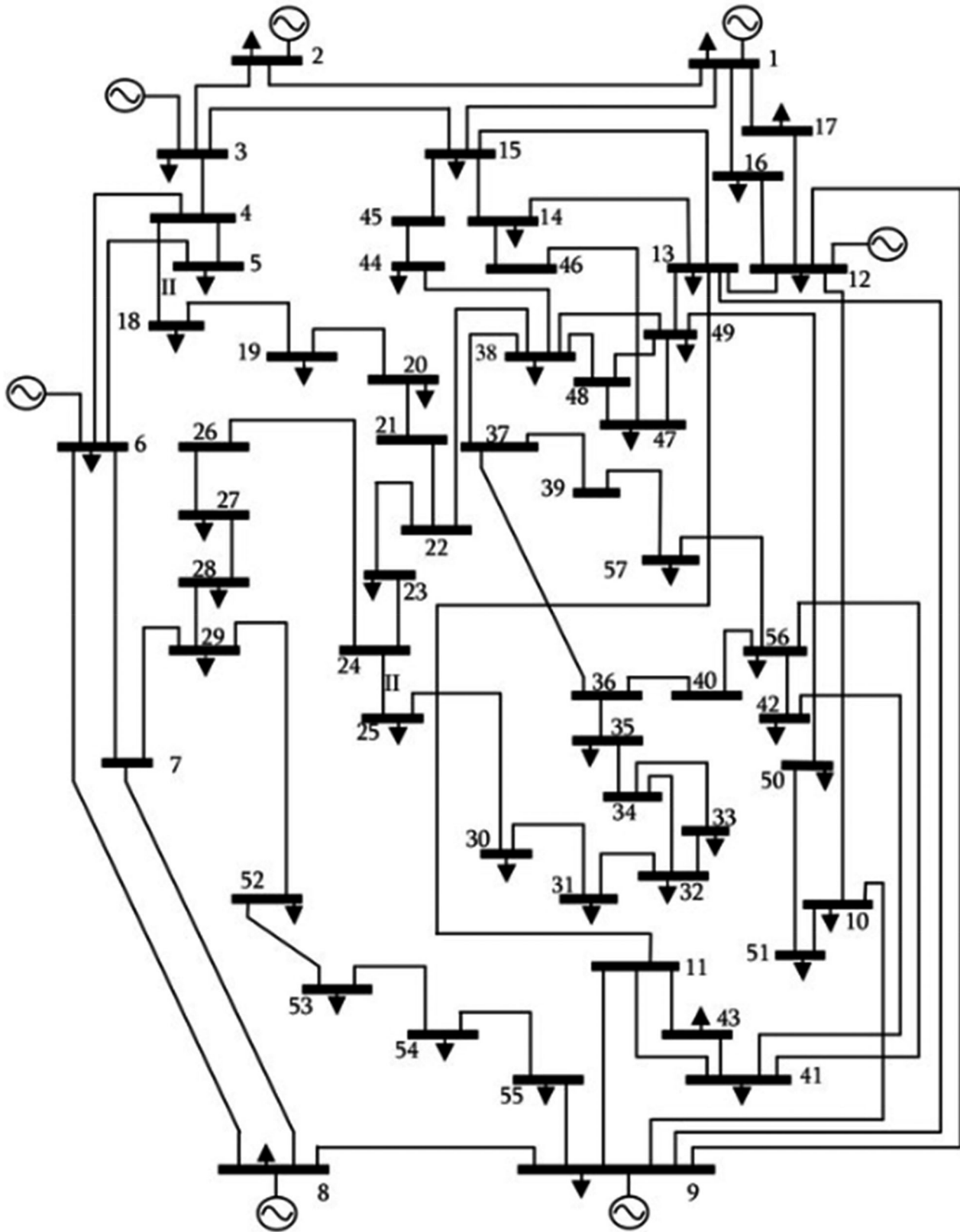


FIGURE 4. Single-line diagram for IEEE 57-bus test system.

Name	Objective functions	Test system
Case 1	Minimization of basic fuel cost	IEEE 30
Case 2	Minimization of power loss	
Case 3	Minimization of voltage deviation	
Case 4	Minimization of fuel cost and active power losses	IEEE 57
Case 5	Minimization of fuel cost and voltage deviation	
Case 6	Minimization of cost, losses, and voltage deviation	
Case 7	Minimization of basic fuel cost	
Case 8	Minimization of fuel cost and voltage deviation	
Case 9	Minimization of cost, losses, and voltage deviation	

TABLE 1. Working conditions in test systems.

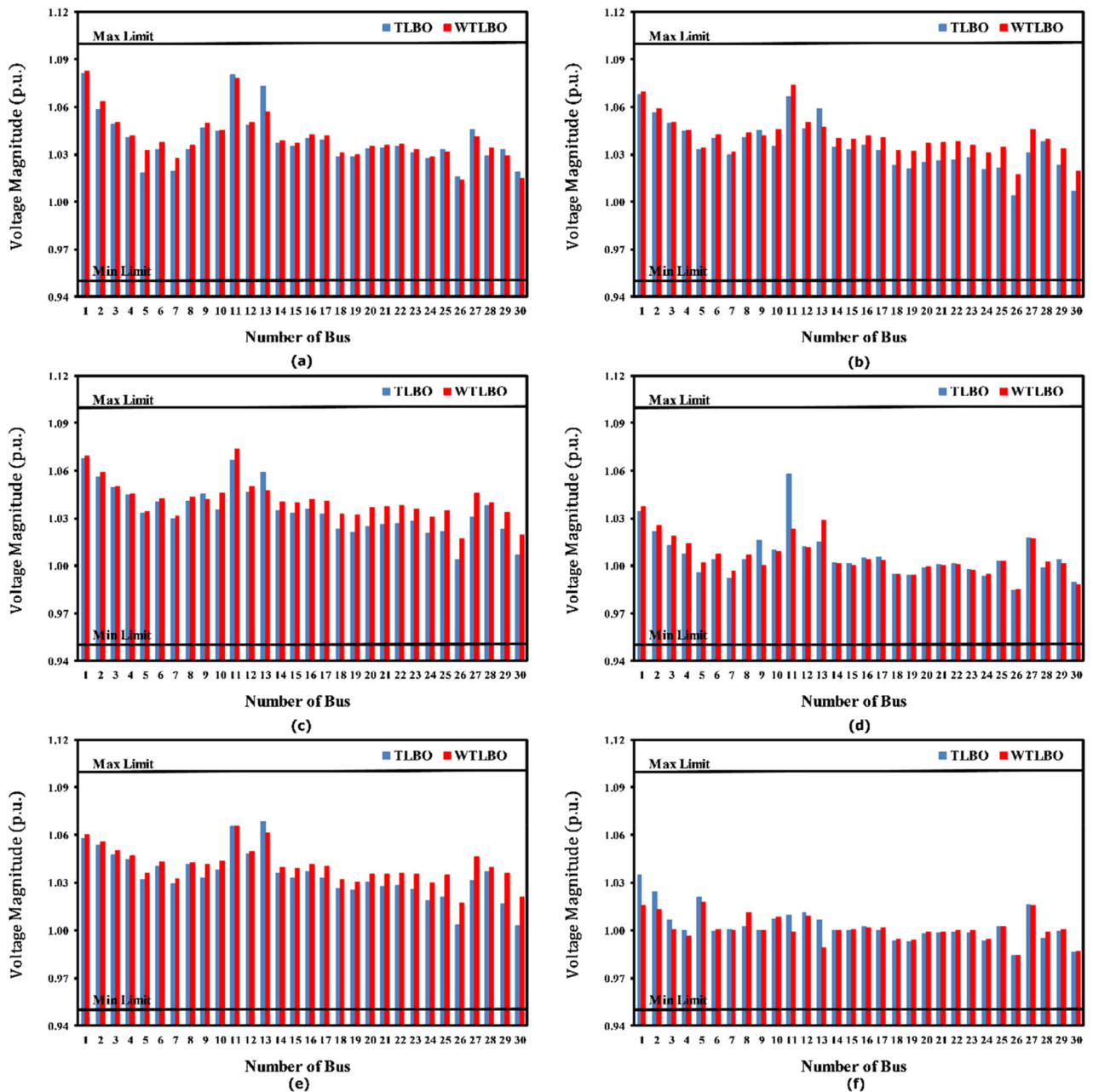


FIGURE 5. Bus voltage graph for IEEE 30 bus: (a) case 1, (b) case 2, (c) case 3, (d) case 4, (e) case 5, and (f) case 6.

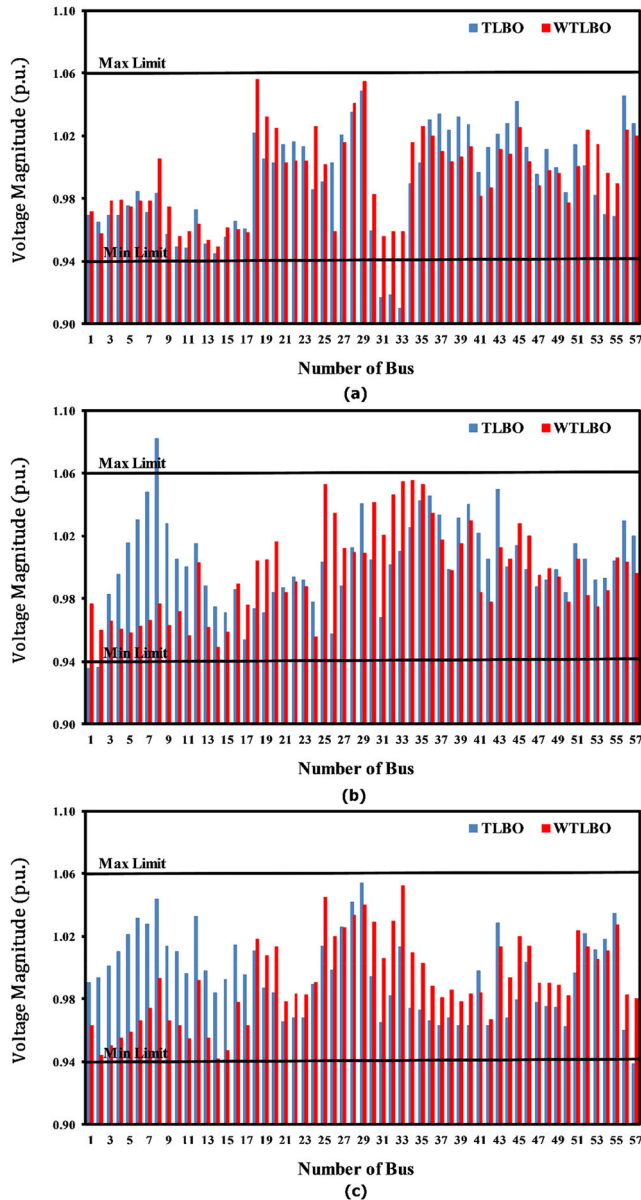


FIGURE 6. Bus voltage graph for IEEE 30 bus: (a) case 7, (b) case 8, and (c) case 9.

to the objective function is given in Eq. (20). TLBO and WTLBO minimum fuel cost and minimum active power loss values are presented in Table 7 as 858,880 \$/hr, 4.772 MW and 853,530 \$/hr, 4.555 MW, respectively. The convergence graph is given in Figure 5(d). The comparison of the results obtained with other methods reported in the literature is shown in Table 8.

**4.1.5. Case 5: Minimization of Fuel Cost and Voltage Deviation.** In this case, as in Eq. (21), the fuel cost and voltage deviation values are presented in the OPF solution

as a multi-purpose function. According to the simulation results of the fuel cost and voltage deviation in Table 7, TLBO is 804.720 \$/hr and 0.122 p.u., and WTLBO is 803.410 \$/hr and 0.106 p.u, respectively. The convergence of the developed algorithm is presented in Figure 5(e) and its superiority over other algorithms is in Table 9.

**4.1.6. Case 6: Minimization of Cost, Losses, and Voltage Deviation.** In this case, three single-purpose functions (fuel cost, active power loss, and voltage bias) are simultaneously realized as a multi-purpose OPF solution. The function equation is shown in Eq. (22). The fuel cost, active power loss, and voltage deviation values of the algorithms are presented in Table 7 as 870,880 \$/hr, 4.796 MW, and 0.156 p.u for TLBO, and 863,570 \$/hr, 4.674 MW and 0.144 p.u for WTLBO. In Figure 5(f), the convergence advantage of the proposed algorithm over TLBO is presented.

## 4.2. IEEE 57-Bus System

In the second stage of the simulation, the IEEE 57 bus system was used to demonstrate the efficiency of the TLBO and WTLBO algorithms. Busbar data, line data, cost coefficients, and limits of power generation (active/reactive power) are taken from Ref. [48]. The test system includes seven generators on buses 1, 2, 3, 6, 8, 9, and 12, 80 transmission lines, and 15 branches under load tap setting transformer branches. It consists of shunt reactive power supplies in 18, 25, and 53 buses. The limits of the step-setting transformer control variables are considered to be 1.1–0.9. The maximum/minimum values of the reactive power (shunt) are 0 and 30 MVAR. In this study, the minimum/maximum voltage values of all buses are taken as 0.94 and 1.06 p.u. The load demand of the IEEE 57 bus test system is 1250.8 MW and 336.4 MVAR [30, 49]. To demonstrate the effectiveness of the proposed algorithm, different cases with various purposes are considered for OPF solutions in IEEE 57 bus system as follows:

**4.2.1. Case 7: Minimization of Basic Fuel Cost.** In this case, the minimization of the total fuel cost of the production system is calculated according to Eq. (17). The generator cost coefficients in the IEEE 57 bus system are given in Table 10 [30]. The fuel costs obtained by the original TLBO and proposed WTLBO algorithms are given in Table 11 as 41150.10 \$/hr for TLBO and 39651.85 \$/hr for WTLBO. The convergence graph of the total fuel cost objective function is shown in Figure 6(a). The proposed WTLBO convergence provides better performance than

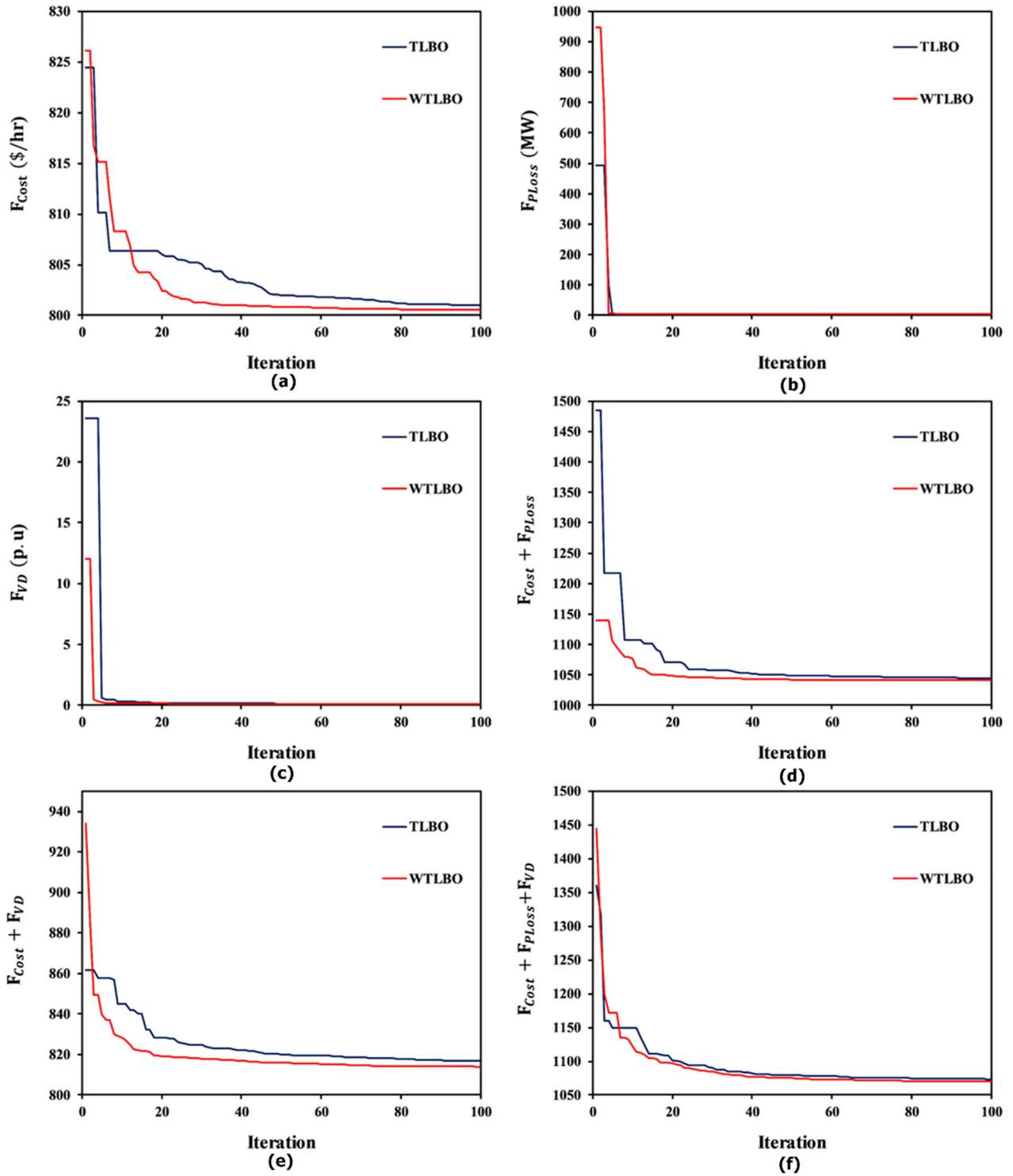


FIGURE 7. Iteration graphs of objective functions for IEEE 30 bus: (a) case 1, (b) case 2, (c) case 3, (d) case 4, (e) case 5, and (f) case 6.

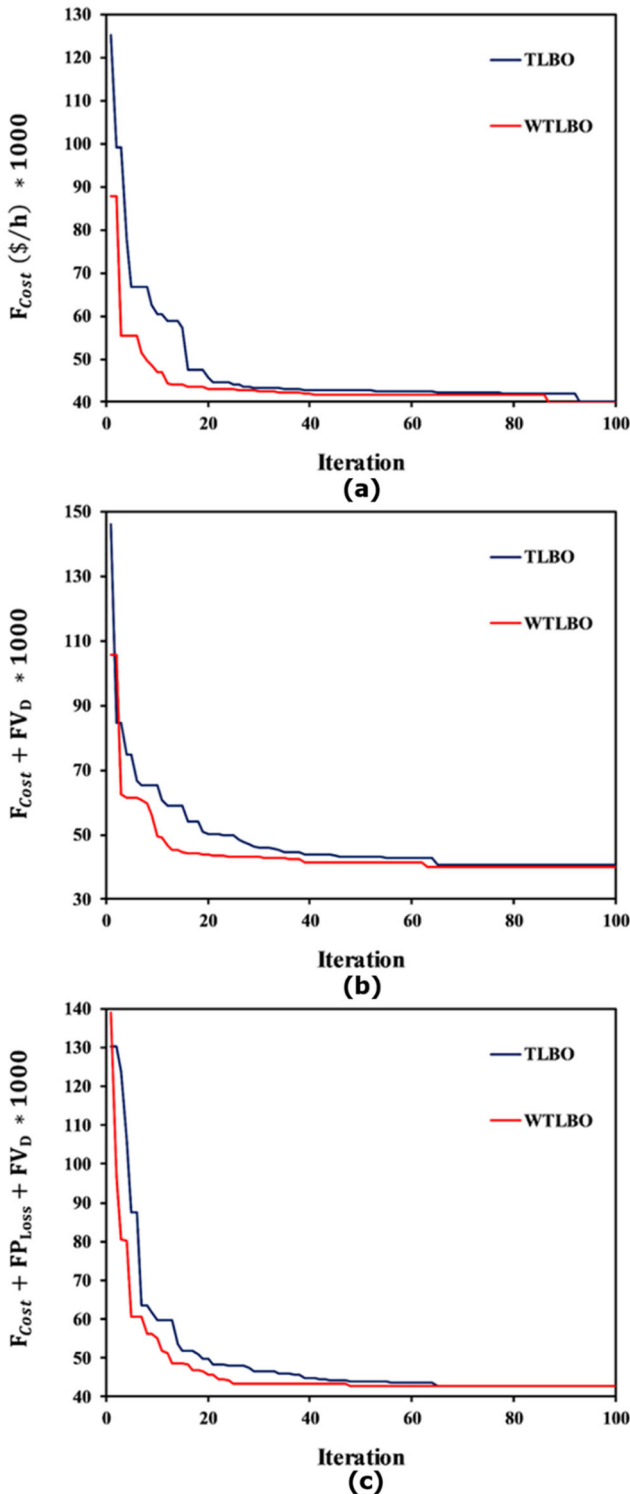


FIGURE 8. Iteration graphs of objective functions for IEEE 57 bus: (a) case 7, (b) case 8, and (c) case 9.

Bus No.	Cost coefficients		
	<i>a</i>	<i>b</i>	<i>c</i>
1	0.00	2.00	0.00375
2	0.00	1.75	0.01750
5	0.00	1.00	0.06250
8	0.00	3.25	0.00834
11	0.00	3.00	0.02500
13	0.00	3.00	0.02500

TABLE 2. Generator cost coefficients (IEEE 30 bus).

TLBO. To evaluate the effectiveness of the WTLBO algorithm, its comparisons with the total fuel cost results in the literature are given in Table 12. WTLBO achieves better solutions than the original TLBO and other methods.

4.2.2. Case 8: Minimization of Fuel Cost and Voltage Deviation. In this case, the fuel cost and voltage deviation values are calculated as in the multi-purpose function Eq. (21). According to the fuel cost and voltage deviation simulation results in Table 11, TLBO and WTLBO values are shown as 42321.60 \$/hr and 1.1707 p.u, and 41685.41 \$/hr and 0.7536 p.u, respectively. The convergence of the developed algorithm in Figure 6(b) and its superiority over other algorithms in Table 12 are presented.

4.2.3. Case 9: Minimization of Cost, Losses, and Voltage Deviation. In this case, the fuel cost, active power loss, and voltage deviation multi-purpose OPF solution are implemented in Eq. (22). The fuel cost, active power loss, and voltage deviation values of the algorithms are presented in Table 11 as 42084.43 \$/hr, 24.3953 MW and 1.1795 p.u for TLBO, 41936.39 \$/hr, 21.5621 MW and 1.1286 p.u for WTLBO. In Figure 6(c), the convergence advantage of the proposed algorithm over TLBO is presented.

## 5. CONCLUSION

In this study, the WTLBO algorithm is proposed as a new solution method by making adjustments to this algorithm with a teaching–learning based optimization algorithm for the single/multi-purpose OPF problem in power systems. These objective functions are studied in detail to realize OPF solutions with a total of six objective functions: single-objective three-state and multi-purpose three-case. The

	Limits		Case 1		Case 2		Case 3	
	Min	Max	TLBO	WTLBO	TLBO	WTLBO	TLBO	WTLBO
$P_{g1}$ (MW)	50	250	178.0078	176.6458	52.3556	51.56	148.2998	82.2206
$P_{g2}$ (MW)	20	80	47.978	48.778	79.445	80.000	61.582	80.000
$P_{g2}$ (MW)	20	80	47.978	48.778	79.445	80.000	61.582	80.000
$P_{g5}$ (MW)	15	50	21.433	21.470	49.941	49.999	25.977	38.979
$P_{g8}$ (MW)	10	35	19.842	21.602	34.990	34.960	20.747	34.953
$P_{g11}$ (MW)	10	30	12.774	11.856	29.968	29.992	22.706	30.000
$P_{g13}$ (MW)	12	40	12.554	12.056	39.880	39.997	12.412	22.238
$V_{g1}$ (p.u)	0.95	1.1	1.081	1.083	1.058	1.0602	1.035	1.016
$V_{g2}$ (p.u)	0.95	1.1	1.058	1.064	1.053	1.056	1.024	1.014
$V_{g5}$ (p.u)	0.95	1.1	1.018	1.032	1.032	1.036	1.021	1.018
$V_{g8}$ (p.u)	0.95	1.1	1.033	1.036	1.042	1.043	1.002	1.011
$V_{g11}$ (p.u)	0.95	1.1	1.081	1.078	1.066	1.066	1.010	0.999
$V_{g13}$ (p.u)	0.95	1.1	1.073	1.057	1.068	1.061	1.007	0.989
$T_{6-9}$ (p.u)	0.9	1.1	1.014	1.005	1.047	1.027	1.020	1.012
$T_{6-10}$ (p.u)	0.9	1.1	0.915	0.966	0.908	0.955	0.900	0.904
$T_{4-12}$ (p.u)	0.9	1.1	1.006	0.981	1.008	1.002	0.955	0.943
$T_{28-27}$ (p.u)	0.9	1.1	0.967	0.979	0.983	0.976	0.954	0.959
$Q_{C10}$ (MVar)	0	0.05	1.540	3.305	2.975	4.994	2.694	4.478
$Q_{C12}$ (MVar)	0	0.05	0.233	2.360	1.893	0.822	0.820	1.869
$Q_{C15}$ (MVar)	0	0.05	1.541	1.999	2.426	4.926	2.396	5.000
$Q_{C17}$ (MVar)	0	0.05	1.647	4.859	1.696	5.000	0.000	1.473
$Q_{C20}$ (MVar)	0	0.05	3.634	4.080	4.871	4.631	4.865	4.933
$Q_{C21}$ (MVar)	0	0.05	1.901	4.954	2.466	5.000	4.433	3.947
$Q_{C23}$ (MVar)	0	0.05	2.468	2.695	2.433	3.231	4.354	5.000
$Q_{C24}$ (MVar)	0	0.05	4.352	4.963	2.873	5.000	4.727	4.975
$Q_{C29}$ (MVar)	0	0.05	2.306	2.490	1.562	3.130	1.171	1.730
$F_{cost}$ (\$/hr)			801.000	800.570	965.980	967.600	814.470	891.100
$P_{loss}$ (MW)			9.188	9.008	3.179	3.109	8.324	4.991
$V_D$ (p.u)			0.8269	0.8556	0.7133	0.8945	0.1077	0.0933
CPU time (s)			34.98	34.48	37.05	34.93	35.50	34.40

TABLE 3. OPF test results for TLBO and WTLBO algorithms in cases 1–3 (IEEE 30 bus).

Algorithm	$F_{cost}$ (\$/hr)	$P_{loss}$ (MW)	$V_D$ (p.u)	Algorithm	$F_{cost}$ (\$/hr)	$P_{loss}$ (MW)	$V_D$ (p.u)
WTLBO	800.57	9.008	0.8556	WTLBO	967.6000	3.1087	0.8945
TLBO	801	9.1875	0.8269	TLBO	965.9766	3.1794	0.7133
PSO [31]	800.5912	9.2229	0.8534	MFO [26]	967.6785	3.1111	0.91558
ABC [32]	800.66	9.0328	0.9209	GA [31]	967.5342	3.1342	0.8739
MFO [26]	800.6863	9.1492	0.7577	MDE [26]	967.6543	3.1619	0.76781
GA [31]	800.7666	8.9882	0.8228	EM [40]	954.315	3.1775	
GOA [31]	800.7806	8.9882	0.6909	EGA [12]	967.86	3.2008	
MDE [26]	800.8399	8.8365	0.7762	GPU-PSO [41]	944.7613	3.2601	
GWO [33]	800.9308	9.0558	0.724	GOA [31]	963.099	3.3141	0.7678
CEO [19]	800.9771	9.0598	1.17	PSO [35]	954.3483	3.318	
DE [34]	801.23	9.22		DE [42]	968.23	3.38	
GAMS [35]	801.5198	9.4		BHBO [43]	932.8176	3.5035	0.7993
IPSO [36]	801.978	9.4282		FPA [26]	967.1138	3.5661	0.3893
MSLFA [3]	802.287	9.6991					
ACO [37]	802.578	9.851					
FPA [26]	802.7983	9.5406	0.3679				
I-NSGA-II [38]	803.1292	9.6091	0.1212				
IWO [39]	807.474	10.078					

TABLE 4. Comparison of OPF solutions for case 1.

TABLE 5. Comparison of OPF solutions for case 2.

proposed algorithm is transformed into a single objective function by using the weighted sum method of multi-objective functions in OPF solutions. These methods have

been tested on power systems with IEEE 30 bus and IEEE 57 bus. The presented WTLBO algorithm shows that in each case comparison, it provides a better solution for the OPF problem compared to both the original TLBO algorithm and other algorithms in the literature. However, it has been determined that WTLBO has less convergence in objective function solutions than TLBO. The proposed

Algorithm	$F_{cost}$ (\$/hr)	$P_{loss}$ (MW)	$V_D$ (p.u)
WTLBO	891.1000	4.9911	0.0933
TLBO	814.4700	8.3236	0.1077
BBO [44]	805.7582	10.18	0.0951
DSA [45]	803.8274		0.0977
MSA [26]	803.3125	9.7206	0.1084
AGA [46]	805.8096	10.6097	0.1207
BHBO [43]	804.5975	9.5778	0.1262
DE [47]	805.2619		0.1357

TABLE 6. Comparison of OPF solutions for case 3.

WTLBO algorithm shows that it is efficient and high-quality for OPF solutions. For example, it is seen that the WTLBO algorithm reduces the active power losses in the IEEE 30 busbar power system by about 3% compared to

Algorithm	$F_{cost}$ (\$/hr)	$P_{loss}$ (MW)	$V_D$ (p.u)
TLBO	853.5300	4.5553	0.7207
WTLBO	858.8800	4.7718	0.9026
FPA [26]	855.2706	4.7981	1.0143
MGOA [31]	858.4536	4.4435	0.826
MFO [26]	858.5812	4.5772	0.89944
PSO [31]	858.6638	4.5724	0.9225
MSA [26]	859.1915	4.5404	0.92852
GA [31]	859.5349	4.5651	0.8689
MPSO [26]	859.5841	4.5409	0.94718
CEO [19]	860.076	4.483	1.1703
GOA [31]	861.8316	4.6645	0.7143
MDE [26]	868.7138	4.3891	0.87816

TABLE 8. Comparison of OPF solutions for case 4.

	Limits		Case 4		Case 5		Case 6	
	Min	Max	TLBO	WTLBO	TLBO	WTLBO	TLBO	WTLBO
$P_{g1}$ (MW)	50	250	103.1342	102.9321	172.9091	176.01	94.6771	100.4851
$P_{g2}$ (MW)	20	80	58.115	54.619	49.505	48.667	57.144	60.658
$P_{g5}$ (MW)	15	50	34.457	37.847	21.991	21.511	38.810	38.969
$P_{g8}$ (MW)	10	35	34.667	35.000	18.117	21.604	34.985	34.973
$P_{g11}$ (MW)	10	30	30.000	29.999	13.967	13.082	29.999	29.896
$P_{g13}$ (MW)	12	40	27.798	27.558	16.591	12.261	26.772	28.901
$V_{g1}$ (p.u)	0.95	1.1	1.068	1.069	1.048	1.051	1.034	1.037
$V_{g2}$ (p.u)	0.95	1.1	1.056	1.059	1.027	1.028	1.021	1.025
$V_{g5}$ (p.u)	0.95	1.1	1.033	1.034	1.016	1.008	0.996	1.002
$V_{g8}$ (p.u)	0.95	1.1	1.041	1.043	0.994	1.005	1.004	1.006
$V_{g11}$ (p.u)	0.95	1.1	1.067	1.074	1.094	1.029	1.058	1.023
$V_{g13}$ (p.u)	0.95	1.1	1.059	1.047	0.995	0.989	1.015	1.029
$T_{6-9}$ (p.u)	0.9	1.1	0.995	1.037	1.081	1.043	1.016	1.046
$T_{6-10}$ (p.u)	0.9	1.1	0.986	0.930	0.914	0.900	0.950	0.909
$T_{4-12}$ (p.u)	0.9	1.1	1.000	0.982	0.941	0.938	0.985	1.021
$T_{28-27}$ (p.u)	0.9	1.1	0.996	0.977	0.960	0.960	0.957	0.958
$Q_{C10}$ (MVar)	0	0.05	0.225	3.775	2.047	4.145	1.521	3.437
$Q_{C12}$ (MVar)	0	0.05	3.105	4.992	2.044	0.965	2.064	4.921
$Q_{C15}$ (MVar)	0	0.05	3.830	4.425	1.215	3.396	4.269	2.774
$Q_{C17}$ (MVar)	0	0.05	4.283	2.524	2.505	0.370	4.220	3.043
$Q_{C20}$ (MVar)	0	0.05	2.551	4.353	5.000	4.999	3.847	5.000
$Q_{C21}$ (MVar)	0	0.05	3.098	5.000	2.058	4.738	4.025	5.000
$Q_{C23}$ (MVar)	0	0.05	3.842	3.142	4.771	4.982	3.177	2.795
$Q_{C24}$ (MVar)	0	0.05	4.924	4.956	4.980	4.993	3.271	4.846
$Q_{C29}$ (MVar)	0	0.05	3.960	2.625	2.152	1.529	2.131	1.285
$F_{cost}$ (\$/hr)			858.880	853.530	804.720	803.410	870.870	863.570
$P_{loss}$ (MW)			4.772	4.555	9.679	9.735	4.796	4.674
$V_D$ (p.u)			0.7207	0.9026	0.1218	0.106	0.1561	0.1443
Obj. Fun.			1044.39	1041.13	816.89	814.41	1073.42	1069.84
CPU time (s)			40.07	35.03	34.74	40.84	34.19	33.88

TABLE 7. OPF test results for TLBO and WTLBO algorithms in cases 4–6 (IEEE 30 bus).

Algorithm	$F_{cost}$ (\$/hr)	$P_{loss}$ (MW)	$V_D$ (p.u)	Bus No.	Cost coefficients		
					$a$	$b$	$c$
WTLBO	803.4100	9.7352	0.1060				
TLBO	804.7200	9.6786	0.1218				
GOA [31]	803.4488	9.7749	0.1709	1	0	20	0.00375
PSO [31]	803.6227	9.9354	0.111	2	0	17.5	0.0175
FPA [26]	803.6638	9.9252	0.13659	3	0	30	0.025
GA [31]	803.9156	9.6615	0.1257	6	0	20	0.00375
MPSO [26]	803.9787	9.9242	0.1202	8	0	10	0.0625
CEO [19]	804.6734	10.0675	0.1102	9	0	17.5	0.0195
BBO [44]	804.9982	9.95	0.102	12	0	32.5	0.00834
DE [47]	805.2619	10.4412	0.1357				

TABLE 10. Generator cost coefficients (IEEE 57 bus).

TABLE 9. Comparison of OPF solutions for case 5.

	Case 7		Case 8		Case 9	
	TLBO	WTLBO	TLBO	WTLBO	TLBO	WTLBO
$P_{g1}$	161.094	151.747	194.019	167.169	163.568	186.182
$P_{g2}$	42.234	62.793	71.889	74.207	69.133	100
$P_{g3}$	75.063	44.743	82.889	75.511	82.271	46.476
$P_{g6}$	49.058	95.899	54.809	99.748	47.139	80.253
$P_{g8}$	480.036	507.139	423.645	419.291	425.134	445.092
$P_{g9}$	58.05	54.631	64.441	45.255	83.186	80.131
$P_{g12}$	406.428	369.09	389.511	399.733	404.764	334.228
$V_{g1}$	0.972	0.969	0.977	0.936	0.963	0.99
$V_{g2}$	0.958	0.965	0.96	0.936	0.944	0.994
$V_{g3}$	0.979	0.969	0.966	0.982	0.95	1.001
$V_{g6}$	0.978	0.985	0.963	1.03	0.966	1.032
$V_{g8}$	1.006	0.983	0.977	1.082	0.993	1.044
$V_{g9}$	0.975	0.957	0.963	1.028	0.966	1.014
$V_{g12}$	0.964	0.973	1.003	1.015	0.992	1.033
$T_{19}$	0.93	0.957	0.938	1.014	0.969	1.05
$T_{20}$	0.97	0.931	0.936	1.049	0.98	1.048
$T_{31}$	0.997	1.035	0.953	0.992	0.958	0.984
$T_{35}$	0.951	0.989	1.017	0.988	0.997	1.016
$T_{36}$	0.979	0.923	0.953	0.985	1.004	1.015
$T_{37}$	1.086	0.983	0.909	1.024	0.968	0.991
$T_{41}$	0.916	0.917	0.959	0.996	0.934	0.969
$T_{46}$	0.968	0.992	0.978	1.014	0.979	0.97
$T_{54}$	0.992	0.917	0.956	0.928	0.975	0.953
$T_{58}$	0.927	0.9	0.922	0.952	0.92	1.016
$T_{59}$	0.935	0.917	0.913	0.969	0.912	0.963
$T_{65}$	0.95	0.927	0.959	0.981	0.931	1.004
$T_{66}$	0.907	0.911	0.927	0.933	0.937	0.986
$T_{71}$	0.934	0.917	0.932	0.94	0.93	0.955
$T_{73}$	0.92	0.951	1.008	1.001	0.959	0.979
$T_{76}$	0.902	0.953	0.969	1.023	0.934	1.059
$T_{80}$	0.991	0.988	0.953	1.021	0.938	0.976
$Q_{C18}$	27.522	10.979	6.582	17.85	22.778	24.975
$Q_{C25}$	7.642	7.875	13.406	14.582	14.957	13.895
$Q_{C53}$	24.022	12.031	14.268	12.06	15.434	12.21
$F_{cost}$	41150.1	39651.85	42321.6	41685.41	42084.43	41963.39
$P_{loss}$	21.163	35.242	30.403	30.114	24.395	21.562
$V_D$	1.129	1.389	1.171	0.754	1.179	1.129

TABLE 11. OPF test results for TLBO and WTLBO algorithms in cases 7–9 (IEEE 57 bus).

Case 7		Case 8		
Algorithm	$F_{\text{cost}}$	Algorithm	$F_{\text{cost}}$	$V_D$
WTLBO	39651.8500	WTLBO	41685.4100	0.7536
TLBO	41150.1000	TLBO	42321.6000	1.1707
IAOA [50]	40911.0000	DSA [51]	41699.4000	0.7620
CKHA [52]	41660.4657	GA [53]	41700.4162	0.8052
IMFO [53]	41667.1497	GOA [49]	41715.1396	0.8260
MGOA [31]	41671.0980	MFO [53]	41719.8471	0.7551
PSO [53]	41673.6339	MALO [54]	41723.1080	0.9382
MSA [26]	41673.7231	EADDE [49]	42051.4400	0.7882
GA [53]	41676.4786	BHOA [50]	48321.0000	1.3805
MPSO [26]	41678.6762	GA [50]	48436.0000	1.3405
MFO [53]	41679.3749			
GOA [31]	41679.6792			
KHA [52]	41684.5567			
GA [31]	41684.5640			

TABLE 12. Comparison of OPF solutions for cases 7–8.

other algorithms examined in the literature. In addition, it has been calculated that the proposed WTLBO algorithm in terms of execution times reaches a solution in a shorter time than the original TLBO algorithm. On the other hand, the proposed algorithm can be developed in future studies for OPF solutions in power systems where renewable energy sources and flexible alternating current transmission systems (FACTS) are integrated.

## DISCLOSURE STATEMENT

No potential conflict of interest was reported by the author(s).

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