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Research Article

Some New Soliton Solutions of Time Fractional Resonant Davey–Stewartson Equations

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In this study, the Bernoulli subequation method (BS-EM) is applied to investigate the traveling wave solutions of the (2 + 1)-dimensional resonant Davey–Stewartson system. By employing a wave transformation, the system's nonlinear partial differential equation is reduced to a nonlinear ordinary differential equation, which is then solved using the BS-EM approach. As a result, several new traveling wave solutions, which have not been previously reported in the literature, have been successfully obtained. These solutions provide new insights into the physical dynamics of the system and also satisfy the (2 + 1)-dimensional time–fractional resonant Davey–Stewartson equation. Furthermore, the analytical and graphical analyses of the obtained solutions have been carried out, and the wave profiles have been examined under various parameter conditions. All computations and graphical visualizations in this study were performed using the Wolfram Mathematica 12 software.

Keywords: resonant Davey–Stewartson equation; the Bernoulli subequation method (BS-EM); the fractional Riemann–Liouville derivative

1. Introduction

Many fields of study, such as physics, mechanics, and material science, use nonlinear evolution equations (NLEEs). The search for solitary wave solutions plays a very important fundamental role in the NLEEs since they describe numerous features of our real-life situations. New research shows the importance and status of the development of soliton types in differential systems [1, 2]. Numerous methods for obtaining the analytical solutions of different types of partial differential equations (PDEs) have been explored. PDEs attract such as simplified Hirota's method [3], $(m + G'/G) -$ expansion method [4], the Bernoulli sub-ODE method [5, 6], symbolic computational method [5–8], multiple Exp-function method [9, 10], the generalized exponential rational function method [11, 12], and many other methods [13–15].

In the literature, researchers have utilized a variety of approaches to find specific types of solutions, such as accurate, numerical solutions. Zhao et al. studied the Davey–Stewartson equation and obtained one and two soliton solutions [16], Tang et al. have applied with the help of the Painlevé test to DS equations [17–19], and by many researches, other methods have been applied to investigate which soliton wave solutions of the resonant Davey–Stewartson equation system will yield [20–23].

This paper has five parts in its most general form: In the second section, general definitions about the overview of conformable fractional derivatives are given. The third sec-

tion is the application steps of the Bernoulli subequation method. In Section 4, resonant Davey–Stewartson equations [24] and its application will be introduced and expressed in more detail. In Section 5, evaluations were written with the result section, and in the last section, discussion was given place.

2. Overview of Conformable Fractional Derivatives

Here, some basic definitions, properties, and theorems about conformable fractional derivatives are discussed [25–27]. Recently, new fractional calculus operators like as the Caputo Fabrizio, the Riemann Liouville, and the Beta derivative have been used to study the characteristics of fractional PDEs. These kinds of models are crucial to solving difficult models in the applied sciences and engineering domains. The conformable fractional derivative is one of these models that helps us understand the nature of the model.

Definition 1. ($R-L$ and $C-F$) A real function $z(t)$, $t>0$ is said to be in space C_ν; $\nu \in \mathbb{R}$ if there exists a real number $k(>\nu)$, such that $u(t)=t^k u_1$, where $u_1(t)\in C[0,\infty)$ which is also in space C_ν^n if and only if $u^{(n)}\in C_\nu, n\in \mathbb{N}.u.$

$$ {}^{RL}D_{0,t}^{\alpha}u(t)=\frac{1}{\Gamma(n-\alpha)}\frac{d^n}{\mathrm{d}t^n}\int_0^t (t-\xi)^{n-\alpha-1}u(\xi)d\xi \tag{1}$$

$$ {}^{C}D_{0,t}^{\alpha}u(t)=\frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\xi)^{n-\alpha-1}\frac{d^n}{\mathrm{d}t^n}u(\xi)d\xi \tag{2}$$
$$ n-1<\alpha\le n$$

Definition 2. (Beta-derivative) Take f be a function such that

$$ f:[\alpha,\infty)\longrightarrow \mathbb{R}$$
$$ {}_0^A D^{\beta}(f(x))=\lim_{\varepsilon\longrightarrow 0}\frac{f\left(x+\xi(x+1/\Gamma(\beta))^{1-\beta}\right)-f(x)}{\varepsilon} \tag{3}$$
$$ x\ge\alpha,\beta\in(0,1]$$

Definition 3. Let $g:(0,\infty)\longrightarrow\mathbb{R}$; then, the conformable fraction derivative of g of order α is defined as

$$ T_\alpha(g)(t)=\lim_{\epsilon\longrightarrow 0}\frac{g\left(t+\epsilon t^{1-\epsilon}\right)}{\epsilon}, t>0, 0<\alpha<1 \tag{4}$$

Here, some basic properties of conformable fractional derivatives [24] are presented.

$$ T_\alpha(bg+ch)=bT_\alpha(g)+cT_\alpha(h), b,c\in\mathbb{R} \tag{5}$$

$$ T_\alpha(t^x)=xt^{x-\alpha}, x\in\mathbb{R} \tag{6}$$

$$ T_\alpha(gh)=gT_\alpha(h)+hT_\alpha(g) \tag{7}$$

$$ T_\alpha\left(\frac{g}{h}\right)=\frac{hT_\alpha(g)-gT_\alpha(h)}{h^2} \tag{8}$$

$$ \text{If } g \text{ is differentiable, then } T_\alpha(g)(t)=t^{1-\alpha}\frac{\mathrm{dg}}{\mathrm{dt}} \tag{9}$$

The chain rule and other significant features are obeyed by the conformable differential operator.

Theorem 1. *Assume that $g_2(x,t)$ is differentiable and well-defined throughout the range of $g_1(x,t)$, and that $g_1(x,t)$ is a α-conformable differentiable function*

$$ T_t^{\alpha}(g_1(t)\circ g_2(t))=1^{1-\alpha}\frac{dg_2}{dt}\frac{d}{dt}(g_1(g_2(t))) \tag{10}$$

3. General Forms of the Bernoulli Subequation Method

In this step, we have expressed the implementation steps of BSEM:

Step 1: Let us consider the equation, the most general form of which is given as Equation (11), as follows:

$$ P(u_x,u_t,u_{xt},u_{xx},\cdots)=0 \tag{11}$$

$$ \phi(x,y,t)=e^{\theta}U(\eta),\quad \varphi(x,y,t)=V(\eta)$$
$$ \theta=i\mu\left(x+y-c\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right),\quad \eta=\alpha x+\beta y+\gamma\frac{t^{\alpha}}{\Gamma(\alpha+1)} \tag{12}$$

where $\gamma\neq 0$. With the help of the wave transform given in Equation (12), the ordinary differential equation given in Equation (3) can be obtained.

$$ N\left(U,U',U'',\cdots\right)=0 \tag{13}$$

Step 2: In the case of Equation (13), the trial equation of solution can be stated as follows:

$$ U(\eta)=\sum_{i=1}^{n}a_iF^i=a_0+a_1F+a_2F^2+\cdots+a_nF^n \tag{14}$$

$$ F'=bF+dF^M, b\neq 0, d\neq 0, M\in\mathbb{R}-\{0,1,2\} \tag{15}$$

Here, $F(\eta)$ symbolize the Bernoulli differential polynomial. Equation (14) and Equation (15) are emplaced into Equation (13), and we get a polynomial $\Omega(F(\eta))$ of $F(\eta)$ as follows:

$$ \Omega(F(\eta))=\rho_sF(\eta)^s+\cdots+\rho_1F(\eta)+\rho_0=0 \tag{16}$$

Due to the principle of balance, we find the relation between n and M.

Step 3: Assuming that the coefficients of $\Omega(F(\eta))$ all be zero, the resulting is an algebraic system of equations:

$$ \rho_1=0, i=0,\cdots,s \tag{17}$$

Solving Equation (17), we evaluate the values $a_0,\cdots,a_n$.

Step 4: Solving the Bernoulli differential equation for Equation (14), we get two solutions due to b and d:

$$F(\eta) = \left[\frac{-d}{b} + \frac{E}{e^{b(M-1)\eta}}\right]^{1/1-M}, b \neq d \tag{18}$$

$$F(\eta) = \left[\frac{(E-1) + (E+1)\tan\left(b(1-M)\eta/2\right)}{1 - \tan\left(b(1-M)\eta/2\right)}\right]^{1/1-M} \tag{19}$$

$b = d, E \in \mathbb{R}$, where $E \neq 0$ is integration constant. Utilizing Wolfram Mathematica software, we obtain the solutions to Equation (13). Giving the proper parameter values to the obtained wave solutions allows three-dimensional wave graphs to be drawn.

4. The $(2+1)$-Dimensional Davey–Stewartson Equations and Implement of BSE Method

In this part of the work, the $(2+1)$-dimensional Davey–Stewartson equations [24] are considered

$$u_t{}^{\alpha} + \sigma^2 u_{xx} + u_{yy} - 2\sigma^2\left(\frac{|u|_{xx}}{|u|} + \sigma^2\frac{|u|_{yy}}{|u|}\right)u - vu + \omega|u|^2 u = 0 \tag{20}$$

$$v_{xx} - \sigma^2 v_{yy} - 2\omega\left(|u|^2\right)_{xx} = 0 \tag{21}$$

The amplitude of a surface wave packet is represented by $u(x,y)$, the velocity potential of the mean flow interacting with the surface wave is represented by $v(x,y)$, and the subscripts indicate the appropriate derivatives. In case of ignoring $y-$ dimension, the RDS system is modified to the resonant nonlinear Schrödinger equation as,

$$u_t{}^{\alpha} + \sigma^2 u_{xx} - 2\sigma^2\left(\frac{|u|_{xx}}{|u|}\right)u - \omega|u|^2 u = 0$$

First of all, the $(2+1)$-dimensional imaginary Davey–Stewartson equations are converted into a system of NLODE to study and analyze its exact solutions.

Assuming the wave transformation, as

$$u(x,y,t) = e^{i}U(\xi), v(x,y,t) = V(\xi)$$
$$\xi = \mu\left(x + y - \frac{\eta t^{\alpha}}{\Gamma(\alpha+1)}\right), \theta = \omega x + \beta y + \frac{\gamma t^{\alpha}}{\Gamma(\alpha+1)} \tag{22}$$

Now applying Equation (22) on the $(2+1)$-dimensional Davey–Stewartson Equations (20) and (21), the following systems are obtained:

$$\mu^2\left(1 - \sigma^2 - 2\sigma^4\right)U'' + \mu\left(\eta - 2\omega\sigma^2 - 2\beta\right)U - UV + \omega U^3 = 0 \tag{23}$$

$$\mu\left(\eta - 2\omega\sigma^2 - 2\lambda\right)iU' = 0 \tag{24}$$

$$-\mu^2\left(1 - \sigma^2\right)V'' + 4\alpha\mu^2\left(UU'' + U'^2\right) = 0 \tag{25}$$

We get Equation (26) by integrating Equation (25) twice with respect to and equaling the integration constant to zero.

$$V = \frac{2\omega}{1 - \sigma^2}U^2 \tag{26}$$

If the Equation (24) is taken as

$$\eta = 2\omega\sigma^2 + 2\lambda \tag{27}$$

Equation (26) is substituted into Equation (23), we get

$$-\mu^2\left(1 - \sigma^2\right)\left(1 - \sigma^2 - 2\sigma^4\right)U'' - \left(1 - \sigma^2\right)\left(\gamma + \alpha^2\sigma^2 + \beta^2\right)U - \left(1 + \sigma^2\right)U^3 = 0 \tag{28}$$

Using the balancing term formula between the terms U'' and U^3, relationship between n and M is examined.

$$M = n + 1 \tag{29}$$

Family 1. When $n = 2$ and $M = 3$ are substituted into Equation (14), the following results are obtained:

$$U = a_0 + a_1 F + a_2 F^2 \tag{30}$$

$$U' = a_1 bF + a_1 dF^3 + 2a_2 bF^2 + 2a_2 F^4 \tag{31}$$

$$U'' = a_1 b^2 F + 4a_2 b^2 F^2 + 4a_1 bdF^3 + 12a_2 bdF^4 + 3a_1 d^2 F^5 + 8a_2 d^2 F^6 \tag{32}$$

where $a_2 \neq 0,\ b \neq 0,\ d \neq 0$.Equations ((30))–((32)) are put into equation to obtain a system of algebraic Equation (28). The following results were obtained by solving this system with Wolfram Mathematica.

Case 1. Suppose that

$$b \neq d, a_0 = -\frac{b\mu\sqrt{2-6\sigma^2+4\sigma^4}}{\sqrt{\omega}}, a_1 = 0, a_2 = -\frac{2d\mu\sqrt{2-6\sigma^2+4\sigma^4}}{\sqrt{\omega}}$$
$$\gamma = -\beta^2 + 2b^2\mu^2\left(-1+\sigma^2+2\sigma^4\right) - \sigma^2\omega^2 \tag{33}$$

we obtain

$$\mathrm{u}_1(x,y,t) = \mathrm{e}^{\mathrm{i}(y\beta+x\omega+\gamma t^\alpha/\Gamma(1+\alpha))}\left(-\frac{a_0}{\sqrt{\omega}} - a_1\left(\left(-\frac{d}{b} + \mathrm{ce}^{-2b\mu\left(x+y-t^\alpha\left(2\beta+2\sigma^2\omega\right)/\Gamma(1+\alpha)\right)}\right)\sqrt{\omega}\right)^{-1}\right) \tag{34}$$

$$v_1(x,y,t) = \frac{2\left(b\mu\sqrt{2-6\sigma^2+4\sigma^4}/\sqrt{\omega} + 2d\mu\sqrt{2-6\sigma^2+4\sigma^4}/\left(-d/b + \mathrm{ce}^{-2b\mu\left(x+y-t^\alpha\left(2\beta+2\sigma^2\omega\right)/\Gamma(1+\alpha)\right)}\right)\sqrt{\omega}\right)^2\omega}{1-\sigma^2} \tag{35}$$

Equations (34) and (35) represent dark solutions as seen in Figure 1.

Case 2. Suppose that

$$b \neq d, \gamma = -\beta^2 + \omega^2, \sigma = -\mathrm{i} \tag{36}$$

we get

$$\mathrm{u}_2(x,y,t) = \mathrm{e}^{\mathrm{i}(y\beta+x\omega+\gamma t^\alpha/\Gamma(1+\alpha))} \times \left(a_0 + \frac{a_2}{-d/b + \mathrm{ce}^{-2b\mu(x+y-\eta t^\alpha/\Gamma(1+\alpha))}} + \frac{a_1}{\sqrt{-d/b + \mathrm{ce}^{-2b\mu(x+y-\eta t^\alpha/\Gamma(1+\alpha))}}}\right) \tag{37}$$

$$v_2(x,y,t) = \left(a_0 + \frac{a_2}{-d/b + \mathrm{ce}^{-2b\mu(x+y-\eta\, t^\alpha/\Gamma(1+\alpha))}} + \frac{a_1}{\sqrt{-d/b + \mathrm{ce}^{-2b\mu(x+y-\eta\, t^\alpha/\Gamma(1+\alpha))}}}\right)^2\omega \tag{38}$$

These solutions are kink solutions as shown in Figure 2.

Case 3. Suppose that $b \neq d$

$$b = \frac{\sqrt{\beta^2+\gamma+\sigma^2\omega^2}}{\sqrt{2}\sqrt{\mu^2\left(-1+\sigma^2+2\sigma^4\right)}}, d = -\frac{\mathrm{i}a_2\sqrt{\omega}\sqrt{\beta^2+\gamma+\sigma^2\omega^2}}{2\mu\sqrt{-2+4\sigma^2}\sqrt{-\left(-1+\sigma^2\right)\left(\beta^2+\gamma+\sigma^2\omega^2\right)}} \tag{39}$$

$$a_0 = \frac{\mathrm{i}\mu\sqrt{-1+2\sigma^2}\sqrt{-\left(-1+\sigma^2\right)\left(\beta^2+\gamma+\sigma^2\omega^2\right)}}{\sqrt{\mu^2\left(-1+\sigma^2+2\sigma^4\right)}\sqrt{\omega}}, a_1 = 0$$

we get

$$\mathrm{u}_3(x,y,t) = \mathrm{e}^{\mathrm{i}(y\beta+x\omega+t^\alpha\gamma/\Gamma(1+\alpha))}\left(\frac{a2}{-d/b + \mathrm{ce}^{-2b\mu(x+y-t^\alpha\eta/\Gamma(1+\alpha))}} + \frac{\mathrm{i}\mu\sqrt{-1+2\sigma^2}\sqrt{\left(1-\sigma^2\right)\left(\beta^2+\gamma+\sigma^2\omega^2\right)}}{\sqrt{\mu^2\left(-1+\sigma^2+2\sigma^4\right)}\sqrt{\omega}}\right), \tag{40}$$

$$v_3(x,y,t) = \frac{2\omega\left(a2/-d/b + \mathrm{ce}^{-2b\mu(x+y-t^\alpha\eta/\Gamma(1+\alpha))} + \mathrm{i}\mu\sqrt{-1+2\sigma^2}\sqrt{\left(1-\sigma^2\right)\left(\beta^2+\gamma+\sigma^2\omega^2\right)}/\sqrt{\mu^2\left(-1+\sigma^2+2\sigma^4\right)}\sqrt{\omega}\right)^2}{1-\sigma^2} \tag{41}$$

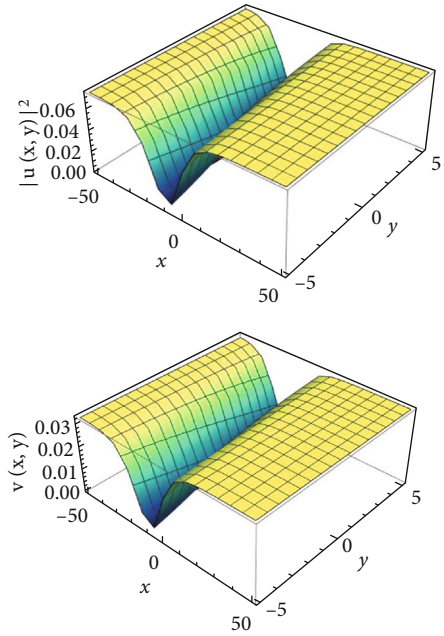

Figure 1: The 3-dimensional figures are drawn for solutions of Equations (34) and (35), when $c = 0.1, b = 1, d = -2, \mu = 0.1, \eta = 0.3, \omega = 0.2, \beta = 0.1, \alpha = 0.2, \sigma = 0.3, t = 1/2$.

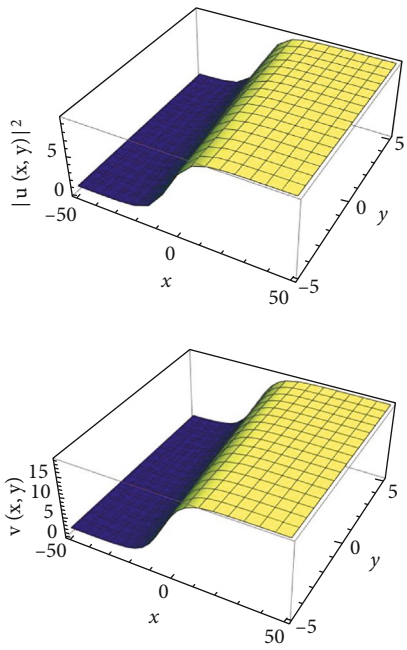

Figure 2: The 3-dimensional figures are drawn for solutions of Equations (37) and (38), when $a_0 = 1, a_1 = 0.1, a_2 = 2, c = 0.2, b = 1, d = -1, \mu = 0.1, \omega = 2, \beta = 0.1, \alpha = 0.2, \sigma = 0.3, t = 1/2$.

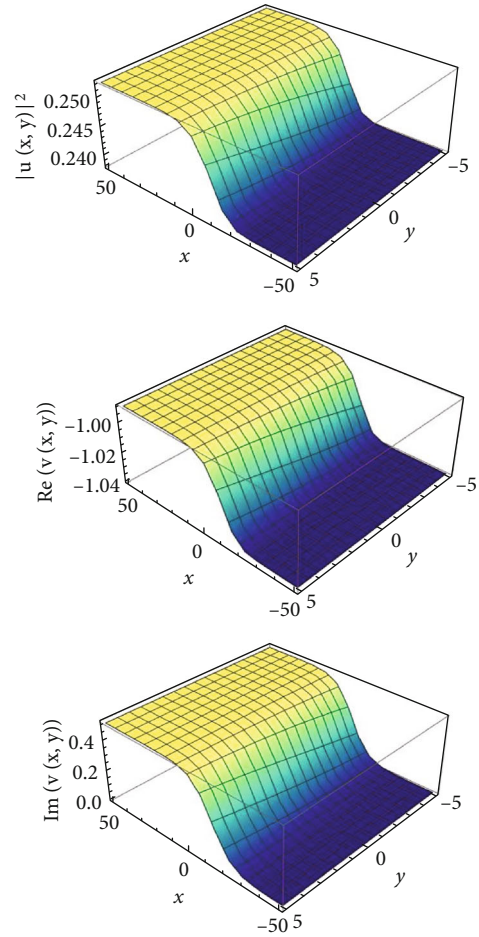

Figure 3: The 3-dimensional figures are drawn for solutions of Equations (40) and (41) when $a_2 = 0.12, c = 0.2, b = 1, d = -1, \mu = 0.1, \omega = 2, \beta = 0.1, \gamma = 0.2, \alpha = 0.2, \sigma = 0.3, t = 1/2$.

These solutions are kink solutions as presented in Figure 3.

Case 4. Suppose that $b \neq d$.

$$a_0 = -\frac{\mu\sqrt{1-3\sigma^2+2\sigma^4}\sqrt{\beta^2+\gamma+\sigma^2\omega^2}}{\sqrt{\mu^2(-1+\sigma^2+2\sigma^4)}\sqrt{\omega}}, a_1 = 0$$
$$a_2 = -\frac{2d\mu\sqrt{2-6\sigma^2+4\sigma^4}}{\sqrt{\omega}}, b = \frac{\sqrt{\beta^2+\gamma+\sigma^2\omega^2}}{\sqrt{2}\sqrt{\mu^2(-1+\sigma^2+2\sigma^4)}} \tag{42}$$

we get

$$u_4(x,y,t) = e^{i(y\beta+x\omega+t^\alpha\gamma/\Gamma(1+\alpha))}\left(-2d\mu\sqrt{2-6\sigma^2+4\sigma^4}\right.$$
$$\left.\cdot\left(\left(-\frac{d}{b}+ce^{-2b\mu(x+y-\eta t^\alpha/\Gamma(1+\alpha))}\right)\sqrt{\omega}\right)^{-1}-a_0\right) \tag{43}$$

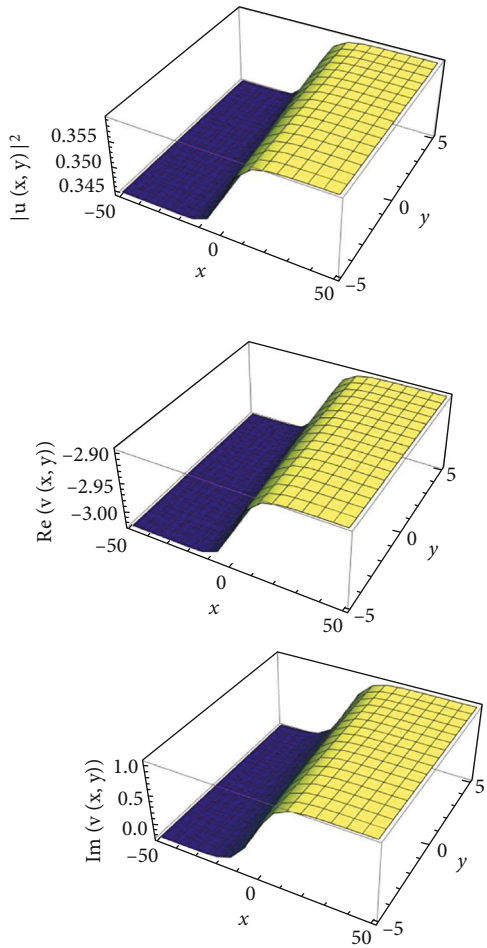

FIGURE 4: The 3-dimensional figures are drawn for solutions of Equation (43) and (44), when $a2 = 0.12$; $c = 0.2$; $b = 1$; $d = -1$; $\mu = 0.1$; $\eta = 0.3$; $\omega = 4$; $\beta = 0.1$; $\gamma = 0.2$; $\alpha = 0.2$; $\sigma = 0.3$; $t = 1/2$.

$$v_4(x,y,t) = \frac{2\omega\left(-2d\mu\sqrt{2-6\sigma^2+4\sigma^4}\left(\left(-d/b + ce^{-2b\mu(x+y-t^\alpha\eta/\Gamma([1+\alpha]))}\right)\sqrt{\omega}\right)^{-1} - a_0\right)^2}{1-\sigma^2} \tag{44}$$

As shown in Figure 4, these solutions are kink solutions.

Case 5. Suppose that $b \neq d$.

$$a_1 = 0, b = \frac{a_0\sqrt{\omega}}{\sqrt{2}\sqrt{\mu^2(1-3\sigma^2+2\sigma^4)}}, d = \frac{a_2\sqrt{\omega}}{2\sqrt{2}\sqrt{\mu^2(1-3\sigma^2+2\sigma^4)}}$$
$$\gamma = -\beta^2 + \omega\left(\frac{a0^2\left(1+\sigma^2\right)}{-1+\sigma^2} - \sigma^2\omega\right) \tag{45}$$

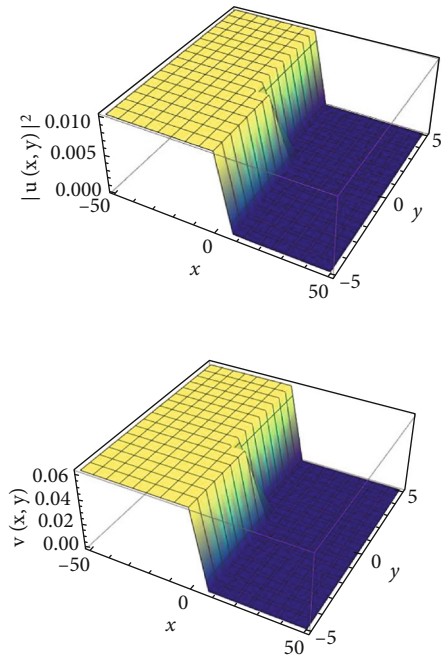

FIGURE 5: The 3-dimensional figures are drawn for solutions of Equations (46) and (47), when $a_0 = 0.1, a_2 = -0.2, c = 0.2, b = 1, d = -2$, $\mu = 6, \omega = 3, \beta = 2$, $\alpha = 0.2, \sigma = 0.2, t = 1/2$.

$$u_5(x,y,t) = e^{i(y\beta + x\omega + t^\alpha\gamma/\Gamma(1+\alpha))} \times \left(a0 + a2\left(-\frac{d}{b} + ce^{-2b\mu(x+y-t^\alpha\eta/\Gamma(1+\alpha))}\right)^{-1}\right) \tag{46}$$

$$v_5(x,y,t) = \frac{2\left(a_0 + a_2\left(-d/b + ce^{-2b\mu(x+y-t^\alpha\eta/\Gamma(1+\alpha))}\right)^{-1}\right)^2\omega}{1-\sigma^2} \tag{47}$$

As shown in Figure 5, these solutions are kink solutions.

Case 6. Suppose that $b \neq d$.

$$a_0 = \frac{b\mu\sqrt{2-6\sigma^2+4\sigma^4}}{\sqrt{\omega}}, a_1 = 0, a_2 = \frac{2d\mu\sqrt{2-6\sigma^2+4\sigma^4}}{\sqrt{\omega}},$$
$$\gamma = -\beta^2 + 2b^2\mu^2\left(-1+\sigma^2+2\sigma^4\right) - \sigma^2\omega^2 \tag{48}$$

we get

$$u_6(x,y,t) = e^{i(y\beta + x\omega + t^\alpha\gamma/\Gamma(1+\alpha))}\left(\frac{b\mu\sqrt{2-6\sigma^2+4\sigma^4}}{\sqrt{\omega}} + 2d\mu\sqrt{2-6\sigma^2+4\sigma^4} \cdot \left(\left(-\frac{d}{b} + c\,e^{-2b\mu(x+y-t^\alpha\eta/\Gamma(1+\alpha))}\right)\sqrt{\omega}\right)\right) \tag{49}$$

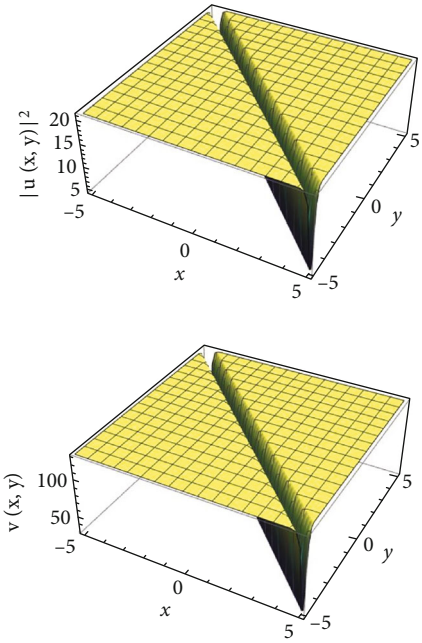

Figure 6: The 3-dimensional figures are drawn for solutions of Equations (49) and (50), when $a_0 = 0.1, a_2 = -0.2, c = 0.2, b = 1, d = -2, \mu = 6, \eta = 0.3, \omega = 3, \beta = 2, \alpha = 0.2, \sigma = 0.2, t = 1/2$.

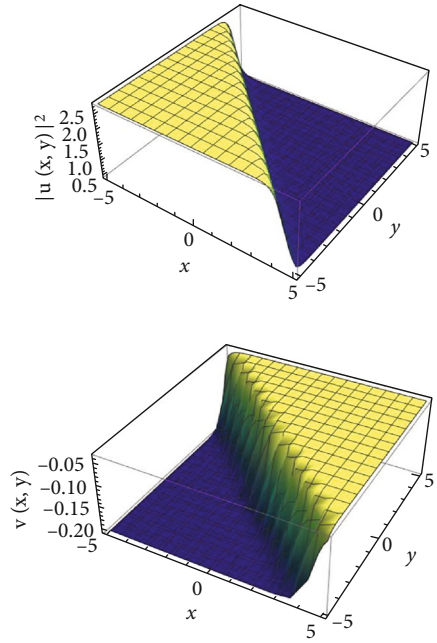

Figure 7: The 3-dimensional figures are drawn for solutions of Equations (52) and (53), when $a_2 = -2, c = 0.1, b = 1, d = -2, \mu = 2, \omega = 0.3, \beta = 0.2, \gamma = 0.2, \alpha = 0.2, \sigma = 3, t = 1/2$.

$$v_6(x,y,t) = \frac{2\left(b\mu\sqrt{2-6\sigma^2+4\sigma^4}/\sqrt{\omega} + 2d\mu\sqrt{2-6\sigma^2+4\sigma^4}/\left(-d/b + ce^{-2b\mu(x+y-t^\alpha\eta/\Gamma(\alpha+1))}\right)\sqrt{\omega}\right)^2\omega}{1-\sigma^2} \tag{50}$$

As shown in Figure 6, solutions presented by Equations (49) and (50) are dark solutions.

Case 7. Suppose that $b \neq d$.

$$a_0 = \frac{\sqrt{\mu^2(1-3\sigma^2+2\sigma^4)}\sqrt{\beta^2+\gamma+\sigma^2\omega^2}}{\sqrt{\mu^2(-1+\sigma^2+2\sigma^4)}\sqrt{\omega}}, a_1 = 0, b = -\frac{\sqrt{\beta^2+\gamma+\sigma^2\omega^2}}{\sqrt{2}\sqrt{\mu^2(-1+\sigma^2+2\sigma^4)}}$$
$$d = -\frac{a_2\sqrt{\omega}}{2\sqrt{2}\sqrt{\mu^2(1-3\sigma^2+2\sigma^4)}} \tag{51}$$

we get

$$u_7(x,y,t) = e^{i(y\beta+x\omega+t^\alpha\gamma/\Gamma(1+\alpha))}\left(\frac{a_2}{-d/b + ce^{-2b\mu(x+y-t^\alpha\eta/\Gamma(1+\alpha))}} + a_0\right) \tag{52}$$

$$v_7(x,y,t) = \frac{2\omega\left(a_2\left(-d/b + ce^{-2b\mu(x+y-t^\alpha\eta/\Gamma(1+\alpha))}\right)^{-1} + a_0\right)^2}{1-\sigma^2} \tag{53}$$

As shown in Figure 7, solutions presented by Equations (52) and (53) are kink solutions.

5. Conclusion

In this investigation, we have studied that in the Resonant DS, equation plays a significant role in the dynamics and stability of fluid movement which in hydrodynamics, surface tension, and other capillary effects. We obtained exponential and complex wave solutions by applying the BSE method to (2 + 1)-dimensional time-fractional resonant Davey–Stewartson system equation. We have expressed by examining the structural parts of these exponential and complex solutions. Using this BSE method gives kink solutions for Equations (37) and (38), Equations (40) and (41), Equations (43) and (44), Equations (46) and (47), and Equation (52) and (53). The dark solution part is given by the Equation ((34)-(35)) and Equation ((49)–(50)). The graphs drawn in the article were re-examined by changing the values taken in the Mathematica program and the solutions were obtained in the same way as the kink-soliton and dark solution graphs. When we changed the given values in the Mathematica program, it gave a kink-soliton solution again. Three-dimensional surface graphics are drawn and supported separately for the real and imaginary parts of all solutions obtained.

6. Discussion

In this study, we use the Bernoulli subequation technique (BS-EM) to study a system, the (2 + 1)-dimensional resonant Davey–Stewartson. For this, we compare the solutions found using the usual (2 + 1)-dimensional resonant Davey–Stewartson system in the prior work with the solutions produced using the BS-EM. Comparing our answers leads us to the conclusion that they are distinct and more general. It provides many solutions when using the BS-EM. Depending on the coefficient values taken, the new solutions are different from those presented in the literature. In case of choosing $b \neq d$, $a_0 = -b\mu\sqrt{2-6\sigma^2+4\sigma^4}/\sqrt{\omega}, a_1 = 0, a_2 = -2d\mu\sqrt{2-6\sigma^2+4\sigma^4}$

$/\sqrt{\omega}, \gamma = -\beta^2 + 2b^2\mu^2(-1 + \sigma^2 + 2\sigma^4) - \sigma^2\omega^2$, value in the system, different kink-soliton solutions emerge. It is possible to see that every answer that was found was unique. Additionally, as shown in Figures 1, 2, 3, 4, 5, 6, and 7, we provide 3D surface profiles and counterplots to help visualize the Resonant DS equation of certain derived solutions. After analysis, it was shown that the constraints that ensure the existence of soliton and kink solutions are connected to the coefficients $\omega, \beta, \gamma, \eta$.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Ethics Statement

Ethical approval is not applicable, because this article does not contain any studies with human or animal subjects.

Conflicts of Interest

The authors declare no conflicts of interest.

Author Contributions

Esin İlhan: conceptualization, methodology investigation, and editing. **Muhammed Yiğider:** conceptualization, methodology investigation. **Ercan Çelik:** visualization, supervision, editing. **Hasan Bulut:** writing–original draft preparation and editing. All authors read and approved the final manuscript.

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