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Strait soft sets and strait rough sets with applications in decision making

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Abstract

In this paper, two new uncertainty modeling concepts, namely strait soft set and strait rough set, between the structures of rough sets and soft sets, which will bring new perspectives to both theoretical and practical aspects, are presented. A reduction method of alternatives is given using strait soft sets. Another convenience arising from the structure of strait soft sets is that they allow the parameters to be combined. Thus, the fusion of parameters is defined and its use in soft set operations is demonstrated. Strait rough sets naturally contain the characteristics of the rough sets and also allow the parameters to be characterized. The strait soft set and strait rough set are supported by many examples and comparisons. In addition, a new decision-making approach based on the strait soft set and strait rough set is proposed and then followed by real-life applications to illustrate the computational processes.

Keywords Soft set · Rough set · Strait soft set · Strait rough set · Decision making

1 Introduction

In 1980s, Pawlak (1982) proposed rough set theory that was to introduce some approximations of sets, to deal with the problems of uncertainty and vagueness. Rough set theory has a fundamental importance in many research areas, especially in artificial intelligence and cognitive sciences where technological development is built today, and further in research areas such as machine learning, intelligent systems, inductive reasoning, pattern recognition, mereology, information discovery, decision making, game theory and expert systems (see Pawlak 2002; Pawlak and Skowron 2007a, b; Sun et al. 2019). Chen and Ziarko (2011) presented some experiments with rough set approach to face recognition. Algebraic structures connected to rough sets were studied by many researchers (Bonikowaski 1995; Iwinski 1987; Pomykala and Pomykala 1998).

In 1999, Molodtsov (1999) proposed a completely novel sophisticated approach called soft set theory for modeling ambiguous or not clearly described information. Since this approach is free from the problem of setting the membership function, this theory can be easily applied to many different fields including the smoothness of functions, operations research, game theory, Riemann integration, Perron integration, measurement theory and probability theory. Some of these applications have already been indicated, using Molodtsov's soft sets. At present, works on soft set theory are progressing rapidly in both theoretical and practical studies. In Ali et al. (2009), Çağman and Enginoğlu (2010b), Maji et al. (2003) and Sezgin and Atagün (2011), the authors introduced several operations and gave some of algebraic properties of soft sets. In 2008, Aktaş and Çağman (2007) compared soft sets to fuzzy sets and rough sets, gave some basic concepts of soft set theory and defined the concept of soft group. In the following years, many authors have studied the soft algebraic structures such as soft semirings (Feng et al. 2008), soft rings (Acar et al. 2010; Aygün and Kamacı 2019, 2021), soft ideals (Kamacı 2020; Sezer 2012; Sezer etal. 2015), soft BCK/BCI/BCH algebras (Jun and Park 2008; Jun et al. 2009; Kazancı et al. 2010), soft Lie algebras (Akram 2013; Akram and Feng 2013), soft near-rings (Sezgin et al. 2011) and soft lattices (Karaaslan et al. 2012; Susanta etal. 2017; Zhan and Xu 2011). Considering the ease of storing

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and manipulating matrices, Çağman and Enginoğlu (2010a) introduced soft matrix, a matrix representation of soft set. They defined soft matrices and some related set-theoretic operations and then constructed a soft max–min decision-making method. Atagün et al. (2018) and Kamacı et al. (2018a) described some soft matrix operations and improved soft distributive max–min decision-making methods. Many researchers were successfully applied soft set theory (Feng et al. 2020; Kamacı et al. 2019; Karaaslan and Çağman 2022; Maji et al. 2002) and soft matrix theory (Kamacı et al. 2018; Petchimuthu et al. 2020; Petchimuthu and Kamacı 2019) to decision making. Gong et al. (2010) proposed a new type of soft set called bijective soft set, every element is only mapped into one parameter, and also, the union of partition by parameter set is universe. In the following years, these sets were developed in both theoretical and practical aspects (Gong et al. 2013; Kamacı et al. 2018b; Tiwari et al. 2017). Feng et al. (2010) introduced full and partition forms of soft sets and created a covering soft set from a full soft set. Several scientists studied on canonical soft sets and their derived types (Feng et al. 2022).

In recent decade, some researchers have investigated the combined types soft sets and rough sets. Accordingly, the several concepts of soft rough sets and rough soft sets were introduced (Ali 2011; Feng et al. 2010, 2011). These sets has been broadly applied in various fields such as business, medicine, engineering, education and multimedia. Ali et al. (2019) described soft dominance rough sets and applied them to deal with multi-agent conflict analysis decision problem. Alcantud et al. (2020) presented a fresh insight into rough set theory from the point of view of the N-soft set, which is the N-expansion of the soft set in binary form. Moreover, Inbarani et al. (2018) proposed a hybrid intelligent system that combines bijective soft set and rough set, and so initiated the theory of rough bijective soft sets.

In this study, we introduce a new approach that naturally carries the properties of both soft and rough sets and enables us to take advantage of these two uncertain modeling theories. Strait soft sets are naturally soft sets whose domain is the support set of a soft set and takes value in the partitions of the alternatives set. At the same time, strait soft sets are full soft sets (Feng et al. 2010) due to their nature and in some cases bear the characteristics of bijective soft sets (Gong et al. 2010). A strait universe of a soft set is introduced, and thus, a new method for reducing alternatives is presented. It is also possible to combine parameters thanks to strait soft sets, as well as the role it plays in reducing alternatives. Therefore, the concept fusion of parameters is introduced and its use in soft set operations is given. Strait rough sets are a kind of soft rough set structure built using strait soft sets. However, since the strait soft sets take value from the partition set of the alternatives and they are full soft sets, they are naturally connected with the rough set structure and many features of rough sets occur spontaneously and also allow the parameters to be characterized. Moreover, this study provides a new perspective on the multi-attribute decision making and thus presents an exclusive decision-making method based on strait soft sets and strait rough sets. All of these highlight the main contributions of this paper.

The remaining parts of this paper is as follows: In Sect. 2, we give some basic concepts of soft sets and rough sets. In Sect. 3, we describe strait soft sets and present reduction of alternatives and fusion of parameters for these sets. Section 4 introduces a new rough set structure using strait soft sets. Section 5 presents a new approach to multi-attribute decision making that includes primary evaluations under the environments of strait soft set and strait rough set. Furthermore, in this section, two examples are given to demonstrate the effectiveness and stability of the proposed approach. Lastly, a concrete conclusion is drawn in Sect. 6.

2 Preliminaries

In this section, we briefly review some essential notions of soft sets and rough sets.

Definition 1 (Molodtsov 1999) Let $\mathcal{V}$ be an initial universe set, $\mathfrak{Q}$ be a set of parameters and $P(\mathcal{V})$ be the power set of $\mathcal{V}$ and $\mathfrak{P} \subseteq \mathfrak{Q}$. A soft set $(\Omega, \mathfrak{P})$ or simply $\Omega_{\mathfrak{P}}$ on the universe $\mathcal{V}$ is defined by the ordered pairs

$$(\Omega, \mathfrak{P}) = \{(\wp, \Omega(\wp)) \mid \wp \in \mathfrak{P}, \Omega(\wp) \in P(\mathcal{V})\},$$

where Ω is a mapping from $\mathfrak{P}$ to $P(\mathcal{V})$.

In other words, a soft set over $\mathcal{V}$ is a parameterized family of subsets of the universe $\mathcal{V}$.

Definition 2 (Pei and Miao 2005) Let $(\Omega_1, \mathfrak{P}_1)$ and $(\Omega_2, \mathfrak{P}_2)$ be soft sets over $\mathcal{V}$.

(a) If $\mathfrak{P}_1 \subseteq \mathfrak{P}_2$ and $\Omega_1(\wp) \subseteq \Omega_2(\wp)$ for all $\wp \in \mathfrak{P}_1$, then $(\Omega_1, \mathfrak{P}_1)$ is a soft subset of $(\Omega_2, \mathfrak{P}_2)$, denoted by $(\Omega_1, \mathfrak{P}_1) \widetilde{\subseteq} (\Omega_2, \mathfrak{P}_2)$.

(b) If $(\Omega_1, \mathfrak{P}_1) \widetilde{\subseteq} (\Omega_2, \mathfrak{P}_2)$ and $(\Omega_2, \mathfrak{P}_2) \widetilde{\subseteq} (\Omega_1, \mathfrak{P}_1)$, then $(\Omega_1, \mathfrak{P}_1)$ and $(\Omega_2, \mathfrak{P}_2)$ is said to be soft equal and denoted by $(\Omega_1, \mathfrak{P}_1) = (\Omega_2, \mathfrak{P}_2)$.

Definition 3 Let $(\Omega_1, \mathfrak{P}_1)$ and $(\Omega_2, \mathfrak{P}_2)$ be soft sets over $\mathcal{V}$.

(a) The restricted intersection of $(\Omega_1, \mathfrak{P}_1)$ and $(\Omega_2, \mathfrak{P}_2)$ is a soft set denoted and defined by $(\Omega_1, \mathfrak{P}_1) \cap_R (\Omega_2, \mathfrak{P}_2) = (\Omega_3, \mathfrak{P}_3)$, where $\mathfrak{P}_3 = \mathfrak{P}_1 \cap \mathfrak{P}_2$ such that $\mathfrak{P}_1 \cap \mathfrak{P}_2 \neq \emptyset$ and $\Omega_3(\wp) = \Omega_1(\wp) \cap \Omega_2(\wp)$ for all $\wp \in \mathfrak{P}_3$ (Pei and Miao 2005).

(b) The restricted union of $(\Omega_1, \mathfrak{P}_1)$ and $(\Omega_2, \mathfrak{P}_2)$ is a soft set denoted and defined by $(\Omega_1, \mathfrak{P}_1) \cup_R (\Omega_2, \mathfrak{P}_2) =$

$(\Omega_3, \mathfrak{P}_3)$, where $\mathfrak{P}_3 = \mathfrak{P}_1 \cap \mathfrak{P}_2$ such that $\mathfrak{P}_1 \cap \mathfrak{P}_2 \neq \emptyset$ and $\Omega_3(\wp) = \Omega_1(\wp) \cup \Omega_2(\wp)$ for all $\wp \in \mathfrak{P}_3$ (Ali et al. 2009).

(c) The restricted difference of $(\Omega_1, \mathfrak{P}_1)$ and $(\Omega_2, \mathfrak{P}_2)$ is a soft set denoted and defined by $(\Omega_1, \mathfrak{P}_1) \sim_R (\Omega_2, \mathfrak{P}_2) = (\Omega_3, \mathfrak{P}_3)$, where $\mathfrak{P}_3 = \mathfrak{P}_1 \cap \mathfrak{P}_2$ such that $\mathfrak{P}_1 \cap \mathfrak{P}_2 \neq \emptyset$ and $\Omega_3(\wp) = \Omega_1(\wp) - \Omega_2(\wp)$ for all $\wp \in \mathfrak{P}_3$ (Ali et al. 2009).

(d) The restricted symmetric difference of $(\Omega_1, \mathfrak{P}_1)$ and $(\Omega_2, \mathfrak{P}_2)$ is a soft set denoted by $(\Omega_1, \mathfrak{P}_1)\widetilde{\triangle}(\Omega_2, \mathfrak{P}_2) = [(\Omega_1, \mathfrak{P}_1) \cup_R (\Omega_2, \mathfrak{P}_2)] \sim_R [(\Omega_1, \mathfrak{P}_1) \cap_R (\Omega_2, \mathfrak{P}_2)]$ (Sezgin and Atagün 2011).

Definition 4 (Feng et al. 2008) Let $(\Omega, \mathfrak{P})$ be soft set over $\mathcal{V}$. Then the set

$$supp(\Omega, \mathfrak{P}) = \{\wp \in \mathfrak{P} \mid \Omega(\wp) \neq \emptyset\}$$

is called the *support* of the soft set $(\Omega, \mathfrak{P})$. The *null soft set* is a soft set with empty support and we denote it by $\emptyset_\Omega$. A soft set $(\Omega, \mathfrak{P})$ is called *non-null* if $supp(\Omega, \mathfrak{P}) \neq \emptyset$.

Definition 5 (Feng et al. 2010) Let $(\Omega, \mathfrak{P})$ be soft set over $\mathcal{V}$. Then $(\Omega, \mathfrak{P})$ is said to be a full soft set if $\bigcup_{\wp \in \mathfrak{P}} \Omega(\wp) = \mathcal{V}$.

Definition 6 (Feng et al. 2010) A full soft set $(\Omega, \mathfrak{P})$ over $\mathcal{V}$ is called a covering soft set if $\Omega(\wp) \neq \emptyset$ for all $\wp \in \mathfrak{P}$.

Definition 7 (Feng et al. 2010) Let $(\Omega, \mathfrak{P})$ be soft set over $\mathcal{V}$. This soft set is called a partition soft set if $\{\Omega(\wp) : \wp \in \mathfrak{P}\}$ forms a partition of $\mathcal{V}$.

Definition 8 (Gong et al. 2010) Let $(\Omega, \mathfrak{P})$ be soft set over $\mathcal{V}$ such that $\mathfrak{P}$ is a non-empty parameter set. We say that $(\Omega, \mathfrak{P})$ is a bijective soft set, if $(\Omega, \mathfrak{P})$ such that

(a) $\bigcup_{\wp \in \mathfrak{P}} \Omega(\wp) = \mathcal{V}$,

(b) $\Omega(\wp_i) \cap \Omega(\wp_j) = \emptyset$ for all $\wp_i, \wp_j \in \mathfrak{P}$ such that $\wp_i \neq \wp_j$.

Definition 9 (Pawlak 1982) Let $\mathfrak{R}$ be an equivalence relation on the universe $\mathcal{V}$, then $(\mathcal{V}, \mathfrak{R})$ is a Pawlak approximation space. Let $X \subseteq \mathcal{V}$. Two sets are defined as $\mathfrak{R}^*X = \{v \in \mathcal{V} \mid [v]_\mathfrak{R} \cap X \neq \emptyset\}$ and $\mathfrak{R}_*X = \{v \in \mathcal{V} \mid [v]_\mathfrak{R} \subseteq X\}$, where $[v]_\mathfrak{R}$ is an equivalence class of v. Then a subset X of $\mathcal{V}$ is called definable if $\mathfrak{R}^*X = \mathfrak{R}_*X$; in the opposite case, i.e., if $\mathfrak{R}^*X - \mathfrak{R}_*X \neq \emptyset$, X is said to be a rough set.

If the set $X \subseteq \mathcal{V}$ is defined by a predicate $\mathcal{P}$ and $v \in \mathcal{V}$, then there are the following:

1. $v \in \mathfrak{R}_*X$ means that v certainly has the property $\mathcal{P}$.
2. $v \in \mathfrak{R}^*X$ means that v possibly has the property $\mathcal{P}$.
3. $v \in \mathcal{V} - \mathfrak{R}^*X$ means that v definitely does not have the property $\mathcal{P}$.

Considering that the reader knows the basic properties about soft sets and rough sets, the information given in the preliminaries section is finished here. However, the explanations about the new definitions and their properties have been made in detail and supported with many examples.

3 Strait soft sets

From now on, $PA(\mathcal{V}) = \bigcup_{k \in I} \mathcal{V}_k$ is the set of all partitions of $\mathcal{V}$, $|\mathcal{V}_k|$ is the number of elements in $\mathcal{V}_k$, and if it would not cause any confusion, we express supp$(\Omega, \mathfrak{P})$ shortly with $S\mathfrak{P}$.

Definition 10 Let $(\Omega, \mathfrak{P})$ be a soft set over $\mathcal{V}$. For the soft set $(\Omega, \mathfrak{P})$, if

$$\Omega : S\mathfrak{P} \to PA(\mathcal{V})$$

is a set-valued function such that $\Omega(S\mathfrak{P}) = \mathcal{V}_k$ for a fixed $k \in I$, then the soft set $(\Omega, S\mathfrak{P})$ is called a strait soft set of $(\Omega, \mathfrak{P})$. A strait soft set over $\mathcal{V}$ can be represented by the set of ordered pairs

$$(\Omega, S\mathfrak{P}) = \{(\wp, Y) \mid \wp \in S\mathfrak{P}, Y \in \mathcal{V}_k \text{ for a fixed } k \in I\}.$$

From now on, the set of all strait soft sets over $\mathcal{V}$ will be shown with $SS(\mathcal{V})$.

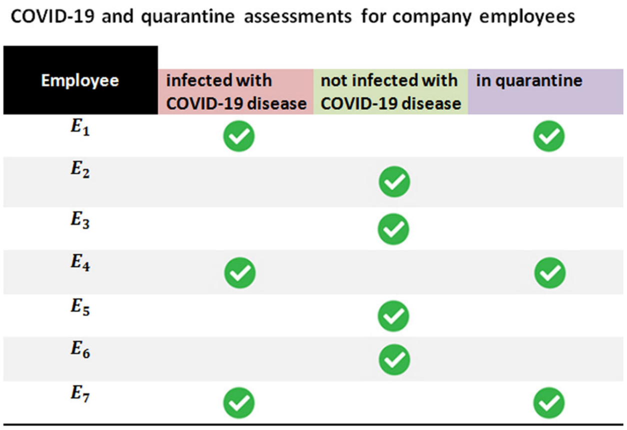
Remark 1 Let $(\Omega, S\mathfrak{P})$ be a strait soft set over $\mathcal{V}$ such that $\Omega(S\mathfrak{P}) = \mathcal{V}_k$, where $\mathcal{V}_k$ is a partition of $\mathcal{V}$.

1. By Definition 10, $\bigcup_{\wp \in S\mathfrak{P}} \Omega(\wp) = \mathcal{V}$. It means that a strait soft set $(\Omega, S\mathfrak{P})$ is also a full soft set over $\mathcal{V}$.
2. If $|S\mathfrak{P}| = |\mathcal{V}_k| = |\mathcal{V}|$, then $\Omega(\wp_i) \cap \Omega(\wp_j) = \emptyset$ for all $\wp_i, \wp_j \in S\mathfrak{P}$ such that $\wp_i \neq \wp_j$. It means that if $|S\mathfrak{P}| = |\mathcal{V}_k| = |\mathcal{V}|$, then $(\Omega, S\mathfrak{P})$ is a bijective soft set over $\mathcal{V}$.
3. $\bigcap_{\wp \in S\mathfrak{P}} \Omega(\wp) = \emptyset \Leftrightarrow |\mathcal{V}_k| \neq 1$ and $\bigcap_{\wp \in S\mathfrak{P}} \Omega(\wp) \neq \emptyset \Leftrightarrow |\mathcal{V}_k| = 1$.

Definition 11 Let $(\Omega_1, S\mathfrak{P}_1)$ and $(\Omega_2, S\mathfrak{P}_2)$ be strait soft sets of the soft sets $(\Omega_1, \mathfrak{P}_1)$ and $(\Omega_2, \mathfrak{P}_2)$ over common universe $\mathcal{V}$, respectively. If $S\mathfrak{P}_1 \subseteq S\mathfrak{P}_2$ and $\Omega_1(\wp) \subseteq \Omega_2(\wp)$ for all $\wp \in S\mathfrak{P}_1$, then $(\Omega_1, S\mathfrak{P}_1)$ is said to be strait soft subset of $(\Omega_2, S\mathfrak{P}_2)$ and denoted by $(\Omega_1, S\mathfrak{P}_1) \subseteq_s (\Omega_2, S\mathfrak{P}_2)$.

Note that for $(\Omega_1, S\mathfrak{P}_1)$ to be a strait subset of $(\Omega_2, S\mathfrak{P}_2)$ does not imply $(\Omega_1, \mathfrak{P}_1)$ to be a soft subset of $(\Omega_2, \mathfrak{P}_2)$. But if $(\Omega_1, \mathfrak{P}_1)\tilde{\subseteq}(\Omega_2, \mathfrak{P}_2)$, then $(\Omega_1, S\mathfrak{P}_1) \subseteq_s (\Omega_2, S\mathfrak{P}_2)$. We have the following example:

Example 1 Let the universe $\mathcal{V} = \{v_1, v_2, v_3\}$, the parameter set $\mathfrak{Q} = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7\}$ and two subsets of $\mathfrak{Q}$ be $\mathfrak{P}_1 = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5\}$ and $\mathfrak{P}_2 =$



COVID-19 and quarantine assessments for company employees

Employee	infected with COVID-19 disease	not infected with COVID-19 disease	in quarantine
E_1	✓		✓
E_2		✓	
E_3		✓	
E_4	✓		✓
E_5		✓	
E_6		✓	
E_7	✓		✓

Fig. 1 An evaluation of the company employees according to the criteria

$\{\wp_1,\wp_2,\wp_3,\wp_4,\wp_5,\wp_6\}$. Suppose that corresponding soft sets of $\mathfrak{P}_1$ and $\mathfrak{P}_2$ are

$$(\Omega_1,\mathfrak{P}_1)=\{(\wp_1,\{v_1,v_2\}),(\wp_2,\{v_1,v_2\}),\\ (\wp_3,\{v_3\}),(\wp_4,\{v_1,v_2\}),(\wp_5,\emptyset)\}$$

and

$$(\Omega_2,\mathfrak{P}_2)=\{(\wp_1,\{v_1\}),(\wp_2,\{v_2\}),(\wp_3,\{v_3\}),\\ (\wp_4,\emptyset),(\wp_5,\emptyset),(\wp_6,\emptyset)\}$$

Then the support sets of these soft sets are

$S\mathfrak{P}_1=\operatorname{supp}(\Omega_1,\mathfrak{P}_1)=\{\wp_1,\wp_2,\wp_3,\wp_4\}$, $S\mathfrak{P}_2=\operatorname{supp}(\Omega_2,\mathfrak{P}_2)=\{\wp_1,\wp_2,\wp_3\}$.

The partitions of $\mathcal{V}$ are $\mathcal{V}_1=\{\{v_1\},\{v_2\},\{v_3\}\}$, $\mathcal{V}_2=\{\{v_1,v_2\},\{v_3\}\}$, $\mathcal{V}_3=\{\{v_1,v_3\},\{v_2\}\}$, $\mathcal{V}_4=\{\{v_1\},\{v_2,v_3\}\}$ and $\mathcal{V}_5=\{\{v_1,v_2,v_3\}\}$, and so $PA(\mathcal{V})=\{\mathcal{V}_1,\mathcal{V}_2,\mathcal{V}_3,\mathcal{V}_4,\mathcal{V}_5\}$. Then the strait soft sets $\Omega(S\mathfrak{P}_1)=\mathcal{V}_2$ and $\Omega(S\mathfrak{P}_2)=\mathcal{V}_1$ such that

$$(\Omega_1,S\mathfrak{P}_1)=\\ \{(\wp_1,\{v_1,v_2\}),(\wp_2,\{v_1,v_2\}),(\wp_3,\{v_3\}),(\wp_4,\{v_1,v_2\})\},$$

and

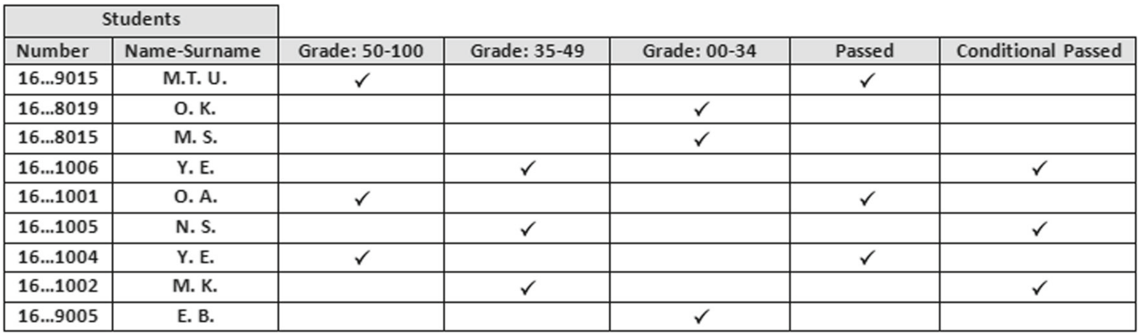
$$(\Omega_2,S\mathfrak{P}_2)=\{(\wp_1,\{v_1\}),(\wp_2,\{v_2\}),(\wp_3,\{v_3\})\}.$$

It is seen that $\bigcup_{\wp\in S\mathfrak{P}_1}\Omega_1(\wp)=\mathcal{V}$ and $\bigcap_{\wp\in S\mathfrak{P}_1}\Omega_1(\wp)=\emptyset$. But $(\Omega_1,S\mathfrak{P}_1)$ is not a bijective soft set over $\mathcal{V}$, since $|S\mathfrak{P}_1|\neq|\mathcal{V}_2|$. Similarly, $\bigcup_{\wp\in S\mathfrak{P}_2}\Omega_2(\wp)=\mathcal{V}$, $\bigcap_{\wp\in S\mathfrak{P}_2}\Omega_2(\wp)=\emptyset$ and $|S\mathfrak{P}_2|=|\mathcal{V}_1|=|\mathcal{V}|$, and so $(\Omega_2,S\mathfrak{P}_2)$ is also a bijective soft set over $\mathcal{V}$.
Also, we see that $(\Omega_2,\mathfrak{P}_2)\underset{\sim}{\nsubseteq}(\Omega_1,\mathfrak{P}_1)$ since $\mathfrak{P}_2\nsubseteq\mathfrak{P}_1$, but $(\Omega_2,S\mathfrak{P}_2)\subseteq_s(\Omega_1,S\mathfrak{P}_1)$.

3.1 Authentic life motivation of strait soft sets

Now, we address some realistic real-life examples of strait soft sets.
While evaluating alternatives according to parameters in real life, alternatives matching some parameters must be the same. These evaluation parameters are not the same, but interact with each other. In some cases, we may consider one of these parameters as the cause or consequence of the other. Under these explanations, consider the following examples.

1. For several years, countries have been trying to cope with the COVID-19 epidemic. Currently, in Turkey, people infected with COVID-19 disease (coronavirus) are kept in quarantine as a precaution, and quarantine is not required for those who do not suffer from COVID-19 disease (coronavirus). A company in Turkey intents to determine whether the employees are suffering from the COVID-19 disease or are in quarantine. Thus, it aims both to monitor the health status of its employees and to seek temporary solutions for vacant positions so that the work is not disrupted. As a result of the evaluation of the seven employees working in the company according to the criteria determined for COVID-19, these criteria, "infected with COVID-19 disease," "not infected with COVID-19 disease" and "in quarantine," are presented with Fig. 1. Figure 1 shows an instance of strait soft set.
2. For the Abstract Mathematics and Logic course conducted in the 2021–2022 fall semester in the Mathematics Department, Fig. 2 is considered regarding the grade level (GPA) of eight students and whether they passed the course (e.g., passed or conditional passed). Figure 2 is created by considering the information about the Abstract Mathematics and Logic course from the Student Information Management System (https://obs.bozok.edu.tr/) of Yozgat Bozok University (in Turkey) on April 15, 2022. This is an instance of strait soft set.
3. Generally, the articles sent to scientific journals for publication are subject to at least two peer reviews, and the journal either accepts the article for publication, or rejects it, or proposes a revision and states that it will reach a decision after revision. Sometimes the number of reviewers can be up to five or six. It is known that when the reviewers propose accept or reject as the final decision on the publication of the manuscript, they avoid re-examining the new version of the manuscript. However, if the reviewer suggested that the manuscript be revised, he/she is willing to review the new version to check whether it includes these revisions. Generally, journals make the review of new version mandatory, not optional, for reviewers who suggest revisions. In other words, when the revision is selected in the journal system, the reviewing of the new version of the manuscript



Students						
Number	Name-Surname	Grade: 50-100	Grade: 35-49	Grade: 00-34	Passed	Conditional Passed
16...9015	M.T. U.	✓			✓	
16...8019	O. K.			✓		
16...8015	M. S.			✓		
16...1006	Y. E.		✓			✓
16...1001	O. A.	✓			✓	
16...1005	N. S.		✓			✓
16...1004	Y. E.	✓			✓	
16...1002	M. K.		✓			✓
16...9005	E. B.			✓		

Fig. 2 Students' grades and pass status for Abstract Mathematics and Logic course

is marked as mandatory. Figure 3 gives the recommendations of six reviewers who have reviewed a manuscript. Figure 3 shows that the reviewers recommending revision of the manuscript are willing to review the new version of the manuscript in the future. Figure 3 shows the mathematical form as a strait soft set.

These figures are just a few examples that illustrate the authentic life motivation of the strait soft sets. It is possible to encounter strait soft set structures in parametric classification of objects/alternatives in many areas.

3.2 Strait universe: a reduction method of alternatives and fusion of parameters

Using the concept strait soft set, a method for reducing the alternatives depending on a soft set will be given in this section. This will play a role in shortening the process and facilitating calculations in decision-making mechanisms.

Definition 12 Let $(\Omega, \mathfrak{P})$ be a soft set over Z and $Z \subset \mathcal{V}$. If $(\Omega, S\mathfrak{P})$ is a strait soft set over Z, but it is not a strait soft set over $\mathcal{V}$, then the sub-universe Z is called a strait universe depending on the soft set $(\Omega, \mathfrak{P})$. In other words, Z is a strait universe depending on the soft set $(\Omega, \mathfrak{P})$ iff $Z \subset \mathcal{V}$, $\Omega(S\mathfrak{P}) \in PA(Z)$ and $\Omega(S\mathfrak{P}) \notin PA(\mathcal{V})$.

Example 2 Let the universe $\mathcal{V} = \{v_1, v_2, v_3, v_4\}$, the parameter set $\mathfrak{Q} = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7\}$ and two subsets of $\mathfrak{Q}$ be $\mathfrak{P}_1 = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5\}$ and $\mathfrak{P}_2 = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_6\}$. Suppose that corresponding soft sets of $\mathfrak{P}_1$ and $\mathfrak{P}_2$ are

$$(\Omega_1, \mathfrak{P}_1) = \{(\wp_1, \{v_1, v_2\}), (\wp_2, \{v_1, v_2\}), (\wp_3, \{v_3\}), (\wp_4, \{v_1, v_2\}), (\wp_5, \emptyset)\}$$

and

$$(\Omega_2, \mathfrak{P}_2) = \{(\wp_1, \{v_1\}), (\wp_2, \{v_2\}), (\wp_3, \{v_3, v_4\}), (\wp_4, \emptyset), (\wp_6, \{v_1, v_2\})\}.$$

Then the support sets of these soft sets are

$$S\mathfrak{P}_1 = \mathrm{supp}(\Omega_1, \mathfrak{P}_1) = \{\wp_1, \wp_2, \wp_3, \wp_4\}$$

and

$$S\mathfrak{P}_2 = \mathrm{supp}(\Omega_2, \mathfrak{P}_2) = \{\wp_1, \wp_2, \wp_3, \wp_6\}.$$

Then

$$(\Omega_1, S\mathfrak{P}_1) = \{(\wp_1, \{v_1, v_2\}), (\wp_2, \{v_1, v_2\}), (\wp_3, \{v_3\}), (\wp_4, \{v_1, v_2\})\}$$

is not a strait soft set over $\mathcal{V}$, but it is a strait soft set over $Z = \{v_1, v_2, v_3\} \subset \mathcal{V}$. Then Z is a strait universe depending on the soft set $(\Omega_1, \mathfrak{P}_1)$. Now, we consider the soft set

$$(\Omega_2, S\mathfrak{P}_2) = \{(\wp_1, \{v_1\}), (\wp_2, \{v_2\}), (\wp_3, \{v_3, v_4\}), (\wp_6, \{v_1, v_2\})\}.$$

It is seen that there is no partition of $\mathcal{V}$ or of any subset of $\mathcal{V}$; then, there is no strait universe depending on the soft set $(\Omega_2, \mathfrak{P}_2)$. Now, the restrictive intersection of $(\Omega_1, \mathfrak{P}_1)$ and $(\Omega_2, \mathfrak{P}_2)$ is the soft set

$$(\Omega_1, \mathfrak{P}_1) \cap_R (\Omega_2, \mathfrak{P}_2) = (\Omega_3, \mathfrak{P}_1 \cap \mathfrak{P}_2) = \left\{ \begin{array}{l} (\wp_1, \{v_1\}), (\wp_2, \{v_2\}), \\ (\wp_3, \{v_3\}), \quad (\wp_4, \emptyset) \end{array} \right\}.$$

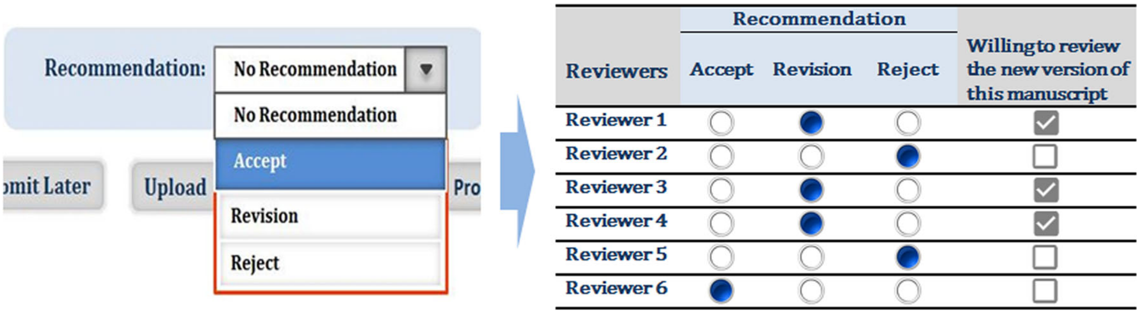
and

$$(\Omega_3, S(\mathfrak{P}_1 \cap \mathfrak{P}_2)) = \{(\wp_1, \{v_1\}), (\wp_2, \{v_2\}), (\wp_3, \{v_3\})\}$$

is also a strait soft set over Z; this means Z is a strait universe depending on the soft set $(\Omega_1, \mathfrak{P}_1) \cap_R (\Omega_2, \mathfrak{P}_2)$.

Since the images of the parameters in strait soft sets form a partition of the alternatives, this gives us an opportunity to shorten the processes by fusing the parameters. This method saves time and does not damage the calculations since it does not affect the results of soft set operations.

Definition 13 Let $(\Omega, S\mathfrak{P})$ be a soft set over $\mathcal{V}$ such that $(\Omega, S\mathfrak{P}) = \mathcal{V}_k$, where $\mathcal{V}_k = \{Y_1, Y_2, ..., Y_r\}$ is a partition of



Reviewers	Recommendation			Willing to review the new version of this manuscript
	Accept	Revision	Reject	
Reviewer 1	○	●	○	☑
Reviewer 2	○	○	●	☐
Reviewer 3	○	●	○	☑
Reviewer 4	○	●	○	☑
Reviewer 5	○	○	●	☐
Reviewer 6	●	○	○	☐

Fig. 3 Recommendations of reviewers for paper and willingness to review the new version

$\mathcal{V}$ and $S\mathfrak{P} = \{\wp_1, \wp_2, ..., \wp_t\}$. If $\Omega(\wp_i) = \Omega(\wp_j) = Y_p$ for $i, j \in \{1, 2, ..., t\}$ and $p \in \{1, 2, ..., r\}$, then $\wp_i$ and $\wp_j$ are called fusible parameters and denoted by $\wp_{(i)(j)}$.
By using fusible parameters, the strait soft set $(\Omega, S\mathfrak{P})$ can be written as $\{(\wp_{(i_m)(j_m),...,(z_m)}, Y_m) \mid \Omega(\wp_{i_m}) = \Omega(\wp_{j_m}) = ... = \Omega(\wp_{z_m}) = Y_m \in \mathcal{V}_k$ forall $m \in \{1, 2, ..., r\}\}$, and we call this notation of a strait soft set is fusion form.

Remark 2 If $(\Omega, S\mathfrak{P})$ is a bijective soft set over $\mathcal{V}$, then $\Omega(\wp_i) \cap \Omega(\wp_j) = \emptyset$ for all $\wp_i, \wp_j \in S\mathfrak{P}$ such that $\wp_i \neq \wp_j$. Therefore, there are no fusible parameters in bijective soft sets.

Proposition 1 *Let $(\Omega, S\mathfrak{P}) \in SS(\mathcal{V})$. There are fusible parameters in $S\mathfrak{P}$ iff $(\Omega, S\mathfrak{P})$ is not a bijective soft set over $\mathcal{V}$.*

Proof The result is given in Remarks 1 and 2. □

Proposition 2 *The notation of strait soft sets by using fusible parameters does not affect the results of soft set operation restrictive intersection (union, difference and symmetric difference).*

Proof Let $PA(\mathcal{V}) = \bigcup_{i \in I} \mathcal{V}_i$ is the set of all partitions of $\mathcal{V}$. Let $(\Omega_1, S\mathfrak{P}_1) = \{(\wp_{(i_m)(j_m),...,(z_m)}, Y_m) \mid \Omega_1(\wp_{i_m}) = \Omega_1(\wp_{j_m}) = ... = \Omega_1(\wp_{z_m}) = Y_m \in \mathcal{V}_k$for all $m \in \{1, 2, ..., r\}\}$ and $(\Omega_2, S\mathfrak{P}_2) = \{(\wp_{(i_s)(j_s),...,(z_s)}, T_s) \mid \Omega_2(\wp_{i_s}) = \Omega_2(\wp_{j_s}) = ... = \Omega_2(\wp_{z_s}) = T_s \in \mathcal{V}_q$ for all $s \in \{1, 2, ..., w\}\}$ be strait soft sets over $\mathcal{V}$ such that $\Omega_1(S\mathfrak{P}_1) = \mathcal{V}_k$ and $\Omega_2(S\mathfrak{P}_2) = \mathcal{V}_q$ where $\mathcal{V}_k = \{Y_1, Y_2, ..., Y_r\}$ and $\mathcal{V}_q = \{T_1, T_2, ..., T_w\}$.
Then the restrictive intersection $(\Omega_1, S\mathfrak{P}_1) \cap_R (\Omega_2, S\mathfrak{P}_2) = (\Omega_3, S\mathfrak{P}_1 \cap S\mathfrak{P}_2)$ is the set of pairs

$$(\Omega_3, S\mathfrak{P}_1 \cap S\mathfrak{P}_2) = \{(\wp_{(i_v)(j_v),...,(z_v)}, Y_v \cap T_v) \mid \Omega_3(\wp_{i_v}) = \Omega_3(\wp_{j_v}) = ... = \Omega_3(\wp_{z_v})\}. \quad (1)$$

On the other hand, these strait soft sets can be written as $(\Omega_1, S\mathfrak{P}_1) = \{(\wp, Y) | \wp \in S\mathfrak{P}_1, Y \in \mathcal{V}_k$ for a fixed $k \in I\}$ and $(\Omega_2, S\mathfrak{P}_2) = \{(\wp, T) | \wp \in S\mathfrak{P}_2, T \in \mathcal{V}_q$ for a fixed $q \in I\}$. Then their restrictive intersection is

$$(\Omega_1, S\mathfrak{P}_1) \cap_R (\Omega_2, S\mathfrak{P}_2) = \{(\wp, Y \cap T) | \wp \in S\mathfrak{P}_1 \cap S\mathfrak{P}_2, Y \in \mathcal{V}_k, T \in \mathcal{V}_q \text{ for fixed } k, q \in I\}. \quad (2)$$

It is seen that the domains are the same, that is, $S\mathfrak{P}_1 \cap S\mathfrak{P}_2$, and the images are the same, that is, $Y \cap T$ where $Y \in \mathcal{V}_k$, $T \in \mathcal{V}_q$. Then, we have (1) = (2). Hence, the proof is done for the operation restrictive intersection. The rest of the proof can be obtained similarly. □

Example 3 Let $\mathcal{V} = \{v_1, v_2, v_3, v_4\}$ be the universe set, $\mathfrak{Q} = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7\}$ be the set of parameters and two subsets of $\mathfrak{Q}$ be $\mathfrak{P}_1 = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5\}$ and $\mathfrak{P}_2 = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6\}$. Suppose that corresponding soft sets of $\mathfrak{P}_1$ and $\mathfrak{P}_2$ are

$$(\Omega_1, \mathfrak{P}_1) = \{(\wp_1, \{v_1, v_2\}), (\wp_2, \{v_1, v_2\}), (\wp_3, \{v_3\}), (\wp_4, \{v_3\}), (\wp_5, \{v_4\})\}$$

and

$$(\Omega_2, \mathfrak{P}_2) = \{(\wp_1, \{v_1\}), (\wp_2, \{v_2\}), (\wp_3, \emptyset), (\wp_4, \{v_3, v_4\}), (\wp_5, \{v_3, v_4\}), (\wp_6, \emptyset)\}.$$

Then the support sets of these soft sets are

$$S\mathfrak{P}_1 = supp(\Omega_1, \mathfrak{P}_1) = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5\}$$

and

$$S\mathfrak{P}_2 = supp(\Omega_2, \mathfrak{P}_2) = \{\wp_1, \wp_2, \wp_4, \wp_5\}.$$

Then for the partitions of $\mathcal{V}$, $\mathcal{V}_1 = \{\{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}\}$, $\mathcal{V}_2 = \{Y_1 = \{v_1, v_2\}, Y_2 = \{v_3\}, Y_3 = \{v_4\}\}$, $\mathcal{V}_3 = \{T_1 = \{v_1\}, T_2 = \{v_2\}, T_3 = \{v_3, v_4\}\}$, the strait soft sets of $(\Omega_1, \mathfrak{P}_1)$ and $(\Omega_2, \mathfrak{P}_2)$ are $(\Omega_1, S\mathfrak{P}_1) = \mathcal{V}_2$ and $(\Omega_2, S\mathfrak{P}_2) = \mathcal{V}_3$ such that

$$(\Omega_1, S\mathfrak{P}_1) = (\Omega_1, \mathfrak{P}_1)$$

and

$$(\Omega_2, S\mathfrak{P}_2) = \{(\wp_1, \{v_1\}), (\wp_2, \{v_2\}), (\wp_4, \{v_3, v_4\}), (\wp_5, \{v_3, v_4\})\}.$$

By Definition 13, these strait soft sets are written in the fusion form as $(\Omega_1, S\mathfrak{P}_1) = \{(\wp_{(1)(2)}, Y_1), (\wp_{(3)(4)}, Y_2), (\wp_5, Y_3)\}$ and $(\Omega_2, S\mathfrak{P}_2) = \{(\wp_1, T_1), (\wp_2, T_2), (\wp_{(4)(5)}, T_3)\}$. Then their restrictive intersection is obtained as follows:
$(\Omega_1, S\mathfrak{P}_1) \cap_R (\Omega_2, S\mathfrak{P}_2)=\{(\wp_1, Y_1 \cap T_1), (\wp_2, Y_1 \cap T_2), (\wp_4, Y_2 \cap T_3), (\wp_5, Y_3 \cap T_3)\}$.

4 Strait rough sets

In this section, a new rough set structure is constructed using strait soft sets over universe $\mathcal{V}$.

Definition 14 Let $\mathcal{V}$ be an universe, $X \subseteq \mathcal{V}$ and let $(\Omega, S\mathfrak{P}) \in SS(\mathcal{V})$ such that $\Omega(S\mathfrak{P}) = \mathcal{V}_k \in PA(\mathcal{V})$.

1. The lower approximation of X depending on $(\Omega, S\mathfrak{P})$ is the set
$$\Omega_*(X) = \{\bigcup Y \mid \Omega(\wp) = Y \ for \ \wp \in \mathfrak{P} \ and \ Y \subseteq X\}.$$
2. The upper approximation of X depending on $(\Omega, S\mathfrak{P})$ is the set
$$\Omega^*(X) = \{\bigcup Y \mid \Omega(\wp) = Y \ for \ \wp \in \mathfrak{P} \ and \ Y \cap X \neq \emptyset\}.$$
3. The boundary set of X depending on $(\Omega, S\mathfrak{P})$ is the set
$$B_\Omega(X) = \Omega^*(X) - \Omega_*(X).$$
4. If $\Omega_*(X) = \Omega^*(X)$, X is said to be strait definable; otherwise, i.e., if $B_\Omega(X) \neq \emptyset$, then X is called a strait rough set depending on $(\Omega, S\mathfrak{P})$.

Since a strait soft set is naturally a full soft set, $\Omega_*(X) \subseteq X \subseteq \Omega^*(X)$ is satisfied. Therefore, strait soft sets give a more appropriate approach to rough sets than that given in the article (Feng et al. 2010). As Pawlak's rough set (Pawlak 1982), if X is the set of alternatives that satisfy the property $\mathcal{P}$ in $\mathcal{V}$, then we have
$v \in \Omega_*(X)$ means that v certainly has property $\mathcal{P}$,
$v \in \Omega^*(X)$ means that v possibly has property $\mathcal{P}$,
$v \in \mathcal{V}-\Omega^*(X)$ means that v definitely does not have property $\mathcal{P}$.

Theorem 1 *Strait rough set is a generalization of Pawlak's rough set.*

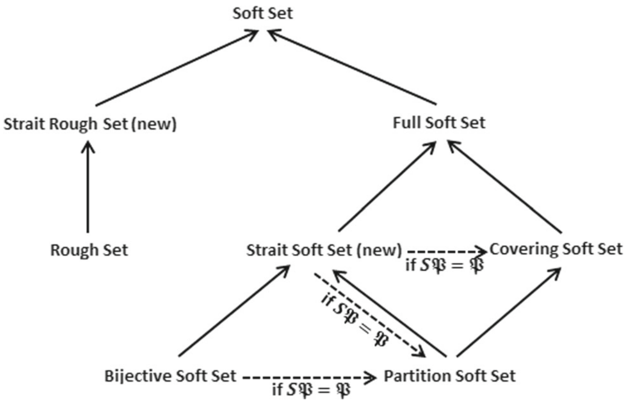


Fig. 4 Relationships among strait soft set, strait rough set, soft set and rough set

Proof Let $X \subseteq \mathcal{V}$ be a rough set, with respect to the equivalence relation R over $\mathcal{V}$. Then the set $\mathcal{V}/R = \{[v]_R \mid v \in \mathcal{V}\} \in PA(\mathcal{V})$, where $[v]_R$ is an equivalence class of v. For convenience, this set can be expressed as $\mathcal{V}/R = \mathcal{V}_k = \{Y_i = [v_i]_R \mid v_i \in \mathcal{V}\}$. Now, consider a soft set $(\Omega, \mathfrak{P})$ over $\mathcal{V}$ such that $\Omega(S\mathfrak{P}) = \mathcal{V}_k$. Then $(\Omega, S\mathfrak{P}) = \{(\wp, Y) \mid \wp \in S\mathfrak{P}, Y \in \mathcal{V}_k\}$ is a strait soft set of $(\Omega, \mathfrak{P})$. Then the upper and lower approximations of X are $R^*(X) = \bigcup_{v \in \mathcal{V}}\{[v]_R \mid [v]_R \cap X \neq \emptyset\} = \{\bigcup Y \mid \Omega(\wp) = Y \ for \ \wp \in \mathfrak{P} \ and \ Y \cap X \neq \emptyset\} = \Omega^*(X)$ and $R_*(X) = \bigcup_{v \in \mathcal{V}}\{[v]_R \mid [v]_R \subseteq X\} = \{\bigcup Y \mid \Omega(\wp) = Y \ for \ \wp \in \mathfrak{P} \ and \ Y \subseteq X\} = \Omega_*(X)$. Since X is a rough set, then $R^*(X) - R_*(X) = \Omega^*(X) - \Omega_*(X) = B_\Omega(X) \neq \emptyset$. Hence, X is a strait rough set, by Definition 14. □

Theorem 2 *Every strait rough set may be considered as a soft set.*

Proof Let X be a strait rough set depending on strait soft set $(\Omega, S\mathfrak{P})$. Then the set X is expressed by approximations $\Omega_*(X)$ and $\Omega^*(X)$. Consider the predicates λ which stands for "$\Lambda = [\Omega(\wp) \subseteq X, \wp \in \mathfrak{P}]$," and γ which stands for "$\Gamma = [\Omega(\wp) \cap X \neq \emptyset, \wp \in \mathfrak{P}]$." The conditions λ and γ may be treated as elements of a parameter set; that is, $\mathcal{E} = \{\lambda, \gamma\} \subset \mathfrak{Q}$. Then we have a soft set defined as $F : \mathcal{E} \to P(\mathcal{V})$, $F(\lambda) = \{\Omega(\wp) \subseteq \mathcal{V} \mid \Lambda \text{ is true}\}$ and $F(\gamma) = \{\Omega(\wp) \subseteq \mathcal{V} \mid \Gamma \, is \, true\}$. Therefore, a strait rough set X depending on strait soft set $(\Omega, S\mathfrak{P})$ may be expressed by the soft set $(F, \mathcal{E}) = \{(\lambda, \Omega_*(X)), (\gamma, \Omega^*(X))\}$. □

The relationships among strait soft set, strait rough set, soft set (full soft set, covering soft set, partition soft set, bijective soft set) and rough set are illustrated in Fig. 4.

In Theorems 1 and 2, we proved that each rough set is a strait rough set and a strait rough set may be considered as a soft set. Feng et al. (2010) stated that every partition soft set is a covering soft set and covering soft set is also in the space of full soft set. Considering Definitions 7, 8 and 10, we can say that strait soft set covers both partition soft set and bijective soft set. Also, if $S\mathfrak{P} = \mathfrak{P}$, i.e., $\Omega(\wp) \neq \emptyset$ for all $\wp \in \mathfrak{P}$, then every strait soft set is a partition soft set and so a covering soft set (from Definitions 6, 7 and 10). Further, since every bijective soft set is a strait soft set, it is obvious that every bijective soft set is a partition soft set and so a covering soft set. On the other hand, a partition soft set is generally not a bijective soft set (from Definition 8 (b)). Also, in the structure of a covering soft set, it may be $\Omega(\wp_i) \cap \Omega(\wp_j) \neq \emptyset$ for $\wp_i, \wp_j \in \mathfrak{P}$ ($\wp_i \neq \wp_j$), and therefore, we can say that the space of strait soft sets does not cover the space of covering soft sets.

We can characterize the parameters for a strait rough set X, with the following definition:

Definition 15 Let X be a strait rough set depending on $(\Omega, S\mathfrak{P}) \in SS(\mathcal{V})$.

1. The definitely set of parameters for X depending on $(\Omega, S\mathfrak{P})$ is the set
$$\Delta_* = \{\wp \in \mathfrak{P} \mid \Omega(\wp) = Y \in \Omega_*(X)\}$$
2. The possibly set of parameters for X depending on $(\Omega, S\mathfrak{P})$ is the set
$$\Delta^* = \{\wp \in \mathfrak{P} \mid \Omega(\wp) = Y \in \Omega^*(X)\}$$
3. The boundary set of parameters for X depending on $(\Omega, S\mathfrak{P})$ is the set
$$B_{\mathfrak{P}}(X) = \Delta^* - \Delta_*$$

By this definition, we have
$\wp \in \Delta_*$ means that alternatives which definitely bear the property $\mathcal{P}$ correspond to the parameter $\wp$,
$\wp \in \Delta^*$ means that alternatives which possibly bear the property $\mathcal{P}$ correspond to the parameter $\wp$.
Clearly, X is a strait definable set depending on $(\Omega, S\mathfrak{P})$ iff $B_{\mathfrak{P}}(X) = \emptyset$ and X is a strait rough set depending on $(\Omega, S\mathfrak{P})$ iff $B_{\mathfrak{P}}(X) \neq \emptyset$.

Example 4 Let the strait soft sets $(\Omega_1, S\mathfrak{P}_1)$ and $(\Omega_2, S\mathfrak{P}_2)$ over the universe $\mathcal{V} = \{v_1, v_2, v_3\}$ given in Example 1. Assume that $X = \{v_1, v_3\}$ is the set of all alternatives which satisfy the property $\mathcal{P}$. Then we have, $(\Omega_1)_*(X) = \{v_3\}$, $(\Omega_1)^*(X) = \{v_1, v_2, v_3\}$, $B_{\Omega_1}(X) = \{v_1, v_2\} \neq \emptyset$ and then X is a strait rough set depending on $(\Omega_1, S\mathfrak{P}_1)$. Furthermore, $\Delta_* = \{\wp_3\}$, i.e., $\wp_3$ is the definitely parameter for X depending on $(\Omega_1, S\mathfrak{P}_1)$, $\Delta^* = \{\wp_1, \wp_2, \wp_3, \wp_4\}$ is the possibly set of parameters for X depending on $(\Omega_1, S\mathfrak{P}_1)$ and $B_{\mathfrak{P}}(X) = \{\wp_1, \wp_2, \wp_4\}$. Now, we consider $(\Omega_2, S\mathfrak{P}_2)$. Then we have, $(\Omega_2)_*(X) = \{v_1, v_3\} = (\Omega_2)^*(X)$, $B_{\Omega_2}(X) = \emptyset$ and then X is a strait definable set depending on $(\Omega_2, S\mathfrak{P}_2)$. Furthermore, $\Delta_* = \Delta^* = \{\wp_1, \wp_3\}$ and $B_{\mathfrak{P}}(X) = \emptyset$ depending on $(\Omega_2, S\mathfrak{P}_2)$.

Proposition 3 *Let $(\Omega, S\mathfrak{P}) \in SS(\mathcal{V})$ such that $\Omega(S\mathfrak{P}) = \mathcal{V}_k \in PA(\mathcal{V})$ and let $X \subseteq \mathcal{V}$. If $|\mathcal{V}_k| = |\mathcal{V}|$, then X is a strait definable set depending on $(\Omega, S\mathfrak{P})$.*

Proof If $|\mathcal{V}_k| = |\mathcal{V}|$, then $\Omega(\wp)$ is a single-valued set for each $\wp \in S\mathfrak{P}$. Let $\wp \in \Omega^*(X)$. Then $\Omega(\wp) \cap X \neq \emptyset$. Since $\Omega(\wp)$ is a single-valued set, then $\Omega(\wp) \subseteq X$. Hence, $\wp \in \Omega_*(X)$. Therefore, $\Omega^*(X) \subseteq \Omega_*(X)$. Since $\Omega_*(X) \subseteq \Omega^*(X)$ is already satisfied, then $B_\Omega(X) = \emptyset$. □

Corollary 1 *Let $(\Omega, S\mathfrak{P}) \in SS(\mathcal{V})$ such that $\Omega(S\mathfrak{P}) = \mathcal{V}_k \in PA(\mathcal{V})$ and let $X \subseteq \mathcal{V}$. If $(\Omega, S\mathfrak{P})$ is a bijective soft set over $\mathcal{V}$, then X is a strait definable set depending on $(\Omega, S\mathfrak{P})$.*

Proof If $(\Omega, S\mathfrak{P})$ is a bijective soft set over $\mathcal{V}$, then $|S\mathfrak{P}| = |\mathcal{V}_k| = |\mathcal{V}|$. Hence, the result follows Proposition 3. □

Proposition 4 *Let $X \subseteq \mathcal{V}$ and $(\Omega, S\mathfrak{P}) \in SS(\mathcal{V})$ such that $\Omega(S\mathfrak{P}) = X \in PA(\mathcal{V})$. Then X is a strait definable set depending on $(\Omega, S\mathfrak{P})$.*

Proof If $\Omega(S\mathfrak{P}) = X \in PA(\mathcal{V})$, then $\Omega_*(X) = X = \Omega^*(X)$. Therefore, $B_{\mathfrak{P}}(X) = \emptyset$. □

Definition 16 Let $\mathcal{V}$ be an universe, $\emptyset \neq X \subseteq \mathcal{V}$ and let $(\Omega, S\mathfrak{P}) \in SS(\mathcal{V})$.

1. The internal measure of X depending on $(\Omega, S\mathfrak{P})$ is the number
$$M_\Omega(X) = |\Omega_*(X)|$$
2. The external measure of X depending on $(\Omega, S\mathfrak{P})$ is the number
$$M^\Omega(X) = |\Omega^*(X)|$$
3. The accuracy of X depending on $(\Omega, S\mathfrak{P})$ is the number
$$A_\Omega(X) = \frac{M_\Omega(X)}{M^\Omega(X)}.$$
4. X is called measurable depending on $(\Omega, S\mathfrak{P})$, if $M_\Omega(X) = M^\Omega(X)$ or $A_\Omega(X) = 1$.

Remark 3 Let $\mathcal{V}$ be an universe, $\emptyset \neq X \subseteq \mathcal{V}$ and let $(\Omega, S\mathfrak{P}) \in SS(\mathcal{V})$ such that $\Omega(S\mathfrak{P}) = \mathcal{V}_k \in PA(\mathcal{V})$. Then $\Omega(\wp) \neq \emptyset$ for all $\wp \in S\mathfrak{P}$. Furthermore, since $\mathcal{V}_k$ is a partition of $\mathcal{V}$, then $\Omega^*(X) \neq \emptyset$. Therefore, $M^\Omega(X) > 0$. Hence, $A_\Omega(X)$ is always determined. But, in Pawlak's rough set (Pawlak 1982), if the external measure of X is different from 0, its accuracy was defined. Obviously, $0 \leq M_\Omega(X) \leq |\mathcal{V}|$,

$0 < M_\Omega(X) \leq |\mathcal{V}|$ and then $0 \leq A_\Omega(X) \leq 1$ for all $\emptyset \neq X \subseteq \mathcal{V}$ and $(\Omega, S\mathfrak{P}) \in SS(\mathcal{V})$. And also $\Omega_*(X) = \emptyset$ iff $A_\Omega(X) = 0$.

Definition 17 Let $\mathcal{V}$ be an universe, $\emptyset \neq X \subseteq \mathcal{V}$ and let $(\Omega, S\mathfrak{P}) \in SS(\mathcal{V})$. If Δ_* and Δ^* are definitely and possibly sets of parameters for X depending on $(\Omega, S\mathfrak{P})$, respectively, then

1. The lower parameter effect factor for X depending on $(\Omega, S\mathfrak{P})$ is the number
$$\underline{\lambda}_\Omega(X) = |\Delta_*|.$$
2. The upper parameter effect factor for X depending on $(\Omega, S\mathfrak{P})$ is the number
$$\overline{\lambda}_\Omega(X) = |\Delta^*|.$$
3. The parameter effect ratio for X depending on $(\Omega, S\mathfrak{P})$ is the number
$$\Lambda_\Omega(X) = \frac{\underline{\lambda}_\Omega(X)}{\overline{\lambda}_\Omega(X)}.$$

By Remark 3, it easily seen that $0 \leq \underline{\lambda}_\Omega(X) \leq |S\mathfrak{P}|$, $0 < \overline{\lambda}_\Omega(X) \leq |S\mathfrak{P}|$ and then $0 \leq \Lambda_\Omega(X) \leq 1$ for all $\emptyset \neq X \subseteq \mathcal{V}$ and $(\Omega, S\mathfrak{P}) \in SS(\mathcal{V})$.
By Definitions 10, 14, 15, 16 and 17, we have the following properties:

Proposition 5 *Let* $(\Omega, S\mathfrak{P}) \in SS(\mathcal{V})$ *such that* $\Omega(S\mathfrak{P}) = \mathcal{V}_k \in PA(\mathcal{V})$ *and let* $\emptyset \neq X \subseteq \mathcal{V}$*. Then,*

1. $\emptyset \subseteq \Omega_*(X) \subseteq X \subseteq \Omega^*(X)$.
2. $0 \leq M_\Omega(X) \leq |X| \leq M^\Omega(X)$.
3. $|\mathcal{V}_k| \leq |S\mathfrak{P}|$.
4. $0 \leq M_\Omega(X) \leq \underline{\lambda}_\Omega(X)$.
5. $0 < M^\Omega(X) \leq \overline{\lambda}_\Omega(X)$.
6. $\emptyset \subseteq \Delta_* \subseteq \Delta^* \subseteq S\mathfrak{P}$.
7. $0 \leq \underline{\lambda}_\Omega(X) \leq \overline{\lambda}_\Omega(X) \leq |S\mathfrak{P}|$.

Proof The proofs are straightforward and therefore omitted. □

Example 5 Let the strait soft sets $(\Omega_1, S\mathfrak{P}_1)$ and $(\Omega_2, S\mathfrak{P}_2)$ over the universe $\mathcal{V} = \{v_1, v_2, v_3\}$ and let $X = \{v_1, v_3\}$ given in Example 4. Then we have the following: $M_{\Omega_1}(X) = 1$, $M^{\Omega_1}(X) = 3$ and $A_{\Omega_1}(X) = \frac{1}{3}$. $\underline{\lambda}_{\Omega_1}(X) = 1$, $\overline{\lambda}_{\Omega_1}(X) = 4$ and $\Lambda_{\Omega_1}(X) = \frac{1}{4}$. $M_{\Omega_2}(X) = M^{\Omega_2}(X) = 2$ and, $A_{\Omega_2}(X) = 1$. $\underline{\lambda}_{\Omega_2}(X) = \overline{\lambda}_{\Omega_2}(X) = 2$ and $\Lambda_{\Omega_2}(X) = 1$.

5 A new decision-making approach based on strait soft sets and strait rough sets

Let $\mathcal{V} = \{v_i \mid i = 1, 2, ..., n\}$ be a set of alternatives, $\mathfrak{Q}_k = \{\wp_j^k \mid j = 1, 2, ..., m_k\}$ $(k = 1, 2, ..., s)$ be the disjoint attribute (parameter) sets and $\mathfrak{P}_k \subseteq \mathfrak{Q}_k$ for each $k \in \{1, 2, ..., s\}$. Also, $(\Omega_k, S\mathfrak{P}_k)$ $(k = 1, 2, ..., s)$ be the strait soft sets of $(\Omega_k, \mathfrak{P}_k)$ $(k = 1, 2, ..., s)$ over $\mathcal{V}$. Assume that X_l $(l = 1, 2, ..., r)$ are subsets of $\mathcal{V}$ and denote primary results (evaluations).

5.1 Decision-making model

The following decision-making model is created by approaching multi-attribute decision making with primary results (evaluations) from a different perspective.

Algorithm

Step 1 Take the primary results (evaluations) X_l $(l = 1, 2, ..., r)$ for the alternatives

Step 2 Determine the subsets $\mathfrak{P}_k$ $(k = 1, 2, ..., s)$ of the disjoint parameter sets $\mathfrak{Q}_k$ $(k = 1, 2, ..., s)$, and then, construct the strait soft sets $(\Omega_k, S\mathfrak{P}_k)$ $(k = 1, 2, ..., s)$

Step 3 Find the strait soft lower and upper approximations of X_l $(l = 1, 2, ..., r)$ depending on $(\Omega_k, S\mathfrak{P}_k)$ $(k = 1, 2, ..., s)$

Step 4 According to the strait soft lower and upper approximations depending on $(\Omega_k, S\mathfrak{P}_k)$ $(k = 1, 2, ..., s)$, determine the definitely set ${}^{k}_{l}\Delta_*$ and the possibly set ${}^{k}_{l}\Delta^*$ of parameters in $S\mathfrak{P}_k$ $(k = 1, 2, ..., s)$ for X_l $(l = 1, 2, ..., r)$

Step 5 For each $S\mathfrak{P}_k$ and X_l, compute (k, l)-lower approximation vector ${}^{k}_{l}\underline{\psi} = ({}^{k}_{l}\underline{\varpi}_1, {}^{k}_{l}\underline{\varpi}_2, ..., {}^{k}_{l}\underline{\varpi}_{m_k})$ and (k, l)-upper approximation vector ${}^{k}_{l}\overline{\psi} = ({}^{k}_{l}\overline{\varpi}_1, {}^{k}_{l}\overline{\varpi}_2, ..., {}^{k}_{l}\overline{\varpi}_{m_k})$, where

$${}^{k}_{l}\underline{\varpi}_j = \begin{cases} 1, \text{ if } \wp_j^k \in {}^{k}_{l}\Delta_* \\ 0, \text{ if } \wp_j^k \notin {}^{k}_{l}\Delta_* \end{cases}$$

and

$${}^{k}_{l}\overline{\varpi}_j = \begin{cases} \frac{1}{\alpha_k}, \text{ if } \wp_j^k \in {}^{k}_{l}\Delta^* \\ 0, \quad \text{if } \wp_j^k \notin {}^{k}_{l}\Delta^* \end{cases}$$

where $\alpha_k \geq 1$ is any real number arbitrarily or specifically (non-arbitrarily) determined by the decision maker (expert).

Step 6 For each $S\mathfrak{P}_k$ and X_l, compute (k, l)-approximation vector

$${}^{k}_{l}\psi = (\max\{{}^{k}_{l}\underline{\psi}, {}^{k}_{l}\overline{\psi}\}) =$$
$$(\max\{{}^{k}_{l}\underline{\varpi}_1, {}^{k}_{l}\overline{\varpi}_1\}, \max\{{}^{k}_{l}\underline{\varpi}_2, {}^{k}_{l}\overline{\varpi}_2\}, ..., \max\{{}^{k}_{l}\underline{\varpi}_{m_k}, {}^{k}_{l}\overline{\varpi}_{m_k}\})$$

Step 7 For each $S\mathfrak{P}_k$, calculate the decision vector ${}^{k}\psi$, formulated as

$${}^{k}\psi = \frac{1}{r}\sum_{l=1}^{r} {}^{k}_{l}\psi$$

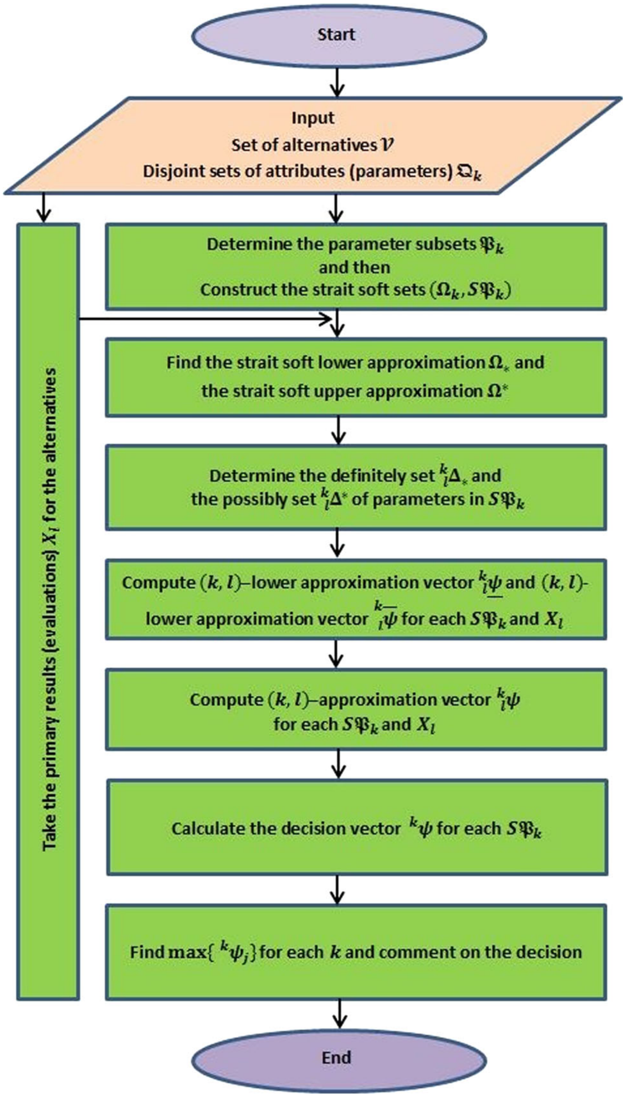


Fig. 5 Procedural steps of algorithm

Step 8 Find $\max\{{}^k\psi_j\}$ for each $k \in \{1, 2, ..., s\}$, and then, comment on the decision
The step-by-step procedure of algorithm is presented in Fig. 5.

5.2 Application of the proposed approach

To illustrate the decision process of the above algorithm, we present the following examples.

Example 6 Let $\mathcal{V} = \{v_1, v_2, ..., v_8\}$ be a set of some smartphones released in 2021. The sets of best-selling smartphones of these smartphones in the countries C_1, C_2 and C_3 are, respectively, X_1, X_2 and X_3 given in Table 1.

An expert group will first determine some attributes or parameters for the smartphones in the set $\mathcal{V}$ and then decide which attributes can be recommended for the smartphones that will be manufactured in the future, considering the best sellers of these smartphones in the countries C_1, C_2, C_3.

Table 1 Primary results (evaluations) for three countries

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
X_1	✓	×	✓	×	×	×	✓	✓
X_2	✓	×	×	×	✓	✓	×	✓
X_3	×	✓	×	×	✓	×	✓	×

In the table, the symbols ✓ and × represent *yes* and *no*, respectively

Table 2 Strait soft set $(\mathfrak{Q}_1, S\mathfrak{P}_1)$

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
$\wp_1^1$	0	0	0	0	0	0	1	1
$\wp_2^1$	1	0	0	0	0	0	0	0
$\wp_3^1$	0	1	0	0	0	1	0	0
$\wp_4^1$	0	0	1	1	1	0	0	0

The sets of attributes for different colors, screen technologies and materials are, respectively,

$$\mathfrak{Q}_1 = \left\{ \begin{array}{l} \wp_1^1 = \text{light color (white, pink, light blue, yellow, etc.)}, \\ \wp_2^1 = \text{dark color (black, brown, smoked, navy blue, etc.)}, \\ \wp_3^1 = \text{golden} - \text{silver color}, \\ \wp_4^1 = \text{mixed color} \end{array} \right\},$$

$$\mathfrak{Q}_2 = \left\{ \begin{array}{l} \wp_1^2 = \text{OLED}, \\ \wp_2^2 = \text{AMOLED}, \\ \wp_3^2 = \text{Super AMOLED}, \\ \wp_4^2 = \text{TFT LCD}, \\ \wp_5^2 = \text{IPS LCD}, \\ \wp_6^2 = \text{PLS LCD} \end{array} \right\},$$

and

$$\mathfrak{Q}_3 = \left\{ \begin{array}{l} \wp_1^3 = \text{plastic} - \text{glass}, \\ \wp_2^3 = \text{plastic} - \text{aluminum}, \\ \wp_3^3 = \text{polycarbonate} - \text{glass}, \\ \wp_4^3 = \text{aluminum} - \text{glass}, \\ \wp_5^3 = \text{aluminum} - \text{polycarbonate} \end{array} \right\}.$$

The expert group determines subsets from these disjoint attribute sets as follows: $\mathfrak{P}_1 = \mathfrak{Q}_1, \mathfrak{P}_2 = \{\wp_1^2, \wp_2^2, \wp_3^2, \wp_4^2, \wp_5^2\} \subseteq \mathfrak{Q}_2$ and $\mathfrak{P}_3 = \mathfrak{Q}_3$. The expert group evaluates the smartphones under these subsets and then creates the strait soft sets as given in Tables 2, 3 and 4.
Note that since $\Omega_3(\wp_5^3) = \emptyset$ it is not included in Table 4.

The primary results (evaluations) of best-selling smartphones in the countries C_1, C_2 and C_3 and expert group's evaluations for smartphones according to the attributes, such as color, screen size and material, are presented in Fig. 6.

We find the strait soft lower and upper approximations of $(\mathfrak{Q}_k, S\mathfrak{P}_k)$ $(k = 1, 2, 3)$ with respect to the primary results (evaluations) X_l $(l = 1, 2, 3)$ as given in Table 5.

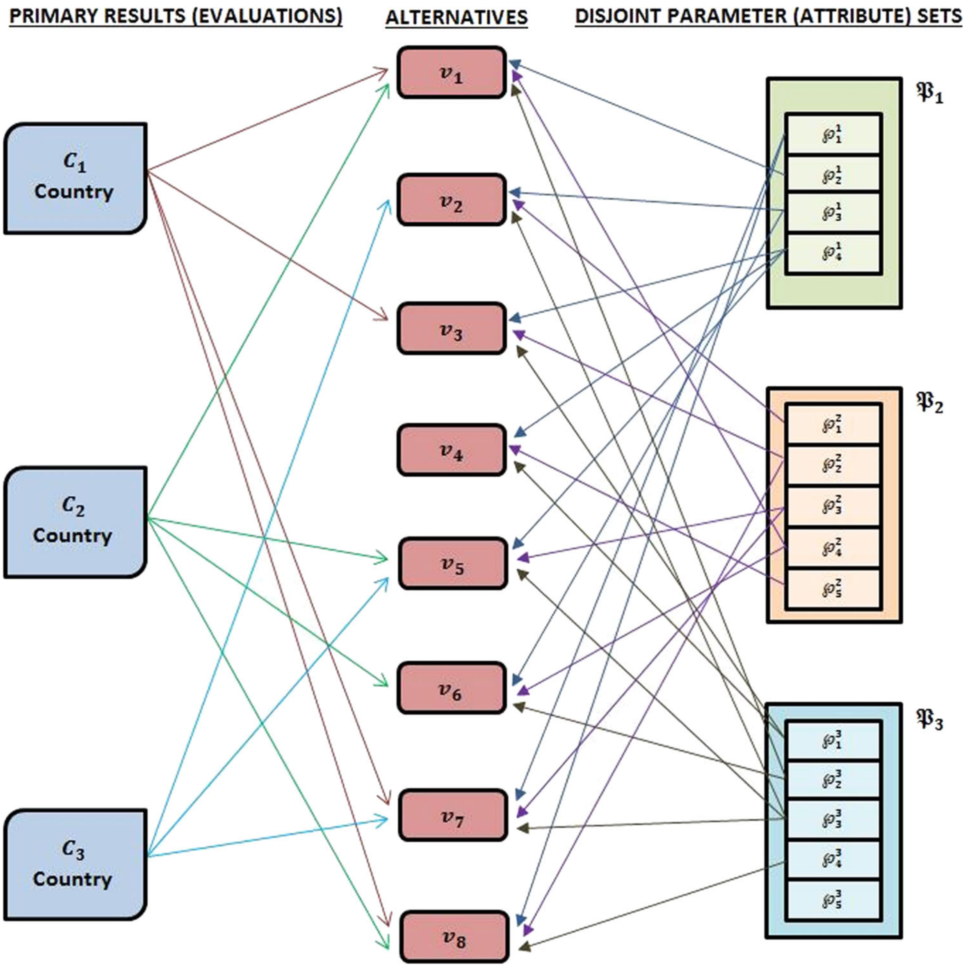


Fig. 6 Figuration of the problem in Example 6

Table 3 Strait soft set $(\Omega_2, S\mathfrak{P}_2)$

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
$\wp_1^2$	0	1	0	0	0	0	0	0
$\wp_2^2$	0	0	1	0	0	0	0	1
$\wp_3^2$	0	0	0	0	1	0	1	0
$\wp_4^2$	1	0	0	0	0	1	0	0
$\wp_5^2$	0	0	0	1	0	0	0	0

Table 4 Strait soft set $(\Omega_3, S\mathfrak{P}_3)$

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
$\wp_1^3$	0	0	1	1	0	0	0	0
$\wp_2^3$	1	0	0	0	0	1	0	0
$\wp_3^3$	0	1	0	0	1	0	1	0
$\wp_4^3$	0	0	0	0	0	0	0	1

By considering Table 5, we create Table 6, which presents the definitely sets ${}^k_l\Delta_*$ and the possibly sets ${}^k_l\Delta^*$ of parameters in $S\mathfrak{P}_k$ $(k = 1, 2, 3)$ for X_l $(l = 1, 2, 3)$.

Table 5 Strait soft lower and upper approximations of X_l depending on $(\Omega_k, S\mathfrak{P}_k)$

X_l	$S\mathfrak{P}_k$	$\Omega_*(X_l)$	$\Omega^*(X_l)$
X_1	$S\mathfrak{P}_1$	$\{v_1, v_7, v_8\}$	$\{v_1, v_3, v_4, v_5, v_7, v_8\}$
	$S\mathfrak{P}_2$	$\{v_3, v_8\}$	$\{v_1, v_3, v_5, v_6, v_7, v_8\}$
	$S\mathfrak{P}_3$	$\{v_8\}$	$\mathcal{V}$
X_2	$S\mathfrak{P}_1$	$\{v_1\}$	$\mathcal{V}$
	$S\mathfrak{P}_2$	$\{v_1, v_6\}$	$\{v_1, v_3, v_5, v_6, v_7, v_8\}$
	$S\mathfrak{P}_3$	$\{v_1, v_6, v_8\}$	$\{v_1, v_2, v_5, v_6, v_7, v_8\}$
X_3	$S\mathfrak{P}_1$	$\emptyset$	$\{v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$
	$S\mathfrak{P}_2$	$\{v_2, v_5, v_7\}$	$\{v_2, v_5, v_7\}$
	$S\mathfrak{P}_3$	$\{v_2, v_5, v_7\}$	$\{v_2, v_5, v_7\}$

Table 7 presents (k, l)-lower approximation vectors ${}^k_l\underline{\psi}$, (k, l)-upper approximation vectors ${}^k_l\overline{\psi}$ and (k, l)-approximation vectors ${}^k_l\psi$, where $\alpha_1 = \alpha_2 = \alpha_3 = 2$.

By considering the approximation vectors ${}^k_l\psi$ in Table 7, we calculate the decision vectors ${}^k\psi$ $(k = 1, 2, 3)$ as follows:

Table 6 Definitely and possibly sets of parameters in $S\mathfrak{P}_k$ for X_l

X_l	$S\mathfrak{P}_k$	${}^k_l\Delta_*$	${}^k_l\Delta^*$
X_1	$S\mathfrak{P}_1$	$\{\wp_1^1, \wp_2^1\}$	$\{\wp_1^1, \wp_2^1, \wp_4^1\}$
	$S\mathfrak{P}_2$	$\{\wp_2^2\}$	$\{\wp_2^2, \wp_3^2, \wp_4^2\}$
	$S\mathfrak{P}_3$	$\{\wp_4^3\}$	$S\mathfrak{P}_3$
X_2	$S\mathfrak{P}_1$	$\{\wp_2^1\}$	$S\mathfrak{P}_1$
	$S\mathfrak{P}_2$	$\{\wp_4^2\}$	$\{\wp_2^2, \wp_3^2, \wp_4^2\}$
	$S\mathfrak{P}_3$	$\{\wp_2^3, \wp_4^3\}$	$\{\wp_2^3, \wp_3^3, \wp_4^3\}$
X_3	$S\mathfrak{P}_1$	$\emptyset$	$\{\wp_1^1, \wp_3^1, \wp_4^1\}$
	$S\mathfrak{P}_2$	$\{\wp_1^2, \wp_3^2\}$	$\{\wp_1^2, \wp_3^2\}$
	$S\mathfrak{P}_3$	$\{\wp_3^3\}$	$\{\wp_3^3\}$

Table 7 Approximation vectors ${}^k_l\psi$ for $S\mathfrak{P}_k$ for X_l

X_l	$S\mathfrak{P}_k$	${}^k_l\underline{\psi}$	${}^k_l\overline{\psi}$	${}^k_l\psi$
X_1	$S\mathfrak{P}_1$	$(1, 1, 0, 0)$	$(\frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2})$	$(1, 1, 0, \frac{1}{2})$
	$S\mathfrak{P}_2$	$(0, 1, 0, 0, 0)$	$(0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0)$	$(0, 1, \frac{1}{2}, \frac{1}{2}, 0)$
	$S\mathfrak{P}_3$	$(0, 0, 0, 1)$	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1)$
X_2	$S\mathfrak{P}_1$	$(0, 1, 0, 0)$	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, 1, \frac{1}{2}, \frac{1}{2})$
	$S\mathfrak{P}_2$	$(0, 0, 0, 1, 0)$	$(0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0)$	$(0, \frac{1}{2}, \frac{1}{2}, 1, 0)$
	$S\mathfrak{P}_3$	$(0, 1, 0, 1)$	$(0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	$(0, 1, \frac{1}{2}, 1)$
X_3	$S\mathfrak{P}_1$	$(0, 0, 0, 0)$	$(\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2})$
	$S\mathfrak{P}_2$	$(1, 0, 1, 0, 0)$	$(\frac{1}{2}, 0, \frac{1}{2}, 0, 0)$	$(1, 0, 1, 0, 0)$
	$S\mathfrak{P}_3$	$(0, 0, 1, 0)$	$(0, 0, \frac{1}{2}, 0)$	$(0, 0, 1, 0)$

$$
{}^1\psi = (\tfrac{2}{3}, \tfrac{2}{3}, \tfrac{1}{3}, \tfrac{1}{2}),\ {}^2\psi = (\tfrac{1}{3}, \tfrac{1}{2}, \tfrac{2}{3}, \tfrac{1}{2}, 0) \text{ and } \\ {}^3\psi = (\tfrac{1}{6}, \tfrac{1}{2}, \tfrac{2}{3}, \tfrac{2}{3}).
$$

Then we obtain

- for $k = 1$, $\max\{{}^1\psi_j\} = \{{}^1\psi_1, {}^1\psi_2\}$
- for $k = 2$, $\max\{{}^2\psi_j\} = \{{}^2\psi_3\}$
- for $k = 3$, $\max\{{}^3\psi_j\} = \{{}^3\psi_3, {}^3\psi_4\}$.

Hence, considering the best-selling smartphones in the countries C_1, C_2 and C_3, we recommend the following attributes for the smartphones that will be manufactured in the future to sell well in these countries:

- Color: light, Screen technology: Super AMOLED, Material: polycarbonate–glass,
- Color: light, Screen technology: Super AMOLED, Material: aluminum–glass,
- Color: dark, Screen technology: Super AMOLED, Material: polycarbonate–glass,
- Color: dark, Screen technology: Super AMOLED, Material: aluminum–glass.

Let us analyze the results according to the arbitrary and special choice of α_k $(k = 1, 2, 3)$.

i. Assume that $\alpha_1 = \alpha_2 = \alpha_3 = 5$. Then, we calculate the decision vectors ${}^k\psi$ $(k = 1, 2, 3)$ as follows:

$$
{}^1\psi = (\tfrac{7}{15}, \tfrac{2}{3}, \tfrac{2}{15}, \tfrac{1}{5}),\ {}^2\psi = (\tfrac{1}{3}, \tfrac{2}{5}, \tfrac{7}{15}, \tfrac{2}{5}, 0), \\ {}^3\psi = (\tfrac{1}{15}, \tfrac{2}{5}, \tfrac{7}{15}, \tfrac{2}{3}),
$$

and so

- for $k = 1$, $\max\{{}^1\psi_j\} = \{{}^1\psi_2\}$
- for $k = 2$, $\max\{{}^2\psi_j\} = \{{}^2\psi_3\}$
- for $k = 3$, $\max\{{}^3\psi_j\} = \{{}^3\psi_4\}$.

That is, we recommend

- Color: dark, Screen technology: Super AMOLED, Material: aluminum–glass.

ii. Assume that $\alpha_1 = \alpha_2 = \alpha_3 = 1$. Then, we calculate the decision vectors ${}^k\psi$ $(k = 1, 2, 3)$ as follows:

$$
{}^1\psi = (1, \tfrac{2}{3}, \tfrac{2}{3}, 1),\ {}^2\psi = (\tfrac{1}{3}, \tfrac{2}{3}, 1, \tfrac{2}{3}, 0), \\ {}^3\psi = (\tfrac{1}{3}, \tfrac{2}{3}, 1, \tfrac{2}{3}),
$$

and so

- for $k = 1$, $\max\{{}^1\psi_j\} = \{{}^1\psi_1, {}^1\psi_4\}$
- for $k = 2$, $\max\{{}^2\psi_j\} = \{{}^2\psi_3\}$
- for $k = 3$, $\max\{{}^3\psi_j\} = \{{}^3\psi_3\}$.

That is, we recommend

- Color: light, Screen technology: Super AMOLED, Material: polycarbonate–glass,
- Color: mixed, Screen technology: Super AMOLED, Material: polycarbonate–glass.

In the parts (i) and (ii), we analyzed the results for the equality of α_1, α_2 and α_3. These values may not be equal. Now let us analyze the results for different values of α_1, α_2 and α_3 depending on special conditions.

iii. Let α_1, α_2 and α_3 be determined as the maximum values of $\overline{\lambda}_{\Omega_1}(X_l) = |{}^1_l\Delta^*|$, $\overline{\lambda}_{\Omega_2}(X_l) = |{}^2_l\Delta^*|$ and $\overline{\lambda}_{\Omega_3}(X_l) = |{}^3_l\Delta^*|$ $(l = 1, 2, 3)$, that is, let $\alpha_1 = \max\{3, 4, 3\} = 4$, $\alpha_2 = \max\{3, 3, 2\} = 3$ and $\alpha_3 = \max\{4, 3, 1\} = 4$. Then, we obtain the decision vectors ${}^k\psi$ $(k = 1, 2, 3)$ as

$$
{}^1\psi = (\tfrac{1}{2}, \tfrac{2}{3}, \tfrac{1}{6}, \tfrac{1}{4}),\ {}^2\psi = (\tfrac{1}{3}, \tfrac{4}{9}, \tfrac{5}{9}, \tfrac{4}{9}, 0), \\ {}^3\psi = (\tfrac{1}{12}, \tfrac{5}{12}, \tfrac{1}{2}, \tfrac{2}{3}),
$$

and so

- for $k = 1$, $\max\{{}^1\psi_j\} = \{{}^1\psi_2\}$
- for $k = 2$, $\max\{{}^2\psi_j\} = \{{}^2\psi_3\}$
- for $k = 3$, $\max\{{}^3\psi_j\} = \{{}^3\psi_4\}$.

Table 8 Strait soft set $(\Omega_4, S\mathfrak{P}_4)$

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
$\wp_1^4$	1	0	0	0	1	0	0	0
$\wp_2^4$	0	1	0	0	0	1	1	0
$\wp_3^4$	0	0	1	1	0	0	0	1
$\wp_4^4$	0	0	1	1	0	0	0	1
$\wp_5^4$	1	0	0	0	1	0	0	0

That is, we recommend

- Color: dark, Screen technology: Super AMOLED, Material: aluminum–glass.

The strait soft sets $(\Omega_1, S\mathfrak{P}_1)$ (Table 2), $(\Omega_2, S\mathfrak{P}_2)$ (Table 3) and $(\Omega_3, S\mathfrak{P}_3)$ (Table 4) in Example 6 are also bijective soft sets. The following example is given to show the applicability of the proposed algorithm for samples that are not bijective soft sets but strait soft sets.

Example 7 Consider the strait soft sets $(\Omega_1, S\mathfrak{P}_1)$, $(\Omega_2, S\mathfrak{P}_2)$ and $(\Omega_3, S\mathfrak{P}_3)$, and the primary results X_1, X_2 and X_3 in Example 6. Also, we determine the attribute set for screen types (of smartphones) as

$$\mathfrak{Q}_4 = \left\{ \begin{array}{l} \wp_1^4 = \text{dual screen}, \\ \wp_2^4 = \text{classic single screen}, \\ \wp_3^4 = \text{expandable single screen}, \\ \wp_4^4 = \text{rollable/sliding screen}, \\ \wp_5^4 = \text{foldable screen} \end{array} \right\}$$

and $\mathfrak{P}_4 = \mathfrak{Q}_4$. For the screen types of smartphones, the strait soft set $(\Omega_4, S\mathfrak{P}_4)$ in Table 8 can be created. (Note that the smartphones with dual screens have both a main screen and a secondary screen. Generally, in case of a second screen, this smartphone is foldable and can be used as a single screen. Also, a rollable/sliding display is an electronic visual display which is expandable in nature, as opposed to the traditional flat screen displays used in smartphones.)

Proceeding with the calculations, we have the decision vectors ${}^1\psi$, ${}^2\psi$, ${}^3\psi$ in Example 6 and

$${}^4\psi = (\tfrac{2}{3}, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{2}{3}).$$

Then, it is obtained $\max\{{}^4\psi_j\} = \{{}^4\psi_1, {}^4\psi_5\}$ and thus deduced as

- Color: light, Screen technology: Super AMOLED, Material: polycarbonate–glass, Screen types: dual screen (and foldable screen),
- Color: light, Screen technology: Super AMOLED, Material: aluminum–glass, Screen types: dual screen (and foldable screen),
- Color: dark, Screen technology: Super AMOLED, Material: polycarbonate–glass, Screen types: dual screen (and foldable screen),
- Color: dark, Screen technology: Super AMOLED, Material: aluminum–glass, Screen types: dual screen (and foldable screen).

Considering the best-selling smartphones in the countries C_1, C_2 and C_3, the above result for the screen type can be interpreted as follows: In these countries, the screen type of smartphone was preferred to be dual screen (and foldable screen) rather than classic or expandable single screen. As a result, it can be said that among the smartphones that will be manufactured in the future, those that are dual screen (and foldable screen) will take priority for preference.

5.3 Advantages and limitations of proposed approach

First, let us talk about the advantages of the proposed decision-making model by comparing it with some existing bijective soft decision-making models. In Gong et al. (2010); Kamacı et al. (2018b), the authors dealt with the multi-attribute decision-making problems under the bijective soft set environments. The multi-attribute decision problems they deal with are the same problems we deal with. It is obvious that the proposed decision-making algorithm can cope with the problems in the papers (Gong et al. 2010; Kamacı et al. 2018b) by introducing some novel operations or further computations. That is, when the problems dealt with in Sect. 4 by Gong et al. (2010) and in Sect. 6 by Kamacı et al. (2018b) are solved by our algorithm, similar outputs are obtained with theirs. But the decision-making models proposed in Gong et al. (2010); Kamacı et al. (2018b) cannot cope with Example 6 in this paper (for cases where bijectivity is not preserved). Thus, we can say that our proposed multi-attribute decision-making approach is more general than bijective soft decision systems (Gong et al. 2010; Kamacı et al. 2018b), as it is based on the amplitude of strait soft sets. Example 7 presented in this paper supports this deduction.

The limitations of our decision-making algorithm are as follows: It cannot currently handle multi-attribute decision-making problems that do not involve primary evaluations, but can be modified to overcome this limitation. Also, in the above examples, the effect ratios/weights of the primary evaluation results and attributes are neglected. It is not always possible in practice to assume that impact ratios/weights are neglected when making decisions. By considering these ratios/weights, the quality of multi-attribute decision-making process may be improved.

6 Conclusions

In this paper, two new concepts, strait soft set and strait rough set, were presented in detail to be used in modeling uncertainties. The strait soft sets were designed to provide a natural connection between the structures of soft set and rough set. In strait soft sets, taking the parameters from the support set of the soft set and their images belonging to a partition of the alternatives plays many facilitating roles such as reducing alternatives and fusion of parameters and constructs a new rough set structure. A strait soft set is also, by definition, a full soft set. If the images of the different parameters are discrete, then a strait soft set is also a bijective soft set (described in the article (Gong et al. 2010)). Therefore, strait soft sets can be thought of as a wider structure than bijective soft sets, and they can also be used in the application areas of bijective soft sets.

Due to the tight connection between the equivalence relations on the universe set and the partitions of the universe set, the structure and properties of Pawlak's rough set are available in strait rough sets. So the structure and properties of Pawlak's rough set naturally occur in strait rough sets. In fact, in strait rough sets, some properties are provided without a certain condition, for example, if the external measure is different from zero in Pawlak's rough set, its accuracy is defined, but in strait rough sets, the external measure is already different from zero. Also, measurements were defined for parameters in strait rough sets. Therefore, in application areas such as decision making, we contributed to a more accurate result by using the effect values of the parameters.

In this paper, the construction and motivation of strait soft set and strait rough set were presented. Also, the relationship between strait soft/rough sets, rough sets, soft sets, full soft sets and bijective soft sets was discussed. A decision-making model based on strait soft/rough sets was constructed and applied to a real-life problem. In future, the applications of strait soft/rough sets needs to be explored in various areas such as pattern recognition, supply chain, risk analysis, game theory, clustering analysis and medical diagnosis. It is worth mentioning that hybrid structures of the proposed strait soft/rough set approaches with other uncertain sets can be developed and their applications in many uncertain environments can be investigated. Algebraic and topological structures for strait soft/rough sets are further research topics for the future. We will endeavor to overcome these open problems in our future work.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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