



Hunting swans: looking to the future through fuzzy LMAW analysis of a gray swan scenario

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Abstract

In an era characterized by growing uncertainties, understanding and managing low-probability but high-impact events has become essential. This study examines such risks through the lens of the Gray Swan metaphor, focusing on earthquake risk in Turkey. To assess potential consequences and critical response factors, the Fuzzy Logarithmic Methodology of Additive Weights (F-LMAW) was used, incorporating insights from nine experts across fields such as disaster response, urban planning, and academia. Eight main criteria and 22 sub-criteria were identified and prioritized. The analysis showed that urban infrastructure (C1: 0.1366), communication systems (C8: 0.1354), and transportation networks (C2: 0.1343) are the most crucial components for effective post-disaster response. Among sub-criteria, debris and damage (C11: 0.0478), medical supply disruptions (C32: 0.0474), and internet access failures (C83: 0.0474) ranked highest. A sensitivity analysis confirmed that the model is robust, with minor weight adjustments having little effect on the overall priority structure. The study offers a practical model for the immediate response phase after a disaster, supporting efficient resource allocation. Additionally, it suggests that scenario-based planning for Gray Swans can help prevent the emergence of new Gray or even Black Swan events. These insights provide a strategic foundation for strengthening local resilience and developing flexible risk management policies to address unforeseen disasters.

Keywords Black swan · Gray swan · Crisis and future scenarios · Fuzzy set · LMAW

1 Introduction

Our rapidly changing and transforming world is surrounded by economic, political and environmental risks. The quality, scope and content of risks are differentiating and increasing day by day. Moreover, the effects of risks today are felt and spread globally. They can affect many more people in a much shorter period in terms of their consequences and the area they cover. Various metaphors, such as “gray swan” and “black swan”, introduced to the literature by Nassim Nicholas Taleb in 2007, offer different perspectives on risk prediction.

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Black Swans were defined by Taleb as events whose existence is unknown, unpredictable, extraordinary and have an extraordinary impact. Grey Swans, on the other hand, are a metaphor used to describe events that, although their existence and probability are known, are unlikely to occur and have the power to cause very important consequences (Taleb, 2007). In the literature, metaphors are named according to the degree of predictability of risks. The probability of the occurrence of grey swans can be predicted in advance, but this probability is considered very small. While the existence of grey swans can be predicted from various data, the existence of black swans is unknown since there is no data. Therefore, prediction models have not been created for such events, and they are unpredictable (Krausmann & Necci, 2021). Since people do not expect such events to occur and do not even know of their existence, they often face a crisis when swans suddenly appear. Throughout history, societies have encountered various economic, political, financial, and social events that, although unlikely to occur, have very effective consequences when they occur. One of the most important of these events has been natural disasters.

Although the existence of disasters is a known fact, it is unpredictable where, when, and how large they will occur. Natural disasters like volcanic eruptions, floods, tsunamis, and earthquakes can result in extremely severe consequences. Although it is very difficult to predict such disasters, it is possible to mitigate their consequences. Today, disaster-related dangers can threaten the future of people, institutions, organizations, and even countries. Black Swans are getting closer with each passing day, making it essential to act swiftly by embracing innovations and continuously enhancing resilience to deal with these challenges (Wu et al., 2022).

Planning for these scenarios helps societies manage uncertainty by preparing for crises and anticipating possible changes during such events. Scenarios differ from predictions, forecasts, and visions. They are vivid images of plausible futures that enable risk management (Lindgren & Bandhold, 2003). Risks are an unavoidable part of life, but they can be reduced through strategic adaptation rather than naive attempts to eliminate them. This perspective suggests that instead of focusing solely on uncertain risks, it is important to foresee their potential outcomes and develop plans to minimize their effects. In particular, the resilience of systems against unexpected, high-impact events, such as Black Swan events, becomes a crucial aspect of risk management. For example, in natural disasters like earthquakes, the occurrence is known, but the exact timing and magnitude are uncertain. This categorizes earthquakes as grey swan events, whose existence is recognized but whose specific effects and timings are unpredictable. Earthquakes in Turkey exemplify grey swans, given the well-known risk but uncertain occurrence. Managing grey swans highlights the importance of preparedness and strategies to anticipate their impacts. This study specifically examines a potential grey swan scenario involving an earthquake disaster in Turkey.

In this study, the grey swan scenario of earthquake disasters is approached as an MCDM problem using information from experts. MCDM is an approach to decision-making that takes into account a number of variables. Experts aim to find the best solution by evaluating various criteria (Çalikoğlu & Łuczak, 2024; Elraaid et al., 2024; Kizielewicz & Sałabun, 2024). In complex and multidimensional events like earthquakes, a more comprehensive analysis is needed by considering more than one criterion since evaluating only one factor is not sufficient. To address such situations, most researchers are using fuzzy logic and its extensions to handle uncertainty, like Wang et al. (2023) and Mishra et al. (2023). The fuzzy F-MCDM method used in this study is an approach designed to make decisions under

uncertainty and imprecise information. Traditional MCDM methods often assume exact and clear data, but F-MCDM approaches manage uncertainty and fuzzy data (Gazi et al., 2024; Hussain & Ullah, 2024). Particularly in predicting and managing large-scale events, fuzzy logic-based methods are very useful when data are uncertain (Kannan et al., 2024). We employed the F-LMAW approach to assess the main and sub-criteria. A summary of the F-LMAW method's advantages (Darıcı et al., 2024; Gholamizadeh et al., 2024; Karakuş, 2024; Nasution et al., 2024; Pamučar et al., 2021; Puška et al., 2022):

The LMAW methodology yields more consistent results than systems reliant on distance calculations between alternatives and reference sites, such as TOPSIS. The efficacy of the results remains constant despite fluctuations in the quantity of possibilities inside the original decision matrix. This improves the method's reliability in dynamic circumstances and ensures consistency in the decision-making process, even when alternatives fluctuate.

The F-LMAW approach successfully mitigates the rank reversal problem commonly seen in decision-making scenarios. This article discusses how the ranking of options varies as alternatives are added or eliminated. The LMAW technique is immune to this problem, preserving the integrity of rankings even when the number of alternatives changes. As a consequence, the outputs are consistent and dependable.

The mathematical framework's invariance: The mathematical structure of the method remains unchanged irrespective of the quantity of alternatives or criteria involved. This enhances the method's adaptability and ensures its consistent application across many issue contexts. It functions effectively in intricate environments with several alternatives and criteria.

The F-LMAW approach facilitates the evaluation of alternatives using qualitative and quantitative criteria. This capacity augments the method's adaptability in scenarios when numerical and verbal criteria are simultaneously employed. The integration of many criteria types enables decision-makers to comprehensively tackle multidimensional scenarios.

Optimal Scenarios for Collective Decision-Making: In collaborative decision-making situations, the F-LMAW technique integrates data using the Bonferroni aggregation operator. This operator enables the effective amalgamation of various perspectives among decision-makers, resulting in more reliable and well-rounded outputs in group decision-making scenarios. The efficient amalgamation of communal assessments facilitates the development of cooperative resolutions to multi-criteria decision-making challenges.

The benefits of the F-LMAW approach allow for dependable and consistent outcomes in decision-making scenarios characterized by uncertainty and complexity. This method holds significant potential for application, particularly in MCDM issues where there is a variety of alternatives and criteria. In the management of uncertain events such as earthquake disasters, the results of different scenarios can be analysed more clearly and proactive measures can be developed against possible risks. According to the analysis results, proactive measures that can be taken against grey swans and the effectiveness of strategies for other grey and black swans in the management of such events will be evaluated. In this framework, proposed strategic measures to mitigate the effects of a possible earthquake disaster will be presented and how existing risk management approaches can be strengthened will be discussed. This approach will identify the necessary steps to ensure more effective risk management at both local government and societal level and provide concrete recommendations for implementation.

In this context, this study aims to develop a methodological framework using fuzzy MCDM techniques to model the impact of foreseeable gray swan events, specifically earthquake disasters. The remainder of the manuscript is structured as follows: Section 3 presents the research methodology, including the F-LMAW approach; Section 4 provides empirical findings and sensitivity analysis; Section 5 discusses practical and managerial implications; and finally, Section 6 concludes with limitations and suggestions for future research.

2 Research gaps

The existing literature on grey swans highlights several areas that require further investigation, particularly in terms of developing more accurate methods and tailored strategies. This study aims to address the following focused gaps:

(1) Improved models and measurement methods for grey swans

Current methods for predicting and assessing the impacts of grey swans are limited in scope, particularly when it comes to managing uncertainties. Existing approaches, such as fuzzy set-based MCDM methods, offer a potential avenue for improvement. However, there is a pressing need for new computational tools and analytical approaches that provide more reliable models to measure and predict the impact of grey swans. This research intends to explore and develop innovative methods that enhance risk management by better addressing uncertainties and improving prediction accuracy.

(2) Tailored strategic management practices for grey swans

Existing strategies largely focus on generic risk management approaches, but they fall short when it comes to customized solutions for specific event types, such as grey swans. A focused gap in the literature is the need to identify and evaluate strategies specifically developed for managing grey swans. This research aims to investigate sector-specific and geographically tailored strategies that can improve resilience against grey swans, particularly in high-risk areas.

(3) Global vs. Local risk management approaches

While global risk management strategies are well-documented, there is a lack of detailed analysis on how local contexts and regional characteristics influence the management of grey swans. This gap is especially significant in high-risk regions like Turkey, where natural disasters such as earthquakes and floods are prevalent. The study seeks to explore the differences between global and local risk management practices and identify region-specific strategies for more effective grey swan management, with a focus on areas with high vulnerability.

(4) Social and economic impacts of grey swans

Research on the social and economic consequences of grey swans remains limited, and much of the existing literature does not provide actionable insights on mitigating these impacts. This research seeks to address this gap by performing comprehensive investigations to comprehend the wider implications of gray swans on society and the economy. It will focus on evaluating how these impacts can be mitigated through effective social policies and economic strategies.

(5) Effectiveness of education and awareness campaigns

Although awareness and education campaigns are often proposed as strategies to reduce the impact of grey swans, there is insufficient evidence on their effectiveness across different societal segments. This study will assess the impact of educational initiatives on public preparedness for grey swans, focusing on how these campaigns can be more effective in raising awareness and prompting action in vulnerable populations.

(6) Technological innovations in grey swan risk management

The role of technology in managing grey swans is an emerging area, yet there is limited research on how innovations like data analytics, artificial intelligence, and early warning systems can transform risk management practices. This study seeks to investigate the capacity of these technologies to increase risk assessment and response tactics, providing insights into how technology improvements can improve the management of grey swans.

2.1 The study's motivation

The main motivation for this research is the need to better understand and manage complex and uncertain risks. In particular, the “grey swan” and “black swan” metaphors express the challenges of risk management and the unpredictability of these risks. In the existing literature, there is no comprehensive study on predicting potential events that may arise from grey swan scenarios and determining whether these scenarios may lead to new grey swans or entirely new black swans. This gap highlights the need to develop innovative approaches and mathematical frameworks to more accurately predict the outcomes of grey swan scenarios.

The purpose of this study is to create a mathematical understanding and to forecast whether different weightings of main and sub-criteria in grey swan scenarios may lead to new grey swans or black swans. In this context, the use of the F-LMAW method, which is one of the fuzzy MCDM methods, is the first study in this field. The application of the F-LMAW method will provide an in-depth mathematical understanding of how grey swan scenarios can evolve and whether they will evolve into new grey swans or black swans during this evolution process. These findings will be integrated into the grey swan metaphor and will contribute significantly to the identification and prevention of future grey and black swans. This integration will enable the development of more effective and robust measures in the field of risk management and strategic planning and will advance the body of knowledge in this field.

3 Research questions

This research seeks to answer specific questions within the context of gray swan scenarios. The research questions (RQs) are as follows:

RQ1: What main and sub-criteria can be identified for gray swan scenarios, particularly earthquake disasters, and how can these criteria be prioritized using the F-LMAW Method?

This inquiry centers on identifying and assessing the essential criteria required for categorizing an earthquake tragedy as a gray swan, followed by evaluating the relative significance of these factors with the F-LMAW Method.

RQ2: How can these identified and prioritized criteria contribute to predicting other gray swan events and preventing the formation of new risk combinations, including black swans?

This question explores the applicability of the identified criteria in forecasting other gray swan scenarios and evaluates whether strategic measures based on these criteria can reduce risks and prevent the emergence of new black swan or gray swan events.

These questions aim to effectively evaluate the main and sub-criteria established to better understand and manage gray swan scenarios. Furthermore, they seek to guarantee that these criteria facilitate the formulation of solutions for tackling other gray swans and black swans.

4 Literature review

4.1 Black swan metaphor

The Black Swan is a metaphor that expresses incongruous and unexpected events that do not indicate before they happen (Bennet, 2014; Castles, 2010). It refers to singular and major events that occur suddenly and are very difficult to predict (Milovanov et al., 2021). These events are unpredictable and statistically unthinkable. There is no indication that a Black Swan event is possible in advance. As Taleb emphasizes, Black Swan events are unprecedented and cannot be envisioned before they happen. There is no data to model them in advance and they are random. This makes them inherently unpredictable. What Taleb refers to as “unknown unknowns” are true black swans. Due to the impossibility of ascertaining such an unknown. We are told that we should be prepared for a surprise, but there is no way to do it. The essence of surprise is surprise. (Alea et al., 2020). Yet they have a huge impact -positive or negative- and have the potential to determine our future (Pretty, 2021). Taleb has also emphasized the fact that although black swans are unpredictable, we are now living in a world shaped by such events that people once considered largely unthinkable (Kramer, 2019). In this respect when Black Swans become predictable, they will no longer be black swans.

The Black Swan offers us a perspective vaccine that will enable us to question what we have known to be true until today and our theories based on generally accepted laws. The Black Swan metaphor, which originated in the 17th century, shattered the idea that all the swans people had seen up to that time were white. The notion that all swans were white was dispelled by the sighting of a Black Swan by explorer Willem de Vlamingh upon his arrival in Australia, contradicting the long-held assumption that swans were exclusively white. The metaphor is predicated on the notion that although white swans are prevalent in nature, black swans also emerge unexpectedly. This shows that generalizations based on limited observation and experimentation will be limited to prove a theory. From this perspective, Taleb's concept has created a new paradigm for managing uncertainty and risk, and as a result, black swan events have become widely popular in today's literature (Bezdrob et al., 2018).

The Black Swan Metaphor refers to the fact that events that have a very low probability of occurring or that are never thought to occur and are therefore unpredictable have the potential to create great effects if they occur. According to Taleb, wars, epidemics, and inventions that changed the history of the world are such events. Therefore, it is possible to characterize Black Swan events as extreme events that change the global society (Sachs et al., 2014).

Notable examples of black swan events encompass the repercussions of the drug Thalidomide, introduced in 1957; the unforeseen housing market collapse during the US financial crisis of 2008, which escalated into a global crisis; the unpredictable hyperinflation of 79.6% in Zimbabwe in 2008; the Dotcom Bubble of 2001, whose collapse was nearly impossible to foresee; and the COVID-19 pandemic, which initiated a worldwide crisis beginning in 2020. These events are regarded as significant black swan occurrences (Aven, 2015; Glette & Aven, 2021; Malajowicz et al., 2023). Although these events were very difficult to predict in advance, they had shocking consequences in terms of their effects. The important point to emphasize about the content of black swan events is that black swan is not only about negative situations. For example, it is very influential and widespread in terms of the consequences of unplanned and unforeseen discoveries such as the computer, the internet and lasers, which have the greatest impact on today's world (Taleb, 2007).

Taleb asserts that an occurrence needs to possess three key elements to qualify as a Black Swan. Firstly, it is atypical as it deviates from standard expectations due to the absence of prior indications of its occurrence. Second, it has an extraordinary power of influence. Thirdly, despite being unexpected, the phenomenon in question leads people to find some explanations that can be explained and made predictable after its occurrence. In other words, it makes the event seem explainable and predictable after it has occurred. The distinctive feature of Black Swan events is their unpredictability. In black swan events, it is very difficult to make predictions based on past experiences. Therefore, statistics based on observations of past events cannot be used as a criterion for predicting black swans. In preparing for the impact of black swans, it is important to estimate the size of the event and the destructiveness of its impact. In this respect, the characteristics of the Black Swan event need to be clearly and unambiguously understood (Hajikazemia et al., 2016).

True Black Swan events are outliers that rarely happen. World War I, initiated by the seemingly random assassination of Archduke Franz Ferdinand in Sarajevo, serves as an illustrative example. In contrast, World War II does not fit the same categorization, as concerns regarding the growing influence of the Nazi Party were repeatedly articulated by figures such as Winston Churchill throughout the 1930s. Situations of sudden character, including financial, cyber, terrorist, pandemic, nuclear safety, industrial, geopolitical, and social instability risks, might have catastrophic consequences (Milovanov et al., 2021). The October 19, 1987, stock market crash, in which the S&P 500 Index dropped more than 20% in a single trading session, is another instance of a Black Swan occurrence. Such a sudden drop was unprecedented and changed all expectations about stock market behavior (Pretty, 2021). Once they occur, Black Swans can cause lasting damage to all societal structures- governance, economy, education, and health. They can significantly affect economic activity, social cohesiveness, and political stability, as well as destabilize global systems, regardless of whether they take place in the health, climatic, social, or financial systems (Parera & Higgins, 2016).

Black swan events have recently been analysed in the literature from various perspectives. For example, De Marzo et al. (2022) provide a scientific approach to quantifying such events using the Zipf-Mandelbrot law, allowing for a better understanding of their occurrence and dynamics. Again, the COVID-19 pandemic has been extensively analysed in terms of its devastating impact on businesses. Popkova et al. (2024) argue that such crises serve as critical catalysts for managing business strategies. This result is in line with that of Ahmad et al. (2021), who point out that the pandemic caused extreme volatility in some industries, impacting stock markets in the US, UK, and Europe. On a broader scale, Parameswar et al. (2021) highlight the growing academic interest in black swan events and the importance of further exploring their multifaceted nature. Zarghami and Dumrak (2021) developed a quantitative methodology to investigate the vulnerability of logistics networks to black swan events.

The Black Swan metaphor is a clever metaphor for today's rapidly transforming world to understand the consequences of these rapid transformations in the future. However, in an environment of uncertainty where conditions are becoming more and more unpredictable, both individuals and organizations are not sufficiently aware of Black Swans. Taleb argues that people behave as if there are no black swans because human nature is not programmed for black swans. Indeed, both individuals and organizations do not want to think about the "negative" when planning for the future. They tend to build the future on optimistic thoughts. However, a rational expectation of the future should also include negative situations.

Ponkin (2019) asserts that the most important responsibility in this regard falls to the state. Regardless of the measures the state implements or actions it undertakes, 'black swan crises' remain inevitable for both the state and society as a whole. However, the state has the capacity to limit the consequences of such crises and enhance society's ability to cope with them. Consequently, policymakers must focus on making society resilient and immune to various dangers, even if the specific black swan events that humanity may face cannot be foreseen.

Black swans may not be foreseeable, but the measures and practices taken can prevent the emergence of other low-probability gray swans and even unknown black swans. In this respect, scenarios are one of the most effective tools for managing crises. The outcomes derived from the theoretical model greatly aid in directing organizational management. While black swan events cannot be predicted, simulating disasters caused by these events provides organizations with a basis for performing stress testing simulations (Martinez, 2024). With the actions and procedures done in this regard, scenarios can produce a condition of behavioral and cognitive preparedness in society and decision-makers (Darıcı, 2020). Preparing for the future with scenarios and taking precautions in advance can prevent the occurrence of unwanted combinations in the pool of possibilities or reduce the severity of the consequences.

4.2 Gray swan metaphor

Gray Swans are defined as "unlikely but impactful events" that are "outside the usual base case and risk scenarios of the community". Grey Swans are events that occur through a chance combination of events that are inherently unlikely to occur and are still considered very unlikely (Akkermans & Van Wassenhove, 2013). A very important event that is unlikely to occur but still possible is referred to as a "gray swan." Stated differently, Grey

Swans are occurrences for which early warning data is accessible but which are not identified, acknowledged, or pursued for a variety of reasons (Engels, 2022). Gray Swans, like Black Swans, are rare, but we know they exist and can make sense of them (Akkermans & Van Wassenhove, 2013). One of the most important elements that distinguishes gray swan events from black swans is that their existence is known. However, the possibility of realization is not perceived as probable. Gray Swan events should be “expected” and pro-active with possible scenarios, as they can have consequences that can shake people, economies, stock markets, states, and even the world when they happen.

The Black Swan metaphor, made popular by Taleb in his book *Black Swan*, served as the model for the gray swan. In this respect, Gray Swans, introduced as the cousin of Black Swans, have become an important object of analysis along with Black Swans in managing unexpected and extraordinary events. According to Taleb, the black swan scenario is one of the ways to mitigate the effects of surprises, to make some swans visible, in other words, to be aware of their consequences, to “gray them out”. The outcomes derived from the theoretical model greatly aid in directing organizational management. While black swan events cannot be predicted, simulating disasters caused by these events provides organizations with a basis for performing stress testing simulations (Woodall et al., 2024).

Gray Swans are the “expected known.” Natural catastrophes like Hurricane Katrina, the UK's unexpected choice to leave the European Union (Brexit), Donald Trump's win in the 2016 US presidential election, or a groundbreaking technological advancement like the internet are a few examples of gray swan events (Toney, 2023). Also, any development that is unpredictable in terms of its consequences can be an example of a Gray Swan. Predicting and planning for Gray Swans can turn them into opportunities. In the case of positive Black and Gray Swans, like the emergence of the Internet and the expansion of personal computing, it's interesting to observe how entrepreneurs, including Bill Gates, Steve Jobs, and Jeff Bezos, established and developed their companies. It is critical to manage the impact of Gray Swans that businesses identify early signs to capitalize on their opportunities in response to such positive Black Swans (Lugtu, 2020).

Many extreme events that have occurred in the past tend to be described as Black Swans, but a closer look reveals that they are Gray Swan events. Below are five such Gray Swan events over the last 20 years, all of which were anticipated, discussed, and warned about with increasing urgency: (Pretty, 2021).

The September 11 attacks Al-Qaeda's threat of a large-scale terrorist attack against the United States was known before the attacks.

2005 Hurricane Katrina In the years before Hurricane Katrina slammed into Louisiana, breached the levees and devastated New Orleans, federal and state officials had many warnings to heed.

The 2008 monetary crisis 2003, when legendary investor Warren Buffet famously referred to complex derivatives as “financial instruments of mass destruction.”

2011 Fukushima The nuclear accident at Fukushima Daiichi was entirely preventable.

2020 COVID The World Health Organization issued cautious and urgent advice that a global pandemic was imminent, noting the potential for consequences. The last 20 years have seen outbreaks such as SARS, H5N1, H1N1, Ebola, and MERS. As the World Health Organization (WHO) summarized in 2019, “the world is not prepared for this situation”. As a result, COVID-19 has destroyed people's lives and means of subsistence all around the world. The distinguishing feature of Black Swans and Gray Swans, which are closely related to each other, is their degree of predictability. The following table compares the characteristics of Black Swans and Gray Swans (Table 1):

Gray Swan events are more likely to occur than Black Swans and we have more data on them. However, while Gray Swans can be expected, their consequences are often overlooked. Gray Swan events occur so infrequently that they are not part of most risk models because they are considered remote possibilities. The following section discusses how to manage Gray Swans so that extreme events can be properly analyzed and their impact contained.

4.3 Managing swans: can scenarios for gray swans prevent other gray swans and black swans?

Admittedly, the trajectory and outcome of swans can often be difficult to predict and accurately measure. But recognizing and capitalizing on these swans is critical to making the threats around us manageable. Identifying gray swans and developing strategies against them can allow us to mitigate the potential impact of other Gray and Black Swans.

Because the threat is so low, governments, companies and individuals tend to ignore these risks or provide limited resources. An example of a gray swan in this case is the 9/11 attack on US soil, as it had an extremely significant cascading impact, even though the warning signs were ignored. Considering the consequences of the 9/11 attack, the importance of strategic foresight becomes clear.

In the world of international trade, unexpected occurrences like natural calamities, economic downturns, and political changes can have significant effects on people, businesses, and communities. These events create difficulty in anticipating situations that are hard or virtually impossible to predict, as they differ greatly from the usual patterns of events, highlighting the shortcomings of conventional statistical models (Nagy & Szentesi, 2024). As a result, Black Swan incidents are challenging to foresee using conventional statistical methods such as correlation, regression, standard deviation, or return intervals. In simpler terms, the lack of ability to forecast the likelihood of Black Swan occurrences obstructs effective risk management practices (Nafday, 2009). Therefore, it is important to identify strategies for foreseeable Grey Swans to mitigate the impact and consequences of other possible Grey Swans and unforeseeable Black Swans. Therefore, scenario-based preparations of organiza-

Table 1 Comparison of black swan and gray swan characteristics

Black Swans	Gray Swans
Unknown unknowns	Known unknowns
Unprecedented, unimagined	Considerable but neglected
Unprecedented, unimagined	Unlikely, big impact
No data, Unpredictable	Limited data, Models and insights

Source: (Pretty, 2021)

tions for possible Grey Swans can prevent other Grey Swans and even Black Swans that may occur in different combinations or reduce the impact of the consequences.

The metaphor of the Black Swan and the Gray Swan provides us with important contributions to the analysis of the uncertainty we face in today's world and the social consequences it brings about. Closely related to the concepts of unpredictability and uncertainty, the black and gray swan metaphor forces us to think about the unthinkable (Antipova, 2021). Not having seen a black swan does not mean that there is no black swan. Characterized as "insidious" by Taleb and Blyth, black swans are "silent risks" that accumulate below the surface. Political leaders and economic policymakers ignore these risks (Taleb & Blyth, 2011). Again, what Taleb refers to as "black swan blindness" implies that the possibility of unlikely events is always ignored. Since the threat in the Grey Swan is quite low, companies and individuals tend to ignore these risks or provide limited resources for their realization (Akkermans & Van Wassenhove, 2018). Today, organizations are unprepared for new risk environments because existing risk management approaches and mechanisms are insufficient.

These situations that turn into crises can have dramatic societal and economic consequences as managers/experts ignore the unlikely event and fail to identify the necessary strategies in advance. All managers and decision-makers should be aware of black and gray swan events and their potential consequences. To overcome a black swan event such as, for example, covid-19, managers need to think ahead (Dogra & Parrey, 2024). They should be prepared for and able to manage their potential impacts. Gray Swans are predictable events. However, they become crises because they can be predicted by people, but no action is taken. The reason why the consequences of Grey Swans lead to a dramatic situation is that people ignore and reject warnings about swans. People are aware of their existence but often ignore their consequences, even when each one could have serious and far-reaching consequences, such as triggering a major crisis or depression.

Swans are seen as dangerous events. Because swans have huge consequences, positive or negative. When they occur, they can destroy companies/brands and reputations. In contemporary risk societies, organizations and governmental bodies must be ready to anticipate and address the possible consequences of overlooked Gray Swans (Lugtu, 2020). Because swans can bring an organization to its knees. Ignoring Gray Swans can cause irreversible damage both on an individual and organizational basis. Modern companies and governmental bodies must navigate and prepare for the threats posed by unexpected Black Swans and Gray Swans. Experts are already warning us about a potential war between Pakistan and India, Apple's possible acquisition of Disney, and gray swans emerging as another global pandemic. Researchers are currently forecasting a new variant of the swine flu virus that may lead to another pandemic in China (Lugtu, 2020).

Risk expert Rade Musulin (2020) explains what to do against swans as follows: In order to lessen the impact of severe events, we should attempt to transfer risk down the ladder and transform unknown unknowns into known unknowns. Grey swan events have a huge impact, which can be positive or negative, says Dr. Deborah Pretty, founder of Pentland Analytics. They are rare, but not inconceivable, so you can prepare for them. These are events that affect the future trajectory of our companies. But more importantly, there is something we can do about it because we can predict them. In light of these assessments, it is possible to say that early and correct intervention on Gray Swans is of strategic importance.

5 Research methodology

In the research, a gray swan scenario was first created on the earthquake disaster. Then, criteria were created with the Delphi method by taking the opinions of experts. Then, experts were asked to evaluate the criteria according to the F-LMAW method scale. With the same method, the criteria were prioritized in order of importance. Based on these results, a mathematical formula for calculating the combinations of the emergence of other gray swans was proposed and it was discussed whether gray and black swans could be prevented. The flow chart of the proposed model for the evaluation of grey swan scenarios is shown in Fig. 1.

5.1 Fuzzy theory set

In order to deal with uncertainty in variables and parameters, Zadeh (1965) introduced the idea of fuzziness. Triangular fuzzy shapes have been used by numerous researchers to translate qualitative claims into quantitative representations. Three number values define each form in a triangle fuzzy shape. The first, second, and third defining values of a fuzzy form represent its least, most likely, and highest values, respectively.

Let $\tilde{A} = (a_l, a_m, a_u)$ and $\tilde{B} = (b_l, b_m, b_u)$ represent two triangular fuzzy numbers. The mathematical operations pertaining to these are delineated in Eqs. (1)-(4) (Biswas et al., 2024).

$$\tilde{A} + \tilde{B} = (a_l + b_l, a_m + b_m, a_u + b_u) \quad (1)$$

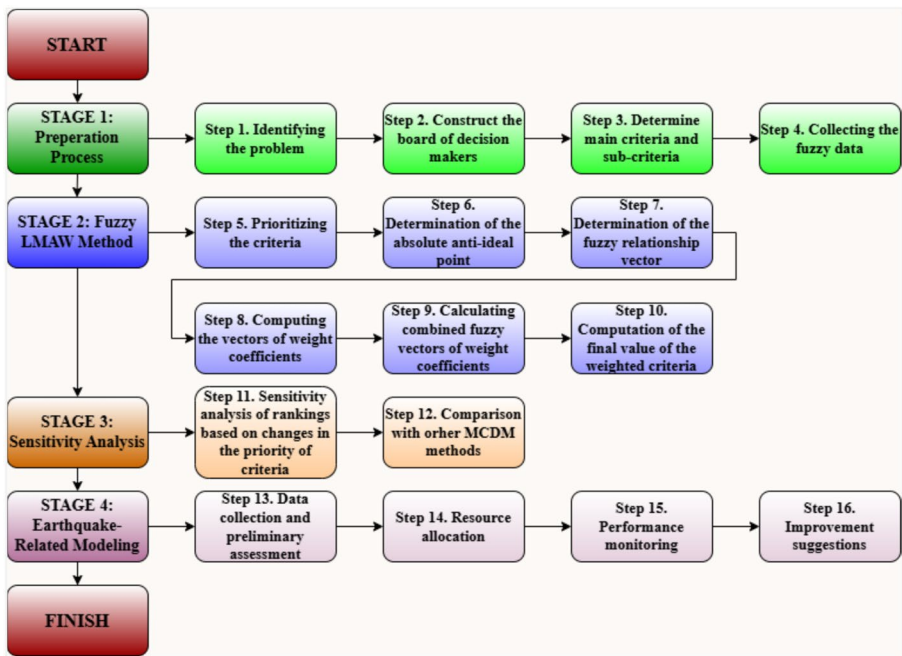


Fig. 1 Flow diagram of the proposed model

$$\tilde{A} - \tilde{B} = (a_l - b_u, a_m - b_m, a_u - b_l) \tag{2}$$

$$\tilde{A}\tilde{B} = (\min(a_l b_l, a_l b_u, a_u b_l, a_u b_u), a_m b_m, \max(a_l b_l, a_l b_u, a_u b_l, a_u b_u)) \tag{3}$$

$$\frac{\tilde{A}}{\tilde{B}} = \left(\min\left(\frac{a_l}{b_l}, \frac{a_l}{b_u}, \frac{a_u}{b_l}, \frac{a_u}{b_u}\right), \frac{a_m}{b_m}, \max\left(\frac{a_l}{b_l}, \frac{a_l}{b_u}, \frac{a_u}{b_l}, \frac{a_u}{b_u}\right) \right) \tag{4}$$

A number of formulas can be used to convert triangular fuzzy integers into accurate numbers. This work utilizes Eq. (5) to defuzzify a fuzzy integer represented as $\tilde{A} = (a_l, a_m, a_u)$ (Narang et al., 2023):

$$A = \frac{a_l + 4a_m + a_u}{6} \tag{5}$$

5.2 F-LMAW method

It was Pamučar et al. who introduced the LMAW method to the world in 2021. It represents one of the novel approaches for establishing criterion weights and ranking options. LMAW guarantees that outcomes remain stable, even with variations in the number of alternatives. The rank reversal issue prevalent in other ranking methodologies is absent in this approach. Its applicability to both qualitative and quantitative criteria enhances the method's versatility.

Given that the LMAW and F-LMAW methods are newly introduced, there are limited studies available. The performance of six logistics service providers was evaluated by Pamučar et al. (2021). The findings indicated that the method provided a consistent ranking of alternatives, unhindered by rank reversal effects. This research marked the introduction of the LMAW method into academic literature. Görçün and Küçükönder (2022) explored the opportunities for transforming production systems in heavy industry sub-sectors via cyber-physical systems. Demir (2022) examined the banking industry's financial results throughout the COVID-19 pandemic. A fuzzy version of the LMAW approach was given by Božanić et al. (2022) and evaluated for military landing operation site selection. The fuzzy LMAW approach in conjunction with Z-numbers was used by Puška et al. (2022) to weight selection criteria for green suppliers. Tešić et al. (2023) used the F-LMAW method to prioritize criteria for selecting a dump truck. Karakuş (2024) assessed ecotourism potential utilizing a GIS-based F-LMAW method for sustainable management of natural resources. Puška et al. (2024) opted for the LMAW method in determining weights for criteria when selecting distribution centers. The steps involved in the method are outlined below (Božanić et al., 2022):

Step 1: Prioritising the criteria.

The experts assessed the criteria by rating them using linguistic terms based on the fuzzy scale shown in Table 2.

Criteria are given significance values based on their relevance by the fuzzy linguistic scale, and vice versa. Each specialist independently produces their priority vectors

$$\tilde{P}^e = (\tilde{\gamma}_{C_1}^e, \tilde{\gamma}_{C_2}^e, \dots, \tilde{\gamma}_{C_n}^e).$$

Step 2: The fuzzy absolute anti-ideal point's determination ($\tilde{\gamma}_{AIP}$).

Table 2 Scale of prioritization

Fuzzy Linguistic Descriptive	Abbreviation	Fuzzy Number
Absolutely low	AL	(1,1,1)
Very low	VL	(1,1.5,2)
Low	L	(1.5,2,2.5)
Medium	M	(2,2.5,3)
Equal	E	(2.5,3,3.5)
Medium–high	MH	(3,3.5,4)
High	H	(3.5,4,4.5)
Very high	VH	(4,4.5,5)
Absolutely high	AH	(4.5,5,5)

Source: (Božanić et al., 2022)

Experts have identified this imprecise figure, which is lower than the minimum value in the whole set of priority vectors. According to the authors of the literature, the approach is $\tilde{\gamma}_{AIP} = (0.5, 0.5, 0.5)$.

Step 3: Assessment of the fuzzy relationship vector (\tilde{R}^e).

The correlation between the components of the priority vector and the associated ideal point is established using Eq. (6).

$$\tilde{\eta}_{C_n}^e = \left(\frac{\tilde{\gamma}_{C_n}^e}{\tilde{\gamma}_{AIP}} \right) = \left(\frac{\gamma_{C_n}^{(l)e}}{\gamma_{AIP}^{(l)}}, \frac{\gamma_{C_n}^{(m)e}}{\gamma_{AIP}^{(m)}}, \frac{\gamma_{C_n}^{(r)e}}{\gamma_{AIP}^{(r)}} \right) \tag{6}$$

Step 4: Weight coefficient vector calculation (w_j^e).

The fuzzy score for the weight coefficients of the criteria provided by each expert is determined using Eq. (7).

$$\tilde{w}_j^e = \left(\frac{\ln(\tilde{\eta}_{C_n}^e)}{\ln(\prod_{j=1}^n \tilde{\eta}_{C_n}^e)} \right) = \left(\frac{\ln(\eta_{C_n}^{(l)e})}{\ln(\prod_{j=1}^n \eta_{C_n}^{(r)e})}, \frac{\ln(\eta_{C_n}^{(m)e})}{\ln(\prod_{j=1}^n \eta_{C_n}^{(m)e})}, \frac{\ln(\eta_{C_n}^{(r)e})}{\ln(\prod_{j=1}^n \eta_{C_n}^{(l)e})} \right) \tag{7}$$

The weight coefficients for all experts are obtained in the form of $w_j^e = (\tilde{w}_1^e, \tilde{w}_2^e, \dots, \tilde{w}_n^e)^T$.

Step 5: Figuring out the weighted coefficients'combined fuzzy vectors.

The Bonferroni aggregator is used to formulate the combined fuzzy vectors of the weight coefficients in accordance with Eq. (8) $w_j = (\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n)^T$.

$$\begin{aligned} \tilde{w}_j &= \left(\frac{1}{k(k-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^k \tilde{w}_i^{e(p)} \tilde{w}_j^{e(q)} \right)^{\frac{1}{p+q}} \\ &= \left\{ \left(\frac{1}{k(k-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^k w_i^{(l_e)p} w_j^{(l_e)q} \right)^{\frac{1}{p+q}}, \left(\frac{1}{k(k-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^k w_i^{(m_e)p} w_j^{(m_e)q} \right)^{\frac{1}{p+q}}, \right. \\ &\quad \left. \left(\frac{1}{k(k-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^k w_i^{(r_e)p} w_j^{(r_e)q} \right)^{\frac{1}{p+q}} \right\} \tag{8} \end{aligned}$$

Step 6: Ultimate value computation using the weighted criteria.

As shown in Eq. (5), the final weight coefficient values for the criteria are obtained by explanation based on $w_j = (w_1, w_2, \dots, w_n)^T$.

6 Empirical analysis and results

In this section, the suggested F-LMAW-based decision-making model is empirically implemented in a gray swan scenario, specifically an earthquake disaster in Turkey. It includes the definition of the decision criteria, data collection from expert evaluations, application of the F-LMAW method, and discussion of sensitivity and comparative analyses. The findings aim to guide strategic resource allocation and proactive disaster management planning.

6.1 Defining the problem

The F-LMAW method was employed to address the weighing issue involving 8 criteria and 22 sub-criteria within a situation characterized as a gray swan. Table 3 provides information regarding the main/sub-criteria.

Table 3 shows the main and sub-criteria and their explanations that can be used to measure the impacts on different sectors after the earthquake.

6.2 F-LMAW method application results

An evaluation group was formed prior to the implementation of the F-LMAW algorithm. Detailed information on the 9 decision makers (DMs) selected to form the evaluation committee is provided in Table 4.

The priority vectors for the criteria, derived from the perspectives of nine distinct decision-makers, are shown in Table 5, as indicated in Table 1.

The fuzzy anti-ideal point that was absolute was determined by the DMs as $\tilde{\gamma}_{AIP} = (0.5, 0.5, 0.5)$. For example, the relationship between the absolute anti-ideal point and the elements of the priority vector established by DM-1 is calculated as follows.

$$\tilde{\eta}_{C1}^{DM-1} = \left(\frac{4,5}{0,5}, \frac{5}{0,5}, \frac{5}{0,5} \right) = (9,10,10), \dots, \tilde{\eta}_{C8}^{DM-1} = \left(\frac{4,5}{0,5}, \frac{5}{0,5}, \frac{5}{0,5} \right) = (9,10,10).$$

For the other decision-makers (DMs), similar calculations are performed. The weight coefficients vector was established using Eq. (6). The weight coefficients' aggregated fuzzy vectors are calculated using Eqs. (7) and (8). The inventors of the approach proposed an absolute anti-ideal point of 0.5 for each criterion. The sum of these values for each of the 8 main criteria is $(8 \times 0.5 = 4)$. This value needs to be normalized in order to be compared with the weight coefficients. In other words, the normalized absolute anti-ideal point value for 8 main criteria was obtained as $0.5/4 = 0.125$. Figure 2 provides this number together with the weight coefficients of the main criterion.

The given absolute anti-ideal point (0,125) was used as a fixed reference point for all criteria. This represents the worst possible outcome for each criterion. The interpretation of this in comparison with the weights is given below:

Table 3 Criteria and sub-criteria for a scenario characterized as a gray swan

Main Criteria	Sub-Criteria	Explanations
City (C1) (min) Assess the damage that may occur in the city after an earthquake and its effects. This criterion aims to minimize the impact of the disaster on the city's infrastructure and quality of life	C11: Debris and damage	Debris and structural damage in the city following an earthquake
	C12: Internal migration	The tendency of people to migrate internally after the earthquake
	C13: Psychological trauma	Psychological traumas caused by the earthquake and its psychological impact on society
Transportation (C2) (max) Assesses the extent to which transportation infrastructure is affected by the earthquake. This criterion aims to keep transportation systems as functional as possible	C21: Public transportation	The availability and condition of public transportation after the earthquake
	C22: Road and railroad	Damage or disruptions in road and railroad transportation following the earthquake
Health (C3) (max) Examines the sustainability of health infrastructure and services. This criterion aims to keep health systems functional and ensure access to healthcare	C31: Residential and health facilities	The condition of residential and health facilities after the earthquake
	C32: Medicines and medical supplies	Disruptions in the supply and distribution of medical supplies and medicines
	C33: Health care services	The availability and continuity of healthcare services after the earthquake
Economy (C4) (max) Assesses the economic impacts of the earthquake, focusing on the sustainability of businesses, trade, and tourism	C41: Workplaces and enterprises	Damage and disruptions in workplaces and commercial activities
	C42: Trade	The extent to which trade is affected and interrupted by the earthquake
	C43: Tourism	The impacts and losses on the tourism sector due to the earthquake
Social Events (C5) (min) Assesses potential social unrest and incidents in society. It aims to maintain social peace	C51: Turmoil and unrest	Social unrest and chaos incidents after the earthquake
	C52: Crime	Increases in crime rates or security issues following the earthquake
	C53: Social aid	Social aid provided after the earthquake and its impact on the community
Education (C6) (max) Focuses on the continuity of the education system. It aims to ensure that schools and educational activities continue without interruption	C61: School Buildings	Damage and destruction in school buildings following the earthquake
	C62: Education and training activities	The status and interruptions of educational activities after the earthquake
Security (C7) (max) Focuses on ensuring the continuity of the education system. It aims to ensure that schools and educational activities continue without interruption	C71: Fire and explosion risk	Fire and explosion risks following the earthquake
	C72: Emergency response	The capacity and effectiveness of emergency response teams
Communication (C8) (max) Assesses the sustainability of communication systems after the earthquake. Communication plays a critical role in post-disaster coordination	C81: Dropped phone lines	Non-functional or disrupted landline phone services after the earthquake
	C82: Collapse of mobile communications	Collapse or disruption of mobile communication networks
	C83: Internet access crashes	Disruptions or problems in internet access
	C84: Lack of face-to-face communication due to loss and damage	Difficulty or total loss of face-to-face communication due to the earthquake

Table 4 Profile of DMs

Decision Maker	Duty	Age
DM-1	Academician	62
DM-2	Academician	51
DM-3	Search and Rescue Specialist	27
DM-4	Search and Rescue Specialist	34
DM-5	City Planner	33
DM-6	Mayor	56
DM-7	NGO Earthquake Expert	37
DM-8	Civil Engineer	46
DM-9	Architect	38

Table 5 Priority vectors of criteria

	C1	C2	C3	C4	C5	C6	C7	C8
DM-1	AH	VH	VH	AH	MH	E	H	AH
DM-2	AH	AH	VH	VH	VH	MH	VH	AH
DM-3	AH	AH	VH	CH	VH	MH	VH	AH
DM-4	AH	AH	AH	E	AH	H	H	VH
DM-5	AH	AH	VH	E	AH	AH	AH	AH
DM-6	AH	VH	VH	MH	AH	VH	AH	AH
DM-7	AH	AH	AH	H	H	AH	VH	AH
DM-8	AH	AH	AH	H	H	E	H	AH
DM-9	AH	AH	AH	MH	VH	MH	AH	AH

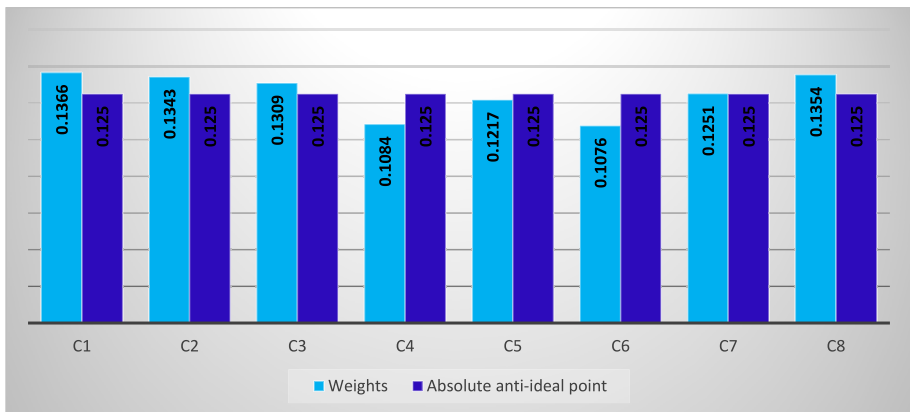


Fig. 2 Absolute anti-ideal point and weight coefficients of main criteria

C1 (City) 0,1366 weight: The deviation of this criterion from the ideal is one of the most important issues, as damage in the city is to be minimised. Its weight is slightly higher than the anti-ideal point, which can have serious consequences if this criterion is not taken into account. C2 (Transport) 0,1343 weight: The continuity of transport systems is also of great importance and is rated with a weight close to the anti-ideal point. A disruption in transport can affect the critical operations of the system. C3 (Health) 0,1309 weight: Disruption of health services has a risk close to the anti-ideal point. This criterion is almost as important

as transport to protect the overall health of the community. C4 (Economy) 0,1084 weight: Economic impacts are slightly less important than the other criteria. However, compared to the anti-ideal point, it is still an issue that needs attention. C5 (Social Events) 0,1217 weight: Minimising social unrest is very close to the anti-ideal point, indicating that social events may pose a major risk after an earthquake. C6 (Education) 0,1076 weight: Keeping the education system functional is of lower importance than the anti-ideal point. Transport, health and safety, which are more critical in the short term, are prioritised. C7 (Security) 0,1251 weight: Security systems were assessed with a weight exactly close to the anti-ideal point. This indicates that security could pose a serious threat in case of post-earthquake disruption. C8 (Communication) 0,1354 weight: Sustainability of communication systems is almost at the highest weight. Disruptions in communication are assessed as one of the risks closest to the anti-ideal point. Maintaining the city's infrastructure and social harmony are the most important aspects, followed by the sustainability of the health, transportation, and communication systems, according to this study based on the weights of the criterion and the absolute anti-ideal point. Social unrest and security are important issues that need to be addressed quickly after an earthquake. Education and economy, on the other hand, have emerged as factors that need to be addressed in the longer term.

Utilizing the opinions of seven distinct experts, the priority vectors obtained for the sub-criteria of the sub-criteria through Table 1 are provided in Table 6.

The weight coefficients vector was determined using Eq. (6). Equations (7) and (8) were used to calculate the weight coefficients'aggregated fuzzy vectors. The sum of absolute anti-ideal point values for each of the 22 sub-criteria is (22*0,5=11). This value needs to be normalized to be compared with the weight coefficients. In other words, the normalized absolute anti-ideal point value for 22 sub-criteria was obtained as 0,5/11=0,0455. Figure 3 provides this number together with the sub-criteria's weight coefficients.

Table 6 Priority vectors of sub-criteria

	C11	C12	C13	C21	C22	C31	C32	C33	C41	C42	C43
DM-1	AH	VH	AH	H	AH	AH	AH	AH	H	VH	AH
DM-2	AH	AH	AH	AH	AH	AH	AH	AH	VH	VH	AH
DM-3	AH	AH	AH	AH	AH	VH	AH	AH	H	VH	AH
DM-4	AH	AH	VH	VH	AH	AH	AH	AH	H	AH	AH
DM-5	AH	VH	AH	MH	AH	AH	AH	AH	MH	AH	AH
DM-6	AH	AH	VH	MH	VH	VH	AH	VH	AH	AH	AH
DM-7	AH	VH	VH	VH	VH	AH	AH	VH	MH	AH	AH
DM-8	AH	AH	VH	VH	AH	VH	AH	AH	VH	VH	AH
DM-9	AH	AH	AH	VH	AH	AH	VH	VH	VH	AH	VH
	C51	C52	C53	C61	C62	C71	C72	C81	C82	C83	C84
DM-1	VH	AH	H	E	H	VH	AH	AH	AH	AH	AH
DM-2	VH	AH	VH	E	AH	VH	AH	AH	AH	AH	AH
DM-3	VH	AH	AH	E	VH	AH	AH	AH	VH	AH	AH
DM-4	AH	AH	AH	MH	VH	VH	AH	AH	AH	AH	AH
DM-5	AH	AH	AH	E	AH	AH	VH	VH	AH	VH	AH
DM-6	AH	VH	H	VH	H	VH	AH	AH	AH	AH	AH
DM-7	AH	VH	VH	AH	H	VH	VH	VH	VH	AH	AH
DM-8	VH	MH	H	VH	VH	AH	AH	AH	AH	AH	AH
DM-9	AH	MH	AH	AH	VH	AH	AH	AH	AH	AH	VH



Fig. 3 Absolute anti-ideal point and weight coefficients of sub- criteria

The given absolute anti-ideal point (0,0455) is a fixed reference value for each sub-criteria. The interpretation of the criteria weights according to this value is given below:

Sub-criteria whose weights are higher than the Anti-Ideal Point: C11 (0,0478), C32 (0,0474), C83 (0,0474), C84 (0,0474), C81 (0,0470), C82 (0,0470), C22 (0,0470): These sub-criteria have higher weights than the anti-ideal point, meaning that poor performance on these issues can lead to serious problems. Risks should be minimised in critical areas such as debris, medical supplies, internet access and mobile communications. Sub-Criteria Close to the Anti-Ideal Point: C13 (0,0462), C41 (0,0416), C43 (0,0468), C53 (0,0441), C61 (0,0369): These sub-criteria are closer to or lower than the anti-ideal point. For example, psychological trauma, tourism, and social assistance are critical but may pose a slightly lower risk than debris or health services.

Criteria where the weights are highest relative to the anti-ideal point: Critical infrastructure elements such as post-earthquake debris, communication systems (internet, telephone, mobile), and medical supplies. Possible disruptions in these areas can have serious conse-

quences. Criteria with lower weights: Criteria such as school buildings, internal migration, and workplaces have lower priorities than others. That is, the anti-ideal scenario may be less critical in these areas. This analysis shows which sub-criteria should be prioritised in post-earthquake response plans. Particular attention needs to be paid to areas such as debris management, communications, and health supplies.

6.3 Sensitivity analysis

This section outlines two distinct sensitivity tests that were carried out to assess the robustness and dependability of the ranking outcomes produced by the suggested methodology. The effects of various weight coefficients are examined in the first sensitivity analysis. The results of the criterion ranking of the suggested method are contrasted with those of earlier MCDM techniques in the second sensitivity analysis.

6.3.1 Variation of criteria weight

We examined how altering the weight of the primary criterion (C1) influences the ranking outcomes. Initially, 15 distinct vectors for weighting factors were generated and organized based on 15 separate scenarios. In every scenario, a new weighting factor vector was developed by decreasing the weight of factor C1 by 2%. To accomplish this, Eq. (9) was utilized (Ecer, 2022).

$$w_{n\beta} = (1 - w_{n\alpha}) \left(\frac{w_{\beta}}{1 - w_n} \right) \quad (9)$$

The newly determined weight values for the criterion are denoted by $w_{n\beta}$. The criteria's reduced value is shown by $w_{n\alpha}$. w_n represents the original value of the lowered criterion, and w_{β} represents the starting value of the criteria. As a result, the range of variation for w_1 is defined as $w_1 \in [0,0048 \ 0,1093]$. In this case, the weighted value in Scenario1 (S1) is represented by $w_1=0,1093$, while the weighted value in Scenario15 (S15) is represented by $w_1=0,0048$. After each change to w_1 , the weights of the other criteria are determined using Eq. (9). For instance, when the actual weight value is lowered by 2%, the S1 value for C1 is 0,1093, or 0,1366 ($0,1366-0,1366 \cdot 0,02=0,1093$). Equation (9) is then used to find the values of the remaining criteria, which are as follows:

$$w_2 = \frac{(1 - 0,1093) * 0,1343}{(1 - 0,1366)} = 0,1385 \dots \dots \dots w_8 = \frac{(1 - 0,1093) * 0,1354}{(1 - 0,1366)} = 0,1397$$

Consequently, Fig. 4 displays the updated weight values for the criteria that were obtained from the 15 scenarios.

The ranking achieved based on the various main criteria weights that differ with each scenario is shown in Table 7.

Table 7 shows how the different criteria (C1-C8) are ranked in the earthquake scenarios and how these rankings change as the scenarios change. The differences between the original scenario (S0) and the rankings from S1 to S15, where the weights of the scenarios are changed, help us to understand which criteria become prioritised in post-earthquake deci-

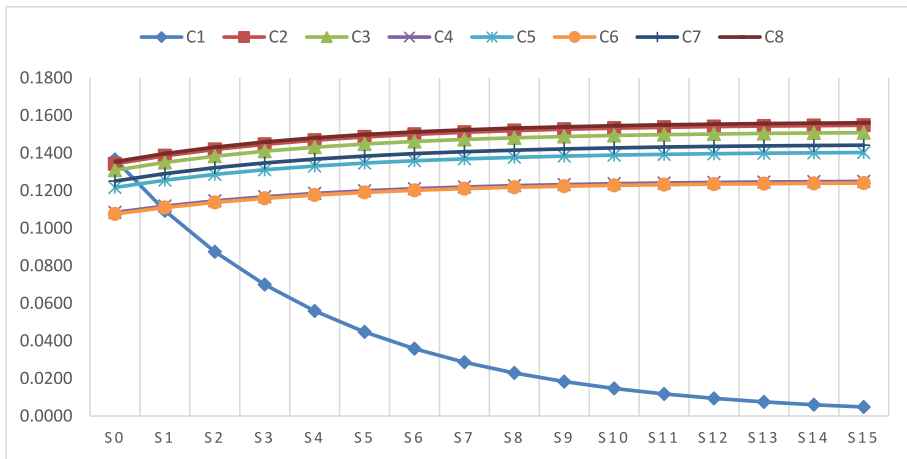


Fig. 4 Weight changes based on different scenarios

sion-making processes. While C1 (City) ranked first in the original scenario (S0), it dropped to eighth place in all scenarios from S1 to S15. This shows that the criterion of minimising damage to the city has a lower priority in the modified scenarios. The fact that C1 was high in the original but did not outrank the other criteria may reflect the fact that city damage was considered less critical after the earthquake. C8 (Communication), although ranked second in the original, has risen to first place in scenarios S1 to S15. This emphasises how important a criterion communication has become after the earthquake. Protecting the communication infrastructure has a critical role for post-disaster coordination and sustainability of information flow. C2 (Transportation) is ranked third in the original scenario and ranked second in the scenarios from S1 to S15. We see that transport is a high priority as keeping transport systems active is important for emergency response and aid distribution. C3 (Health), the health criterion, which ranked fourth in the original ranking, has risen to third place in the scenarios from S1 to S15. The continuity of health services has gained a higher priority as an important criterion for treating the injured and preventing epidemics. C4 (Economy), originally ranked seventh, remained stable at sixth in scenarios S1 to S15. Economic impacts and the sustainability of businesses and trade are important, but not as prioritised as other criteria. C5 (Social Events), ranked sixth in the original, remained stable at fifth in the scenarios from S1 to S15. Minimising social unrest is important for maintaining social order, but not as high a priority as other criteria. C6 (Education), originally ranked eighth, has risen to seventh from S1 to S15. Continuity of the education system is important, but not as urgent as other critical infrastructure. C7 (Security), originally ranked fifth, has risen to fourth from S1 to S15. Maintaining security is seen as an important factor in maintaining social order and preventing social unrest.

6.3.2 Comparison with other MCDM methods

In our study, we compared the ranking results of the F-LMAW technique to those of other strategies including the F-PSI and F-ENTROPY. The following is a list of the reasons various procedures are preferred. Regardless of the number of criteria in the model, the F-PSI

Table 7 Ranking factors in order of importance based on scenarios

	S0	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15
C1	1	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
C2	3	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
C3	4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
C4	7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
C5	6	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
C6	8	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
C7	5	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
C8	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

approach decreases subjectivity by offering a straightforward and intelligible mathematical formula, in contrast to earlier subjective weighting strategies like AHP and BWM. While removing any contradictions that may arise in models that use pairwise comparison methodologies, it provides a great deal of flexibility in defining the preferences of decision makers (Kumar et al., 2023; Zhang et al., 2023). By computing the information entropy based on the degree of variance, the F-ENTROPY technique establishes the weight of each criterion. To make sure that the indicators capture the majority of the original data, the entropy for each criterion may be computed to assess the amount of information. Figure 5 presents the comparison results.

In criteria such as city (C1), transportation (C2), health (C3), security (C7), and communication (C8), the ranking results of all three methods are the same. This shows that different methods can make similar evaluations on these criteria, meaning that these factors have a fundamental and constant importance for the methods. On the other hand, significant ranking differences are observed in criteria such as economy (C4), social events (C5) and education (C6). These differences show that the methods develop different strategies on how they evaluate and prioritize these criteria.

6.4 Benefits of the proposed methodology to grey swans and limitations in grey swans

In grey swan scenarios, MCDM methods can provide significant benefits in providing solutions to complex real-world problems. However, as with any model, such decision-making models have limitations. The benefits of this model and the limitations that may be encountered in practical scenarios are identified.

6.4.1 Benefits to real world problems

(1) Structuring the decision making process

Utility: This model helps to identify which elements are prioritised, especially in complex events such as earthquakes. Having clear weights on criteria and sub-criteria identifies prioritised areas of intervention. For example, infrastructure such as urban, transport and communications are weighted as the most critical areas, which can optimise how limited resources are used.

Use in real world problem: In disaster management, urban planning and crisis response, it is of great benefit to make decisions based on specific criteria and to identify which areas need to be addressed most urgently.

(2) Optimising resource allocation

Utility: The model guides decision makers on which resources should be prioritised and how much. When there are limited resources (labour, financial resources, time), resources can be allocated according to the weight of each criterion. For example, criteria such as debris and communications can be prioritised higher, while relatively less important criteria such as repairing school buildings can be prioritised lower.

Use in a real world problem: Decisions can be made on where emergency response teams and financial resources should be directed more. Especially when the public budget is limited, such a model can guide management.



Fig. 5 Comparison of the MCDM algorithms'ranking performances

(3) Performance monitoring and improvement

Utility: This model can be used not only at the decision stage but also during the intervention process. During the intervention, such weighting models can be used to measure how successful the intervention has been in terms of which criteria. It can be used to monitor progress and to see which areas need improvement.

Use in real world problems: In post-earthquake reconstruction and recovery processes, performance against criteria can be monitored to measure whether progress is effective.

6.4.2 Limitations in practical scenarios

(1) Complexity of real world dynamics

Limitations: The criteria and sub-criteria in the model may not adequately cover all real-world dynamics. In disasters such as earthquakes, uncertainties and unpredictable situations may occur. For example, factors such as aftershocks, secondary disasters (fire, epidemics) or international aid may not be fully represented in the model.

Practical Situation: In each scenario, the criteria and weights determined may be insufficient in the face of unexpected events. Incorrect prioritisation may be made because events that vary in the real world cannot be predicted in the model.

(2) Lack of data and misleading information

Limitation: The quality of the data required to determine the weights may not always be high. The weights assigned to criteria in a model should be based on reliable data. However, data collected during a disaster may be incomplete, inaccurate or delayed. When data quality is low, the results produced by the model may be misleading.

Practical Situation: In the real world, data collection is difficult, especially during disasters. For example, data on health services may be incomplete because the exact number of people under debris is not known.

(3) Constancy of criteria

Limitation: The criteria set in this model are inflexible and may not adapt immediately to dynamic changes. In the real world, the importance of a criterion may change rapidly at a given time. For example, while health services may be the most critical at the beginning, housing and social assistance may become more critical after a few weeks.

Practical Situation: Since the model works with fixed criteria and weights, it may not be flexible in the later stages of the event, leading to wrong decisions.

(4) Ignoring the human factor

Containment: Such models often do not adequately account for more abstract factors such as human behaviour and psychological influences. Factors such as people's emotional reactions, social solidarity and psychological traumas may not be fully quantified by mathematical models.

Practical situation: Although post-earthquake social unrest and psychological trauma (C13) are given certain weights in the model, the complexity of this situation in the real world may be much greater.

(5) Subjectivity of weights

Limitation: The weights of criteria and sub-criteria are usually determined on the basis of expert judgement or historical data. However, the choice of these weights is always subjective and may vary depending on the biases of the decision makers. This may limit the accuracy and objectivity of the model.

Practical situation: Different experts or organisations may set different weights for the same event, which may make the results of the model open to debate.

In terms of benefits, the model is an important aid in structuring decision-making, optimising resources and monitoring performance. In post-earthquake responses, it can be effective in determining which areas should be prioritised. Limitations arise especially in the complexity of real world dynamics, lack of data, inflexibility of criteria, ignoring human factors and subjectivity of weights. Therefore, these models should only be used as a guide for decision-makers, but final decisions should be based on the situation on the ground and the experience of experts.

6.5 Modeling to evaluate scenarios at the first moment of an earthquake

Gray swan scenarios are low probability but high impact events. The first moment of an earthquake is an example of such a scenario. This model was simulated for the first four main criteria in order of importance obtained by the F-MCDM method in order to evaluate an effective intervention and resource allocation at the first moment of an earthquake. The resulting modeling is given in Fig. 6.

The model in Fig. 6 shows how to provide initial response and allocate resources in gray swan scenarios such as earthquakes. Performance monitoring and recommendations for improvement are critical for effective emergency response. Simulation should be continuously updated and improved to take into account real-world dynamics. Furthermore, this modeling provides a basis for emergency planning and resource management for gray swan scenarios.

The first step of the model begins with determining the damage and determining which criteria are the priority. In the second stage, the workforce and financial resources required for each criterion are allocated. C1 and C8 received the most resources because commu-

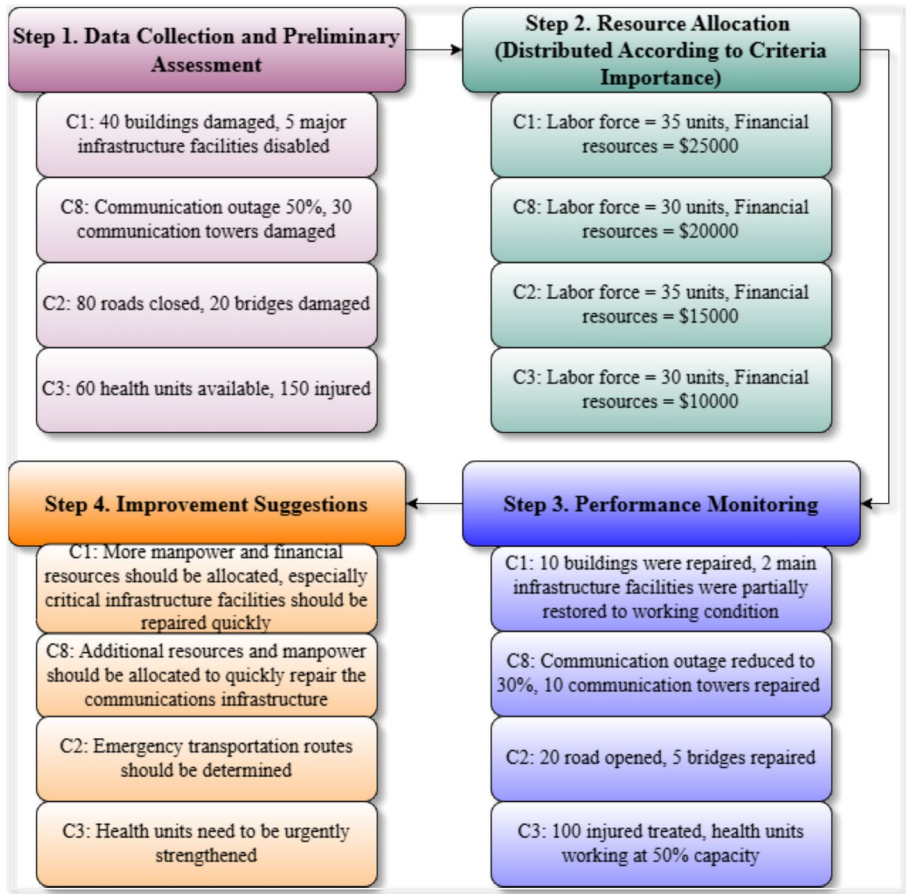


Fig. 6 Modelling for the first moment of the earthquake

nication and infrastructure are the most critical factors for the overall recovery process. In the third stage, the effectiveness of the interventions was monitored and although the initial results seemed positive, it became clear that there was still much work to be done. In the final stage, recommendations were made to achieve better results and improve critical areas. This model provides decision-makers with an important guide on which areas to invest more in and how to use which resources. In addition, it makes it possible to identify deficiencies and improvement opportunities in each step by monitoring performance.

7 Discussion, practical and managerial implications

In this section, the practical relevance of the study's findings is interpreted in the context of real-world risk management. The implications for local authorities, emergency planners, and decision-makers are highlighted. Additionally, the role of stakeholder diversity and model applicability across different gray swan events is explored.

Unexpected, high-impact events such as earthquakes fall under the category of "gray swan" scenarios. In managing these kinds of events, determining the appropriate main and sub-criteria is crucial for effective risk management and post-disaster recovery. The criteria identified in this study, based on expert opinions and literature, offer a structured approach to evaluating the multidimensional effects of a disaster and guiding strategic decision-making in the aftermath. The F-LMAW method allows decision-makers to assess the significance of these criteria under uncertainty, converting qualitative expert evaluations into objective, measurable data using fuzzy logic.

In the case of an earthquake, the protection of urban infrastructure (C1), the functionality of transportation networks (C2), and the continuity of healthcare services (C3) are among the most critical aspects of risk management. Urban resilience and sustainable quality of life are seen as top priorities in disaster planning. Additionally, communication infrastructure (C8) plays a pivotal role in preventing chaos during emergencies by ensuring coordination. These criteria are not limited to earthquake scenarios alone but can be adapted to other gray swan events, such as natural disasters like tsunamis or hurricanes. Furthermore, criteria such as economic factors (C4) and social stability (C5) can be relevant during economic crises, while communication and security (C7, C8) are also critical in cybersecurity threats.

From a management standpoint, implementing these criteria can considerably improve an organization's capacity to respond rapidly and efficiently in crisis circumstances. Investments in areas like transportation (C2), healthcare (C3), and education (C6) will not only improve preparedness but also strengthen long-term risk management capabilities. Similarly, strengthening communication and security infrastructures (C7, C8) will benefit organizations in responding to both natural and human-made disasters, including cyberattacks. However, it is important to acknowledge that while these measures can reduce risks, they cannot eliminate them entirely. Proactive risk management and adaptable strategies are essential to mitigate the impact of such events.

In this study, a multidimensional perspective was obtained by ensuring the participation of decision-makers from different fields of expertise in the evaluation process. The evaluation committee included various stakeholders such as academics, local government representatives, non-governmental organization experts, urban planners, engineers and search and rescue experts. This diversity has made the results of the study more inclusive and applicable.

- Contribution of diversity

Expert Perspectives Academics: (DM-1, DM-2) provided theoretical knowledge and literature-based approaches, while search and rescue experts (DM-3, DM-4) and engineers with field experience (DM-8) provided practical and operational knowledge. In this way, the determination and weighting of the criteria were supported both theoretically and practically. Local Government and Community Perspectives: The mayor (DM-6) and the urban planner (DM-5) brought a broader management perspective to the study, taking into account local needs and challenges related to governance. The NGO representative (DM-7) represented the social dimension of disaster management and contributed to criteria such as social stability (C5).

- The effect of stakeholder engagement on the power of the model

The success of the F-LMAW method used in the study is directly related to the multifaceted contributions of the stakeholders. The evaluation of each decision-maker ensured

the applicability of the model in a broader framework and increased the applicability of the results in real life. Thanks to these stakeholders from various disciplines, the importance of the criteria was evaluated from different perspectives and a more balanced assessment was made of critical elements such as communication infrastructure (C8), continuity of health services (C3) and social stability (C5).

- Recommendations for future studies

The evaluation committee may include representatives of the local community, private sector employees or a wider civil society representative. In particular, this can ensure that different social groups are included in the disaster management process and that decisions become more inclusive. Tools such as a survey or focus group can be used to ensure more in-depth stakeholder involvement in the process. This can make it easier to consider not only the views of decision-makers, but also the challenges they face in the field and their proposed solutions.

- Contributions in policy and strategy development

The involvement of various stakeholders makes strategic plans more realistic and feasible. For example, when assessing the feasibility of proposals, administrators such as the mayor (DM-6); The NGO representative (DM-7) and search and rescue experts (DM-3, DM-4) can identify implementation difficulties in the field. This directly contributes to policy and strategy development processes.

In practical terms, the model helps to allocate resources more efficiently by prioritizing the most critical areas based on the calculated weights of the criteria. Focusing resources on essential aspects such as urban infrastructure, transportation networks, and communication systems can expedite recovery efforts after a disaster. Furthermore, the use of expert-driven, data-based results ensures that risk management policies are grounded in realistic, applicable solutions.

In conclusion, the criteria determined through the F-LMAW method are not only applicable to earthquake scenarios but can also be adapted to other gray swan events. By using these criteria as a strategic guide, decision-makers and managers can create more resilient and flexible risk management systems, better preparing for both natural and man-made disasters. Stakeholder engagement further strengthens these systems by ensuring a diverse range of perspectives and expertise are incorporated, leading to more effective and inclusive risk management strategies.

8 Conclusions, limitations, and future directions

This concluding section summarizes the key outcomes of the study, discusses its theoretical and methodological contributions, outlines the limitations encountered, and proposes directions for future research. The broader implications for gray and black swan risk management strategies are also emphasized.

There is research in the literature on the application of fuzzy logic for catastrophe mitigation via gray swans (Kandel et al., 2014), black swans, and uncertainty management (Last & Kandel, 2010), but they are primarily theoretical. In this context, the starting point of this study is the situation created through the gray swan scenario. In this study, for the first time, the disaster situation related to the gray swan scenario is divided into main and sub-

criteria in detail in line with the relevant expert opinions. Then, the main and sub-criteria determined in the gray scenario for the first time are prioritized with the F-LMAW method. In addition, the potential of the least important criteria to create new gray swans is addressed for the first time in the study, and it is discussed that new gray swans can arise from gray swans.

This study is a fictional scenario of an earthquake disaster that may occur in Turkey. In a fictional gray swan scenario regarding an earthquake disaster that may occur on the same day in 5 different provinces in 5 different regions of Turkey, main and sub-criteria were created based on interviews conducted with experts at the same time on the same day. These criteria were again evaluated by experts according to their importance and then prioritized by analyzing them with the F-LMAW Method. The research on multi-criteria decision making and fuzzy multi-criteria decision making often focuses on evaluating and interpreting key outcomes (Pamučar et al., 2023; Simić et al., 2023; Stanković et al., 2020). Gray Swan occurrences have early warning information accessible, but it is not acknowledged, accepted, or followed for a variety of reasons (Engels, 2022). Gray Swans, like Black Swans, are rare but can be interpreted (Akkermans & Wassenhove, 2013). In this context, according to the gray swan metaphor, the least important outcomes should be evaluated. The least important outcomes can come together in different combinations to form new gray swans.

This study has effectively demonstrated how gray swan scenarios, such as earthquakes, can be managed with a structured risk assessment approach. Identifying the main and sub-criteria supported by expert judgments and fuzzy logic methods provides a comprehensive framework for understanding and mitigating the impacts of such unexpected, high-impact events. Using the F-LMAW method, we were able to quantify the importance of various criteria and provide actionable insights into resource allocation and strategic decision-making. Key findings indicate that urban infrastructure, transportation networks, and healthcare are critical for effective disaster management and recovery. Additionally, communication and security infrastructures are vital for maintaining coordination and minimizing chaos during emergencies. The applicability of these criteria extends beyond earthquakes to other gray swan scenarios, including natural disasters, economic crises, and cybersecurity threats.

For the first research question, the main and sub-criteria help assess the various dimensions of a gray swan disaster, such as an earthquake, and provide strategic guidance for the post-disaster recovery process. Using the F-LMAW Method, the relative importance of these criteria has been determined, highlighting which areas should receive more focus in the risk management process. For instance, a high-weighted criterion like urban infrastructure (C1) emphasizes how crucial it is to minimize damage to cities and maintain the quality of life. For the second question, the applicability of the identified criteria in predicting other gray swan scenarios is examined, and the potential contribution of strategic measures based on these criteria to reducing risks and preventing the emergence of new gray or black swan events is evaluated. The rankings of criteria obtained through the F-LMAW Method can be applied to a wide range of gray swan scenarios, from natural disasters like earthquakes to economic crises and cybersecurity threats. Additionally, preventive measures based on these criteria can help limit the impacts of new gray and black swan risks, such as strengthening infrastructure, raising awareness, and investing in communication and technological infrastructure. These strategic measures can be effective in reducing the impact of these risks, but since black swans are entirely unpredictable, it is not possible to fully eliminate

these risks. Proactive risk management and flexible strategies can help limit the impact of potential disasters.

This study acknowledges several limitations, each of which points to specific directions for future research. First, the complexity of real-world disaster scenarios may not be fully captured by the fixed criteria structure used in the model. Future studies could explore adaptive decision-making models that allow criteria and weights to evolve dynamically during different disaster phases. Second, the model heavily relies on expert opinions, which may be affected by incomplete or delayed data during crises. To address this, future research should consider integrating real-time data sources such as IoT-based environmental sensors or mobile reporting platforms to improve data reliability. Third, the model uses a pre-defined and static set of criteria, which may not be suitable for every evolving disaster context. Accordingly, researchers could develop flexible frameworks capable of updating inputs and outputs based on live scenario feedback. Fourth, human behavioral and psychological dimensions—such as trauma response, panic behavior, or community solidarity—are inherently difficult to quantify in mathematical models. Future work should consider incorporating behavioral modeling approaches, such as agent-based simulations, to better represent human-centered outcomes. Finally, the subjectivity involved in assigning weights through expert judgment introduces potential bias. Future research could mitigate this limitation by combining expert-based F-MCDM approaches with data-driven calibration tools, such as machine learning or statistical validation techniques, to enhance objectivity and replicability.

Authors' contributions SD, GD, FFB and DP performed the conceptualization and made the formal analysis, developed the methodology. They also carried out supervision and validation and wrote the original draft. GD, FFB and DP carried out the data curation and investigation, as well as the conceptualization. Their contribution also reflects in writing the original draft and the review editing. SD conducted the formal analysis and developed the methodology and the software as well; he was also responsible for the visualization and writing the original draft. GD and SD contributed in the formal analysis, supervision, the writing of the original draft, and the review and editing, too. All the authors have read and approved the final manuscript.

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