Research Article

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A new type of soft multi rough sets

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Abstract: Soft multi rough sets which are a hybrid model combining rough sets with soft multisets are defined by using soft multi rough approximation operators. Soft multi rough sets can be seen as a generalized rough set model based on soft multisets. In this paper, we contribute to a recent modelization of uncertainty. To be precise, in order to approach the uncertainty issue, we introduce a novel type of soft multi rough set by means of soft multi neighborhoods and then we use it to improve decision making in a multicriteria group environment. The motivation and method of this paper are as follows. Firstly we establish a soft multi covering approximation space. Secondly we define the soft multi neighborhood of the objects. We also introduce a new type of soft multi upper approximation operator by using soft multi neighborhood of the objects. Based on this new type of soft multi upper approximation operator, we propose a new kind of soft multi rough set. We present its basic properties and provide some illustrative examples. Thirdly, we show that our soft multi upper approximation operator is smaller than other multi upper approximation operator. Finally, we present an application of multicriteria group decision making using new type of soft multi covering rough approximation operators.

Keywords: rough set; soft set; soft multiset; soft multi rough set

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1 Introduction

Some problems in daily life cannot be solved with classical logic tools since they contain uncertain expressions. This is because, classical logic has a structure that operates within certain and specific limits. Many scientists have tried to develop theories to solve uncertain problems by leaving the certain and precise world of classical mathematics. Examples of these are; fuzzy set theory [1], rough set theory [2], soft set theory [3] and multiset theory [4]. These theories have created a natural structure for the generalization of mathematical concepts to fuzzy logic. Fuzzy logic is a broader concept than classical logic. Fuzzy logic is an encryption method that gives machines human-specific abilities and characteristics. The concept of fuzzy logic is used in many fields such as mathematics, sociology, mechatronics, artificial intelligence, robotics coding, medicine and science.

The first of the mathematical models proposed to express uncertainty problems is the fuzzy set theory proposed by Zadeh [1] in 1965. Unlike classical sets that categorize elements as either belonging or not belonging to a set completely, each element has a degree of membership in fuzzy set theory. The degree of membership can be any value in the real interval [0,1]. An element is a member of a set to a certain degree or is not a member to a certain degree. In this respect, this model, which allows graded membership by adapting to varying degrees of uncertainty or ambiguity, has created a natural structure for generalizing mathematical concepts to fuzzy logic.

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This theory has been actively studied by researchers ever since it was established, and it has been found useful in diverse areas of science and technology.

The rough set theory proposed by Pawlak [2] in 1982 is based on information systems. The structure of information systems is closely related to the equivalence relation, which is called the indiscernibility relation. In the approximation space created by the equivalence relation, two sets that are called lower and upper approximations are defined with the help of equivalence classes. Through these approximations, the rough set, which is a subset of a universal set, is expressed. In other words, approximation operators play an important role in the construction of uncertainty. Furthermore, by the fact that a covering is a generalization of a partition, classical rough set theory has been extended to include covering based generalizations of rough sets [5,6].

Rough set theory is derived from the fuzzy logic approach as an important tool that, in combination with data mining techniques, enables the efficient use of available data. Therefore, the rough set approach, which organizes incomplete, inadequate and ambiguous information and makes it suitable for data analysis, also enables the analysis of large databases, especially with rule reduction and classification issues, to be successfully applied in daily life. Despite its rapid development, rough set theory has been inadequate for modeling some uncertain problems because it does not take parameter values into account. In 1999, Molodtsov [3] introduced soft set theory, which can be seen as a completely new approximation method using the concept of parameters to model such problems. Molodtsov defined the soft set as the set of parameterized alternatives by matching the desired parameters with the alternatives to be selected. Soft set theory has attracted the attention of many researchers and many studies have been conducted on it, especially since it is important in terms of its ability to express uncertainty problems in a practical way and to provide convenience for the operations that can be performed on the uncertainty encountered [7–9]. Feng et al. [10,11] took an important step to make parameterization tools available for rough sets and defined a hybrid set model. They obtained the soft approximation space by using the soft set instead of the equivalence relation in rough set theory to find the soft lower and soft upper approximations of a set. By means of the notions of soft lower and upper approximation in the soft approximation space, they defined soft rough sets and analyzed their basic properties. They also showed that Pawlak's rough sets are a special case of soft rough sets. After, Feng [12] developed a decision making method to determine the most suitable object using the concepts of soft lower and upper approximations in the soft approximation space.

In 1999, Blizard [4] introduced the concept of multiset theory and function shells. This theory is known as the generalization of crisp set theory. This theory also acts as a tool for studying and providing solutions to sophisticated problems found in different fields such as computer intelligence, image processing, artificial intelligence, signal processing, data analysis and medicine and many more. Syropoulos [13] defined various operations on multisets in 2001. Building upon multiset theory, soft multiset theory was developed the combination of multiset and soft set theory. Soft multiset theory offers a parameterized structure in which each parameter maps to a multiset rather than a simple subset, enabling a more nuanced representation of vague and repetitive information. After that, multiset theory and soft multiset theory have been studied by many researchers including Herawan and Mustafa [14]; Alkhazaleh et al. [15]; Babitha and John [16]; Balami and Ibrahim [17]; and Tokat and Osmanoglu [18].

In 2020, Riaz et al. [19,20] described the concept of soft multi rough set using soft multiset and rough set. They presented basic properties of soft multi rough approximations and supported them with some illustrative examples. Moreover, they gave an application of soft multi rough approximations in multicriteria group decision making problems.

In the following years, researchers proposed many hybrid models to better express the matters encountered in problems involving uncertainty and to approach the ideal solution as closely as possible. Hybrid sets, which are created by combining set theories developed to address uncertainty, can model uncertainty problems more effectivelly. Because hybrid sets incorporate all the properties of their constituent sets, they enable easier classifications aimed at separating more complex data.

In this paper, we investigate a new hybrid set model inspired by the soft multi rough set. We define the concept of soft multi covering rough sets which are derived from rough sets and covering soft multisets. We establish a soft multi covering approximation space. We define a new type of soft multi upper approximation operator by using soft multi neighborhood of the objects. Based on the new type of soft multi upper approximation operator, we introduce a new kind of soft multi rough set. We present its basic properties and provide some illustrative examples. We supply an example to show that the new type of soft multi upper approximation operator, which is based on covering soft multiset, is smaller than the existing one. Comparing with the other type of soft multi upper operation, our soft multi upper approximation is more accurate and has more properties. We also present an application of multicriteria group decision making by the new type of soft multi covering rough approximation operators.

2 A new type of soft multi rough sets

In this section, we put forth of a new model of soft multi rough set via the concept of soft multi neighborhood. We present its basic and topological properties. Interior and closure operators are two core concepts in classical topology and for Pawlak's rough sets, the lower and upper approximation operations on a set are also the interior and closure operators on this set, respectively. In this paper, we use these topological tools to investigate a new type of soft multi rough sets. We present the similarity and difference between the properties of this type of soft multi rough sets and those of Pawlak's rough sets. For the definitions and fundamental properties of rough sets, soft sets, multisets, soft multisets and soft multi rough sets, we refer the readers to the following sources: [2,3,13,16,19,20]. We only recall the concept of soft multi rough sets proposed by Riaz et al. [19], which we use in this section. Unlike traditional Pawlak's rough set theory that relies on an equivalence relation to partition the universe of discourse, this approach employs a soft multiset to achieve the granulation of the universe. Consequently, this leads to a generalization of the classical Pawlak approximation space, termed as a soft multi approximation space.

Definition 1. [19] Let $G = (\Psi, A)$ be a soft multiset over M. Then the pair $P = (M, G)$ is called a soft multi approximation space. Based on the soft multi approximation space P, we define the following two operations

$$\underline{apr}_P(X) = \left\{ \frac{s}{u} \in M\colon\ \exists a \in A, \left[\frac{s}{u} \in \Psi(a) \subseteq X \right] \right\} = \bigcup_{a\in A} \{\Psi(a)\colon \Psi(a) \subseteq X\},$$
$$\overline{apr}^P(X) = \left\{ \frac{s}{u} \in M\colon\ \exists a \in A, \left[\frac{s}{u} \in \Psi(a),\ \Psi(a) \cap X \neq \emptyset \right] \right\} = \bigcup_{a\in A} \left\{\Psi(a)\colon \Psi(a) \cap X \neq \emptyset\right\}$$

assigning to every whole sub-multiset $X \subseteq M$ two sets $\underline{apr}_P(X)$ and $\overline{apr}^P(X)$, which are called the soft multi P – lower approximation and the soft multi P – upper approximation of X, respectively. In general, we refer to $\underline{apr}_P(X)$ and $\overline{apr}^P(X)$ as soft multi rough approximations of X with respect to P.

In addition,

$$Pos_P(X) = \underline{apr}_P(X)$$
$$Neg_P(X) = -\overline{apr}^P(X)$$
$$Bnd_P(X) = \overline{apr}^P(X) - \underline{apr}_P(X)$$

are called the soft multi P – positive, negative and boundary regions of X, respectively. If $\overline{apr}^P(X) = \underline{apr}_P(X)$, X is said to be soft multi P – definable; otherwise X is called a soft multi P – rough set.

Theorem 1. *[19] Let $G = (\Psi, A)$ be a soft multiset over M and $P = (M, G)$ be a soft multi approximation space. For all whole sub-multiset $X, Y \subseteq M$, then the soft multi P – lower and upper approximations have the following properties:*

1. $\underline{apr}_P(\emptyset) = \overline{apr}^P(\emptyset) = \emptyset$
2. $\underline{apr}_P(M) = \overline{apr}^P(M) = \bigcup_{a\in A} \Psi(a)$
3. $\underline{apr}_P(X \cap Y) \subseteq \underline{apr}_P(X) \cap \underline{apr}_P(Y)$

4. $\underline{\underline{apr}}_P(X \cup Y) \supseteq \underline{\underline{apr}}_P(X) \cup \underline{\underline{apr}}_P(Y)$
5. $\overline{\overline{apr}}^P(X \cup Y) = \overline{\overline{apr}}^P(X) \cup \overline{\overline{apr}}^P(Y)$
6. $\overline{\overline{apr}}^P(X \cap Y) \subseteq \overline{\overline{apr}}^P(X) \cap \overline{\overline{apr}}^P(Y)$
7. $X \subseteq Y \Rightarrow \underline{\underline{apr}}_P(X) \subseteq \underline{\underline{apr}}_P(Y)$
8. $X \subseteq Y \Rightarrow \overline{\overline{apr}}^P(X) \subseteq \overline{\overline{apr}}^P(Y)$.

Proposition 1. *[19] Let $G = (\Psi, A)$ be a soft multiset over M and $P = (M, G)$ be a soft multi approximation space. Then for any whole sub-multiset $X \subseteq M$, X is soft multi P – definable if and only if $\overline{\overline{apr}}^P(X) \subseteq X$.*

Firstly; we use a special kind of soft multiset with rough set, establish a soft multi covering approximation space.

Definition 2. A soft multiset $G = (\Psi, A)$ over M is called a full soft multiset if $\bigcup_{a \in A} \Psi(a) = M$.

Definition 3. A full soft multiset $G = (\Psi, A)$ over M is called a covering soft multiset if $\Psi(a) \neq \emptyset, \forall a \in A$.

We indicate a covering soft multiset with C_G.

Definition 4. Let $G = (\Psi, A)$ be a covering soft multiset over M. Then the pair $P = (M, C_G)$ is called a soft multi covering approximation space.

Definition 5. Let $G = (\Psi, A)$ be a covering soft multiset over M and $P = (M, C_G)$ be a soft multi covering approximation space. For any $\frac{s}{u} \in M$, we define the soft multi neighborhood of $\frac{s}{u}$ as follows:

$$N_P\left(\frac{s}{u}\right) = \cap\left\{\Psi(a): a \in A \wedge \frac{s}{u} \in \Psi(a)\right\}.$$

For better understanding of the soft multi neighborhood concept we illustrate it by the following example.

Example 1. Let $M = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right]$ be a universal multiset and $G = (\Psi, A)$ be a covering soft multiset over M, where $A = \{e_1, e_2, e_3, e_4\} \subseteq E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$,

$$\Psi(e_1) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}\right],$$

$$\Psi(e_2) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_4}{u_4}, \frac{s_6}{u_6}\right],$$

$$\Psi(e_3) = \left[\frac{s_2}{u_2}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_6}{u_6}\right],$$

$$\Psi(e_4) = \left[\frac{s_2}{u_2}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right].$$

Then $P = (M, C_G)$ is a soft multi covering approximation space. Then we obtain from this data

$N_P\left(\frac{s_1}{u_1}\right) = \cap\left\{\Psi(a): a \in A \wedge \frac{s_1}{u_1} \in \Psi(a)\right\} = \Psi(e_1) \cap \Psi(e_2) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}\right]$,
$N_P\left(\frac{s_2}{u_2}\right) = \cap\left\{\Psi(a): a \in A \wedge \frac{s_2}{u_2} \in \Psi(a)\right\} = \Psi(e_1) \cap \Psi(e_2) \cap \Psi(e_3) \cap \Psi(e_4) = \left[\frac{s_2}{u_2}\right]$,
$N_P\left(\frac{s_3}{u_3}\right) = \cap\left\{\Psi(a): a \in A \wedge \frac{s_3}{u_3} \in \Psi(a)\right\} = \Psi(e_1) \cap \Psi(e_3) \cap \Psi(e_4) = \left[\frac{s_2}{u_2}, \frac{s_3}{u_3}\right]$,
$N_P\left(\frac{s_4}{u_4}\right) = \cap\left\{\Psi(a): a \in A \wedge \frac{s_4}{u_4} \in \Psi(a)\right\} = \Psi(e_2) \cap \Psi(e_3) \cap \Psi(e_4) = \left[\frac{s_2}{u_2}, \frac{s_4}{u_4}, \frac{s_6}{u_6}\right]$,
$N_P\left(\frac{s_5}{u_5}\right) = \cap\left\{\Psi(a): a \in A \wedge \frac{s_5}{u_5} \in \Psi(a)\right\} = \Psi(e_4) = \left[\frac{s_2}{u_2}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right]$,
$N_P\left(\frac{s_6}{u_6}\right) = \cap\left\{\Psi(a): a \in A \wedge \frac{s_6}{u_6} \in \Psi(a)\right\} = \Psi(e_2) \cap \Psi(e_3) \cap \Psi(e_4) = \left[\frac{s_2}{u_2}, \frac{s_4}{u_4}, \frac{s_6}{u_6}\right]$.

The new type of soft multi rough set model as follows:

Definition 6. Let $G = (\Psi, A)$ be a covering soft multiset over M and $P = (M, C_G)$ be a soft multi covering approximation space. For all whole sub-multiset $X \subseteq M$, soft multi covering lower and upper approximations are respectively defined as

$$\underline{apr}_{N_P}(X) = \left\{ \frac{s}{u} \in M\colon\ \exists a \in A, \left[\frac{s}{u} \in \Psi(a) \subseteq X \right] \right\},$$
$$\overline{apr}^{N_P}(X) = \underline{apr}_{N_P}(X) \cup \left(\cup \left\{ N_P\left(\frac{s}{u}\right)\colon \frac{s}{u} \in X - \underline{apr}_{N_P}(X) \right\} \right).$$

In addition,

$$Pos_P(X) = \underline{apr}_{N_P}(X)$$
$$Neg_P(X) = -\overline{apr}^{N_P}(X)$$
$$Bnd_P(X) = \overline{apr}^{N_P}(X) - \underline{apr}_{N_P}(X)$$

are called the soft multi covering positive, negative and boundary regions of X, respectively.

Definition 7. Let $G = (\Psi, A)$ be a covering soft multiset over M and $P = (M, C_G)$ be a soft multi covering approximation space. A sub-multiset $X \subseteq M$ is called soft multi covering definable if $\overline{apr}^{N_P}(X) = \underline{apr}_{N_P}(X)$; in opposite case, i.e., if $\overline{apr}^{N_P}(X) \neq \underline{apr}_{N_P}(X)$, X is said to be soft multi covering rough set.

Example 2. Let $M = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right]$ be a universal multiset and $G = (\Psi, A)$ be a covering soft multiset over M, where $A = \{e_1, e_2, e_3, e_4\} \subseteq E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$,

$$\Psi(e_1) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}\right],$$
$$\Psi(e_2) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_4}{u_4}, \frac{s_5}{u_5}\right],$$
$$\Psi(e_3) = \left[\frac{s_2}{u_2}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}\right],$$
$$\Psi(e_4) = \left[\frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right].$$

Then $P = (M, C_G)$ is a soft multi covering approximation space.

For $X_1 = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}\right] \subseteq M$, we have

$$\underline{apr}_{N_P}(X_1) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}\right], \overline{apr}^{N_P}(X_1) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}\right].$$

Thus, $\underline{apr}_{N_P}(X_1) = \overline{apr}^{N_P}(X_1)$ and X_1 is a soft multi covering definable set.

For $X_2 = \left[\frac{s_1}{u_1}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right] \subseteq M$, we have

$$\underline{apr}_{N_P}(X_2) = \left[\frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right], \overline{apr}^{N_P}(X_2) = M.$$

Thus, $\underline{apr}_{N_P}(X_2) \neq \overline{apr}^{N_P}(X_2)$ and X_2 is a soft multi covering rough set.

Remark 1. From the definitions of two types of soft multi covering upper approximation operations, we have $\overline{apr}^{N_P}(X) \subseteq \overline{apr}^{P}(X)$ for all whole sub-multiset set $X \subseteq M$. But $\overline{apr}^{P}(X) \subseteq \overline{apr}^{N_P}(X)$ is not true in general as shown in the following example.

Example 3. Let $M = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}\right]$ be a universal multiset and $G = (\Psi, A)$ be a covering soft multiset over M, where $A = \{e_1, e_2, e_3, e_4\} \subseteq E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$,

$$\Psi(e_1) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}\right],$$
$$\Psi(e_2) = \left[\frac{s_2}{u_2}, \frac{s_3}{u_3}\right],$$
$$\Psi(e_3) = \left[\frac{s_3}{u_3}, \frac{s_4}{u_4}\right],$$
$$\Psi(e_4) = \left[\frac{s_4}{u_4}, \frac{s_5}{u_5}\right].$$

Then $P = (M, C_G)$ is a soft multi covering approximation space.

For $X = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}\right] \subseteq M$, we have

$$\overline{apr}^P(X) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}\right], \overline{apr}^{N_P}(X) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}\right].$$

Thus, we obtain $\overline{apr}^P(X) \subsetneq \overline{apr}^{N_P}(X)$.

Remark 2. It is evident that our soft multi covering upper approximation has the more exactitude than the other one. So our soft multi covering upper approximation is more accurate.

Remark 3. A soft multi rough set is based on a soft multiset in a soft multi approximation space, whereas a soft multi covering rough set is based on a covering soft multiset in a soft multi covering approximation space. We can call the soft multi rough set which is given in [19] as the first type of soft multi covering rough set in a soft multi covering approximation space. From the definitions of two types of soft multi covering lower approximation operations, it is easy to see that the new soft multi covering lower approximation is the same as that in the first type of soft multi covering rough set model. However, their upper approximation operations are different. Therefore, we can give the following results.

Theorem 2. *Let $G = (\Psi, A)$ be a soft multiset over M and $P = (M, G)$ be a soft multi approximation space. Then the following conditions are equivalent:*

1. *G is a full soft multiset*
2. *$\underline{apr}_P(M) = M$*
3. *$\overline{apr}^P(M) = M$*
4. *$X \subseteq \overline{apr}^P(X)$ for all $X \subseteq M$*
5. *$\overline{apr}^P\left(\{\frac{s}{u}\}\right) \neq \emptyset$ for all $\frac{s}{u} \in M$.*

Proof. (1) ⇒ (2) From the property (2) in Theorem 1, we have $\underline{apr}_P(M) = \bigcup_{a\in A}\{\Psi(a) \colon \Psi(a) \subseteq M\} = \bigcup_{a\in A}\Psi(a)$. Since G is a full soft multiset, we have $\bigcup_{a\in A}\Psi(a) = M$. So $\underline{apr}_P(M) = M$.

(2) ⇒ (3) From the property (2) in Theorem 1, we have $\overline{apr}^P(M) = \bigcup_{a\in A}\{\Psi(a) \colon \Psi(a) \cap M \neq \emptyset\} = \bigcup_{a\in A}\Psi(a)$. By condition (2), we have $\bigcup_{a\in A}\Psi(a) = M$. So $\overline{apr}^P(M) = M$.

(3) ⇒ (4) $X \subseteq M \Rightarrow \overline{apr}^P(X) \subseteq \overline{apr}^P(M)$ is implemented because of the property (8) in Theorem 1. Therefore, $X \subseteq \bigcup_{a\in A}\{\Psi(a) \colon \Psi(a) \cap X \neq \emptyset\} \subseteq \bigcup_{a\in A}\{\Psi(a) \colon \Psi(a) \cap M \neq \emptyset\} = \overline{apr}^P(M) = M$ is obtained from Definition 1.

(4) ⇒ (5) Take $X = \{\frac{s}{u}\}$. By condition (4), we have $\{\frac{s}{u}\} \subseteq \overline{apr}^P\left(\{\frac{s}{u}\}\right)$. Thus $\overline{apr}^P\left(\{\frac{s}{u}\}\right) \neq \emptyset$ since $\frac{s}{u} \in \overline{apr}^P\left(\{\frac{s}{u}\}\right)$.

(5) ⇒ (1) For any $\frac{s}{u} \in M$, we have that $\overline{apr}^P\left(\{\frac{s}{u}\}\right) \neq \emptyset$. From definition of the soft multi P – upper approximation, we have $\overline{apr}^P\left(\{\frac{s}{u}\}\right) = \bigcup_{a\in A}\left\{\Psi(a): \Psi(a) \cap \{\frac{s}{u}\} \neq \emptyset\right\}$. Then there exists some $a \in A$ such that $\Psi(a) \cap \{\frac{s}{u}\} \neq \emptyset$. It follows that $\frac{s}{u} \in \Psi(a)$ and so we have that $\frac{s}{u} \in \bigcup_{a\in A} \Psi(a)$. Hence $G = (\Psi, A)$ is a full soft multiset. □

Corollary 1. *Let $G = (\Psi, A)$ be a covering soft multiset over M and $P = (M, C_G)$ be a soft multi covering approximation space. For all whole sub-multiset $X, Y \subseteq M$, then the first type of soft multi covering lower and upper approximations have the following properties:*

1. $\underline{apr}_P(\emptyset) = \overline{apr}^P(\emptyset) = \emptyset$
2. $\underline{apr}_P(M) = \overline{apr}^P(M) = M$
3. $\underline{apr}_P(X) \subseteq X \subseteq \overline{apr}^P(X)$
4. $X \subseteq Y \Rightarrow \underline{apr}_P(X) \subseteq \underline{apr}_P(Y)$
5. $X \subseteq Y \Rightarrow \overline{apr}^P(X) \subseteq \overline{apr}^P(Y)$
6. $\underline{apr}_P(X \cap Y) \subseteq \underline{apr}_P(X) \cap \underline{apr}_P(Y)$
7. $\underline{apr}_P(X \cup Y) \supseteq \underline{apr}_P(X) \cup \underline{apr}_P(Y)$
8. $\overline{apr}^P(X \cup Y) = \overline{apr}^P(X) \cup \overline{apr}^P(Y)$
9. $\overline{apr}^P(X \cap Y) \subseteq \overline{apr}^P(X) \cap \overline{apr}^P(Y)$
10. $\overline{apr}^P\left(\{\frac{s}{u}\}\right) \neq \emptyset$ *for all* $\frac{s}{u} \in M$.

Proof. The proof is a direct consequence of Theorem 1 and Theorem 2. □

Corollary 2. *Let $G = (\Psi, A)$ be a covering soft multiset over M and $P = (M, C_G)$ be a soft multi covering approximation space. Then for any whole sub-multiset $X \subseteq M$, X is soft multi P – definable if and only if $\overline{apr}^P(X) = X$.*

Proof. The proof is a direct consequence of Proposition 1 and Theorem 2. □

Now, we investigate the second type of soft multi covering upper approximation and present a theorem which is necessary to prove the properties of the second type of soft multi covering upper approximation. Actually we give another representation of the second type of soft multi covering upper approximation. From this representation, we see more clearly the importance of the soft multi neighborhood concept.

Theorem 3. *Let $G = (\Psi, A)$ be a covering soft multiset over M and $P = (M, C_G)$ be a soft multi covering approximation space. For all whole sub-multiset $X \subseteq M$, then*

$$\overline{apr}^{N_P}(X) = \cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X\right\}.$$

Proof. It is easy to see

$$\cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X\right\} = \left(\cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in \underline{apr}_{N_P}(X)\right\}\right) \cup \left(\cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X - \underline{apr}_{N_P}(X)\right\}\right).$$

From definition of the soft multi covering lower approximation, for $\forall \frac{s}{u} \in \underline{apr}_{N_P}(X)$, there exists a $\Psi(a)$ such that $\Psi(a) \subseteq X$, thus $N_P\left(\frac{s}{u}\right) \subseteq \Psi(a) \subseteq X$. It is easy to see $X \subseteq \overline{apr}^{N_P}(X)$, therefore, we have

$$\forall \frac{s}{u} \in \underline{apr}_{N_P}(X), N_P\left(\frac{s}{u}\right) \subseteq \overline{apr}^{N_P}(X),$$

so

$$\cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X\right\} \subseteq \overline{apr}^{N_P}(X).$$

On the other hand, from definition of the soft multi neighborhood of $\frac{s}{u} \in M$, we have $\frac{s}{u} \in N_P\left(\frac{s}{u}\right)$, as a result, $\underline{apr}_{N_p}(X) \subseteq \cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in \underline{apr}_{N_p}(X)\right\}$. Thus,

$$\begin{aligned}\overline{apr}^{N_P}(X) &= \underline{apr}_{N_p}(X) \cup \left(\cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X - \underline{apr}_{N_p}(X)\right\}\right)\\ &\subseteq \left(\cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in \underline{apr}_{N_p}(X)\right\}\right) \cup \left(\cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X - \underline{apr}_{N_p}(X)\right\}\right)\\ &= \cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X\right\}.\end{aligned}$$

This shows that $\overline{apr}^{N_P}(X) = \cup\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X\}$. □

Now, we investigate some properties of the new type of soft multi covering upper approximation.

Theorem 4. *Let $G = (\Psi, A)$ be a covering soft multiset over M and $P = (M, C_G)$ be a soft multi covering approximation space. For all whole sub-multiset $X, Y \subseteq M$, then the second type of soft multi covering upper approximation has the following properties:*
1. $\overline{apr}^{N_P}(M) = M$
2. $\overline{apr}^{N_P}(\emptyset) = \emptyset$
3. $X \subseteq \overline{apr}^{N_P}(X)$
4. $\overline{apr}^{N_P}(X \cup Y) = \overline{apr}^{N_P}(X) \cup \overline{apr}^{N_P}(Y)$
5. $\overline{apr}^{N_P}\left(\overline{apr}^{N_P}(X)\right) = \overline{apr}^{N_P}(X)$
6. $X \subseteq Y \Rightarrow \overline{apr}^{N_P}(X) \subseteq \overline{apr}^{N_P}(Y)$
7. $\forall a \in A$, $\overline{apr}^{N_P}\left(\Psi(a)\right) = \Psi(a)$.

Proof. From Definition 6, we can easily prove that properties 1, 2, 3 and 7.

4) From Definition 6 and Theorem 3 we have

$$\overline{apr}^{N_P}(X) = \cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X\right\}$$

and

$$\overline{apr}^{N_P}(Y) = \cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in Y\right\}$$

for all $X, Y \subseteq M$. So

$$\begin{aligned}\overline{apr}^{N_P}(X) \cup \overline{apr}^{N_P}(Y) &= \left(\cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X\right\}\right) \cup \left(\cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in Y\right\}\right)\\ &= \cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X \cup Y\right\}\\ &= \overline{apr}^{N_P}(X \cup Y).\end{aligned}$$

5) By the definition of $N_P\left(\frac{s}{u}\right) = \cap\left\{\Psi(a): a \in A \wedge \frac{s}{u} \in \Psi(a)\right\}$, we have $N_P\left(\frac{s_n}{u_n}\right) \subseteq N_P\left(\frac{s}{u}\right)$ for all $\frac{s_n}{u_n} \in N_P\left(\frac{s}{u}\right)$. This shows that

$$\begin{aligned}\overline{apr}^{N_P}\left(\overline{apr}^{N_P}(X)\right) &= \cup\left\{N_P\left(\frac{s_n}{u_n}\right): \frac{s_n}{u_n} \in \overline{apr}^{N_P}(X)\right\}\\ &= \cup\left\{N_P\left(\frac{s_n}{u_n}\right): \frac{s_n}{u_n} \in N_P\left(\frac{s}{u}\right) \text{ and } \frac{s}{u} \in X\right\}\\ &\subseteq \cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X\right\}\\ &= \overline{apr}^{N_P}(X).\end{aligned}$$

On the other hand, from the property (3), we have $\overline{apr}^{N_P}(X) \subseteq \overline{apr}^{N_P}\left(\overline{apr}^{N_P}(X)\right)$. Thus, we prove that $\overline{apr}^{N_P}\left(\overline{apr}^{N_P}(X)\right) = \overline{apr}^{N_P}(X)$.

6) From Definition 6 and Theorem 3 we have

$$\overline{apr}^{N_P}(X) = \cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X\right\}$$

and

$$\overline{apr}^{N_P}(Y) = \cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in Y\right\}$$

for all $X, Y \subseteq M$. If $X \subseteq Y$, we have

$$\overline{apr}^{N_P}(X) = \cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in X\right\} \subseteq \cup\left\{N_P\left(\frac{s}{u}\right): \frac{s}{u} \in Y\right\} = \overline{apr}^{N_P}(Y).$$

Theorem 5. *Let $G = (\Psi, A)$ be a covering soft multiset over M and $P = (M, C_G)$ be a soft multi covering approximation space. For all whole sub-multiset $X, Y \subseteq M$, then the second type of soft multi covering lower and upper approximations do not have the following properties:*

1. $\underline{apr}_{N_P}(X \cap Y) = \underline{apr}_{N_P}(X) \cap \underline{apr}_{N_P}(Y)$
2. $\underline{apr}_{N_P}\left(-\underline{apr}_{N_P}(X)\right) = -\underline{apr}_{N_P}(X)$
3. $\overline{apr}^{N_P}\left(-\overline{apr}^{N_P}(X)\right) = -\overline{apr}^{N_P}(X)$
4. $\underline{apr}_{N_P}(X) = -\overline{apr}^{N_P}(-X)$
5. $\overline{apr}^{N_P}(X) = -\underline{apr}_{N_P}(-X)$.

The following examples show that the equalities mentioned above do not hold.

Example 4. Let $M = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right]$ be a universal multiset and $G = (\Psi, A)$ be a covering soft multiset over M, where $A = \{e_1, e_2, e_3\} \subseteq E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$,

$$\Psi(e_1) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}\right],$$

$$\Psi(e_2) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}\right],$$

$$\Psi(e_3) = \left[\frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right].$$

Then $P = (M, C_G)$ is a soft multi covering approximation space. Suppose that $X = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}\right] \subseteq M$ and $Y = \left[\frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right] \subseteq M$.

1. $\underline{apr}_{N_P}(X) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}\right]$, $\underline{apr}_{N_P}(Y) = \left[\frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right]$, $\underline{apr}_{N_P}(X) \cap \underline{apr}_{N_P}(Y) = \left[\frac{s_3}{u_3}\right]$ and $\underline{apr}_{N_P}(X \cap Y) = \emptyset$. This shows that $\underline{apr}_{N_P}(X \cap Y) \neq \underline{apr}_{N_P}(X) \cap \underline{apr}_{N_P}(Y)$.
2. $\underline{apr}_{N_P}(X) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}\right]$, $-\underline{apr}_{N_P}(X) = \left[\frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right]$, $\underline{apr}_{N_P}\left(-\underline{apr}_{N_P}(X)\right) = \emptyset$. This shows that $\underline{apr}_{N_P}\left(-\underline{apr}_{N_P}(X)\right) \neq -\underline{apr}_{N_P}(X)$.
3. $\overline{apr}^{N_P}(X) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}\right]$, $-\overline{apr}^{N_P}(X) = \left[\frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right]$, $\overline{apr}^{N_P}\left(-\overline{apr}^{N_P}(X)\right) = \left[\frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right]$. This shows that $\overline{apr}^{N_P}\left(-\overline{apr}^{N_P}(X)\right) \neq -\overline{apr}^{N_P}(X)$.
4. $\underline{apr}_{N_P}(X) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}\right]$, $\overline{apr}^{N_P}(-X) = \left[\frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right]$, $-\overline{apr}^{N_P}(-X) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}\right]$. This shows that $\underline{apr}_{N_P}(X) \neq -\overline{apr}^{N_P}(-X)$.
5. $\overline{apr}^{N_P}(X) = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}\right]$, $\underline{apr}_{N_P}(-X) = \emptyset$, $-\underline{apr}_{N_P}(-X) = M = \left[\frac{s_1}{u_1}, \frac{s_2}{u_2}, \frac{s_3}{u_3}, \frac{s_4}{u_4}, \frac{s_5}{u_5}, \frac{s_6}{u_6}\right]$. This shows that $\overline{apr}^{N_P}(X) \neq -\underline{apr}_{N_P}(-X)$.

3 An application of multicriteria group decision making by new type of soft multi covering approximation operators

Riaz et al. [19,20] applied soft multi rough sets to multicriteria group decision making problems which refines the primary evaluation of the whole expert group and enables us to select the optimal object in a more reliable manner. In this work we use the new type of soft multi covering rough sets at Riaz's method. Riaz's methods can be seen as a first attempt toward the possible application of soft multi rough approximations in multicriteria group decision making under uncertainty. In the proposed schemes, each expert simply gives an initial set consisting of the preferable alternatives in the corresponding expert's point of view. The primary evaluation results of the expert group store in the evaluation soft multiset and then approximate in the original description soft multiset using soft multi rough approximations. It should be noted that the use of our soft multi rough sets could, to some extent, automatically reduce the noise factor caused by the subjective nature of the expert's evaluation. Because when our approaches are used, the boundaries of the primary evaluation results of the whole expert group become smaller and their definability increases. This makes the results clearer. Thus we may expect to gain much more useful information with the help of soft multi covering rough approximations. The soft multi rough set based decision making methods in [19,20] can be summarized as follows:

Algorithm 1. ([19])

1. Step: Write the original description soft multiset $G = (\Psi, A)$ which describes the given data.
2. Step: Consider a group of decision makers $Z = \{D_1, D_2, \ldots, D_n\}$ and construct the evaluation soft multiset $\Omega = (\omega, Z)$ using the primary evaluation results of the expert group Z.
3. Step: Obtain soft multi rough approximations in the form of soft multisets $\Omega_* = (\underline{\omega}_*, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$.
4. Step: Define fuzzy multiset μ_{Ω_*}, μ_Ω and μ_{Ω^*} corresponding to the soft multiset $\Omega_* = (\underline{\omega}_*, Z), \Omega = (\omega, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$ defined by the formulas:

$$\mu_{\Omega_*}\left(\left[\frac{s_k}{u_k}\right]\right) = \frac{1}{n}\sum_{i=1}^{n} C_{\underline{\omega}_* D_i}\left(\left[\frac{s_k}{u_k}\right]\right)$$

$$\mu_{\Omega}\left(\left[\frac{s_k}{u_k}\right]\right) = \frac{1}{n}\sum_{i=1}^{n} C_{\omega D_i}\left(\left[\frac{s_k}{u_k}\right]\right)$$

$$\mu_{\Omega^*}\left(\left[\frac{s_k}{u_k}\right]\right) = \frac{1}{n}\sum_{i=1}^{n} C_{\overline{\omega}^* D_i}\left(\left[\frac{s_k}{u_k}\right]\right)$$

5. Step: Characterize the recommended level of experts in the form of parameter set

$$C = \{\text{not recommended, recommended, highly recommended}\}.$$

6. Step: Define fuzzy soft multiset $R = (\alpha, C)$ using the fuzzy multisets μ_{Ω_*}, μ_Ω and μ_{Ω^*}.
7. Step: Calculate choice value c_i for each object. Select the object having maximum decision choice value.

Algorithm 2. ([20])

1. Step: Write the original description soft multiset $G = (\Psi, A)$ which describes the given data.
2. Step: Consider a group of decision makers $Z = \{D_1, D_2, \ldots, D_n\}$ and construct the evaluation soft multiset $\Omega = (\omega, Z)$ using the primary evaluation results of the expert group Z.
3. Step: Obtain soft multi rough approximations in the form of soft multisets $\Omega_* = (\underline{\omega}_*, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$.
4. Step: Define fuzzy multiset μ_{Ω_*}, μ_Ω and μ_{Ω^*} corresponding to the soft multiset $\Omega_* = (\underline{\omega}_*, Z), \Omega = (\omega, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$ defined by the formulas:

$$\mu_{\Omega_*}\left(\left[\frac{s_k}{u_k}\right]\right) = \frac{1}{n}\sum_{i=1}^{n} C_{\underline{\omega}_* D_i}\left(\left[\frac{s_k}{u_k}\right]\right)$$

$$\mu_{\Omega}\left(\left[\frac{s_k}{u_k}\right]\right) = \frac{1}{n}\sum_{i=1}^{n} C_{\omega D_i}\left(\left[\frac{s_k}{u_k}\right]\right)$$

$$\mu_{\Omega^*}\left(\left[\frac{s_k}{u_k}\right]\right) = \frac{1}{n}\sum_{i=1}^{n} C_{\overline{\omega}^* D_i}\left(\left[\frac{s_k}{u_k}\right]\right)$$

5. Step: Find the final decision set by adding $\Omega_* = (\underline{\omega}_*, Z), \Omega = (\omega, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$ calculated as

$$\mu_{\Omega_*+\Omega+\Omega_*}\left(\left[\frac{s_k}{u_k}\right]\right) = \mu_{\Omega_*}\left(\left[\frac{s_k}{u_k}\right]\right) + \mu_{\Omega}\left(\left[\frac{s_k}{u_k}\right]\right) + \mu_{\Omega^*}\left(\left[\frac{s_k}{u_k}\right]\right) - \left[\mu_{\Omega_*}\left(\left[\frac{s_k}{u_k}\right]\right) \times \mu_{\Omega}\left(\left[\frac{s_k}{u_k}\right]\right) \times \mu_{\Omega^*}\left(\left[\frac{s_k}{u_k}\right]\right)\right]$$

6. Step: Finally the alternative having maximum decision value can be selected as optimal solution.

At the begin of our discussion, we introduce a comparative analysis of decision making methods in [19,20]. By comparing these given algorithms, we decide which one to use in this work. Now we remind the example given in [19]. The proposed scheme was illustrated by a concrete example regarding the selection of humanoid robot.

Example 5. Riaz et al. [19] applied the concept of soft multi rough set for the selection of humanoid robots for hotel staff members. Suppose a multinational hotels company name JAL Hotels Company decided to have AI (Artificial Intelligence) humanoid robot staff members in their hotels. For this purpose the CEO of the company contact with three Artificial Intelligence experts to decide which humanoid robot is better as hotel staff. Let $M = \left[\frac{20}{u_1}, \frac{30}{u_2}, \frac{25}{u_3}, \frac{35}{u_4}\right]$ be a multiset of Humanoid Robots for the selection as a hotels staff member. Here the multiplicity of humanoid robots denotes the number of robots that are required as a staff in hotels. Let $A = \{e_1, e_2, e_3, e_4\}$ be the set of features considered for humanoid robots where,

e_1 = Facial Recognition, e_2 = Conversations Skills, e_3 = Movable, e_4 = Affordable Price.

Construct a soft multiset $G = (\Psi, A)$ which specify the Artificial Intelligence of humanoid robots.

$$\Psi(e_1) = \left[\frac{30}{u_2}, \frac{35}{u_4}\right],$$
$$\Psi(e_2) = \left[\frac{20}{u_1}, \frac{25}{u_3}\right],$$
$$\Psi(e_3) = \left[\frac{30}{u_2}, \frac{35}{u_4}\right],$$
$$\Psi(e_4) = \left[\frac{30}{u_2}, \frac{25}{u_3}\right].$$

Riaz et al. [19] used Algorithm 1 in this example and obtained the optimal choice for the selection as a hotels staff member.

Algorithm 1 results should be as follows [19]:

$$\underset{2,3}{\left[\frac{30}{u_2}\right]} \underset{>2}{>} \left[\frac{35}{u_4}\right] \underset{>1,6}{>} \left[\frac{25}{u_3}\right] \underset{\approx 1,6}{\approx} \left[\frac{20}{u_1}\right]$$

Since $\left[\frac{30}{u_2}\right]$ is the group having maximum decision value. So, $\left[\frac{30}{u_2}\right]$ is the optimal choice for the selection as a hotels staff member.

Let us solve the above example again according to Algorithm 2 given in [20]. It is easy to find that the ranking of the alternatives based on Algorithm 2 and results should be as follows:

$$\left[\frac{30}{u_2}\right] > \left[\frac{35}{u_4}\right] > \left[\frac{25}{u_3}\right] > \left[\frac{20}{u_1}\right]$$
$$1,86 > 1,78 > 1,48 > 1,45$$

Now we give the comparison analysis of between Algorithm 1 and Algorithm 2.

Remark 4. Both algorithms have different formulation strategies we can produced lightly different results but it is great significance mention that both algorithms as given above give the same final optimal choices. So, $\left[\frac{30}{u_2}\right]$ is the robot to be selected for the hotel staff member. However Algorithm 1 in [19] gives little information to support the decision making process. Since Algorithm 2 in [20] gives more precise results, we decide to use it.

Remark 5. In our work, to test the success of this novel approximations, the example given in [20] will be solved again with new approximations. We will use soft multi covering rough approximations instead of soft multi rough approximations in 3. Step at Algorithm 2 [20]. We may expect to gain much more useful information with the help of soft multi covering rough approximations and to reduce possible deviations in the decision-making results.

Now we remind the example given in [20]. The proposed scheme was illustrated by a concrete example regarding the selection of patients suffering from depression by the psychiatrists which has more chances of depression.

Example 6. [20] Let $M = \left[\frac{29}{u_1}, \frac{30}{u_2}, \frac{35}{u_3}, \frac{20}{u_4}, \frac{30}{u_5}, \frac{23}{u_6}\right]$ be the collection of groups of patients suffering from depression, where 29 is the multiplicity of group u_1 which represent number of patients in group u_1. Similarly 30 is multiplicity of u_2 which represent number of patients in group u_1 and so on. Since victimization of depression is associated with more severe symptomatology, decreased quality of life, and high risk of revictimization. As any group of individuals can be victimized, let

$$u_1\text{: woman, } u_2\text{: man, } u_3\text{: young age, } u_4\text{: mature age, } u_5\text{: old age, } u_6\text{: jobless}$$

Assume that the set $Z = \{D_1, D_2, D_3\}$ consist of three psychiatrist (decision makers) who analyze the groups of patients depending upon related attributes. Because psychiatrits are trained medical doctors, they can prescribe medications, and they spend much of their time with patients on medication management as a course of treatment. The group of three psychiatrist is selected to diagnose the patients suffering from depression. The list of criterion/treatments is as follows.

C_1 represent ”methods of treatment”

C_2 represent ”analytical testing”

C_3 represent ”problem − solving techniques”

C_4 represent ”psychological theory”

C_5 represent ”behavioral therapy”

Consider the set of attributes is $A = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}\}$, where

e_1 represent ”Hopeless outlook, not going out anymore”

e_2 represent ”loss of interest, not doing usual enjoyable activities”

e_3 represent ”fatigue and sleep problems”

e_4 represent ”anxiety, unable to concentrate”

e_5 represent ”irritating, felt sad, down or miserable”

e_6 represent ”loss or change of appetite”

e_7 represent ”weight problem, significant weight loss or gain”

e_8 represent ”overwhelmed, guilty, irritable, frustrated”

e_9 represent ”asking help in every matter”

e_{10} represent ”change in behavior”

e_{11} represent ”lacking in confidence, unhappy, indecisive, disappointed”.

Construct a soft multiset $G = (\Psi, A)$ to describe uncertain information of 6 groups patients and their attributes.

$$\Psi(e_1) = \left[\frac{29}{u_1}, \frac{30}{u_2}\right]$$
$$\Psi(e_2) = \left[\frac{35}{u_3}, \frac{20}{u_4}, \frac{30}{u_5}, \frac{23}{u_6}\right]$$
$$\Psi(e_3) = \left[\frac{29}{u_1}, \frac{30}{u_2}\right]$$
$$\Psi(e_4) = \left[\frac{35}{u_3}, \frac{30}{u_5}, \frac{23}{u_6}\right]$$
$$\Psi(e_5) = \left[\frac{20}{u_4}\right]$$
$$\Psi(e_6) = \left[\frac{29}{u_1}, \frac{30}{u_2}, \frac{35}{u_3}, \frac{30}{u_5}\right]$$
$$\Psi(e_7) = \left[\frac{20}{u_4}, \frac{23}{u_6}\right]$$
$$\Psi(e_8) = \left[\frac{30}{u_2}, \frac{35}{u_3}, \frac{30}{u_5}\right]$$
$$\Psi(e_9) = \left[\frac{20}{u_4}, \frac{30}{u_5}\right]$$
$$\Psi(e_{10}) = \left[\frac{20}{u_4}, \frac{30}{u_5}\right]$$
$$\Psi(e_{11}) = \left[\frac{29}{u_1}, \frac{23}{u_6}\right]$$

Riaz et al. [20] used Algorithm 2 in this example and obtained the optimal choice for the selection of patients by the psychiatrists which has more chances of depression.

Algorithm 2 results should be as follows [20]:

$$\begin{array}{ccccccccccc} \left[\frac{30}{u_2}\right] & > & \left[\frac{30}{u_5}\right] & > & \left[\frac{29}{u_1}\right] & \approx & \left[\frac{20}{u_4}\right] & > & \left[\frac{35}{u_3}\right] & \approx & \left[\frac{23}{u_6}\right] \\ 2 & > & 1,667 & > & 1,556 & \approx & 1,556 & > & 1,33 & \approx & 1,33 \end{array}$$

Since $\left[\frac{30}{u_2}\right]$ is the group having maximum decision value. So, $\left[\frac{30}{u_2}\right]$ is selected by the psychiatrists which has more chances of depression.

Now, we show how to use a new type of soft multi covering rough sets to support this group decision making process. Let us solve the above example again according to new type of soft multi covering rough approximation operators.

Example 7. **1. Step:** ([20]) Write the original description soft multiset $G = \left(\Psi, A\right)$ which describes the given data. Since $G = \left(\Psi, A\right)$ be a covering soft multiset over M, then $P = (M, C_G)$ is a soft multi covering approximation space.

2. Step: ([20]) Consider a group of decision makers $Z = \{D_1, D_2, \ldots, D_n\}$ and construct the evaluation soft multiset $\Omega = (\omega, Z)$ using the primary evaluation results of the expert group Z. $Z = \{D_1, D_2, D_3\}$ are the specialist psychiatrists group who analyze the groups of patients depending upon related attributes. X_i is the initial assessment result of the psychiatrists. Now we generate the soft multiset $\Omega = (\omega, Z)$ primary evaluation result of experts are

$$X_1 = \omega(D_1) = \left[\frac{29}{u_1}, \frac{30}{u_2}, \frac{30}{u_5}\right]$$

$$X_2 = \omega(D_2) = \left[\frac{30}{u_2}, \frac{35}{u_3}, \frac{20}{u_4}\right]$$

$$X_3 = \omega(D_3) = \left[\frac{30}{u_2}, \frac{30}{u_5}, \frac{23}{u_6}\right]$$

3. Step: Obtain soft multi covering rough approximations in the form of soft multisets $\Omega_* = (\underline{\omega}_*, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$. Now, we use a new type of soft multi covering rough approximations instead of soft multi rough approximations and we find the soft multi covering rough approximations as

$$\underline{\omega}_*(D_1) = \underline{apr}_{N_P}\left(X_1\right) = \left[\frac{29}{u_1}, \frac{30}{u_2}\right]$$

$$\underline{\omega}_*(D_2) = \underline{apr}_{N_P}\left(X_2\right) = \left[\frac{20}{u_4}\right]$$

$$\underline{\omega}_*(D_3) = \underline{apr}_{N_P}\left(X_3\right) = \emptyset$$

$$\overline{\omega}^*(D_1) = \overline{apr}^{N_P}\left(X_1\right) = \left[\frac{29}{u_1}, \frac{30}{u_2}, \frac{30}{u_5}\right]$$

$$\overline{\omega}^*(D_2) = \overline{apr}^{N_P}\left(X_2\right) = \left[\frac{29}{u_1}, \frac{30}{u_2}, \frac{35}{u_3}, \frac{20}{u_4}, \frac{30}{u_5}\right]$$

$$\overline{\omega}^*(D_3) = \overline{apr}^{N_P}\left(X_3\right) = \left[\frac{29}{u_1}, \frac{30}{u_2}, \frac{30}{u_5}, \frac{23}{u_6}\right]$$

It is great significance mention that lower approximations are the same but upper approximation are the different at above example. Following these soft multi covering rough approximations, we get two soft multisets $\Omega_* = (\underline{\omega}_*, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$ where, $\underline{\omega}_*(D_i) = \underline{apr}_{N_P}\left(X_i\right)$ and $\overline{\omega}^*(D_i) = \overline{apr}^{N_P}\left(X_i\right)$.

4. Step: Now, we define fuzzy multiset μ_{Ω_*}, μ_Ω and μ_{Ω^*} corresponding to the soft multiset $\Omega_* = (\underline{\omega}_*, Z)$, $\Omega = (\omega, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$ defined by the formulas:

$$\mu_{\Omega_*}\left(\left[\frac{s_k}{u_k}\right]\right) = \frac{1}{3}\sum_{i=1}^{3} C_{\underline{\omega}_* D_i}\left(\left[\frac{s_k}{u_k}\right]\right)$$

$$\mu_{\Omega}\left(\left[\frac{s_k}{u_k}\right]\right) = \frac{1}{3}\sum_{i=1}^{3} C_{\omega D_i}\left(\left[\frac{s_k}{u_k}\right]\right)$$

$$\mu_{\Omega^*}\left(\left[\frac{s_k}{u_k}\right]\right) = \frac{1}{3}\sum_{i=1}^{3} C_{\overline{\omega}^* D_i}\left(\left[\frac{s_k}{u_k}\right]\right)$$

Thus, we have

$$\mu_{\Omega_*}\left(\left[\frac{s_k}{u_k}\right]\right)=\left\{\left(\left[\frac{29}{u_1}\right],\frac{1}{3}\right),\left(\left[\frac{30}{u_2}\right],\frac{1}{3}\right),\left(\left[\frac{35}{u_3}\right],0\right),\left(\left[\frac{20}{u_4}\right],\frac{1}{3}\right),\left(\left[\frac{30}{u_5}\right],0\right),\left(\left[\frac{23}{u_6}\right],0\right)\right\}$$
$$\mu_{\Omega}\left(\left[\frac{s_k}{u_k}\right]\right)=\left\{\left(\left[\frac{29}{u_1}\right],\frac{1}{3}\right),\left(\left[\frac{30}{u_2}\right],1\right),\left(\left[\frac{35}{u_3}\right],\frac{1}{3}\right),\left(\left[\frac{20}{u_4}\right],\frac{1}{3}\right),\left(\left[\frac{30}{u_5}\right],\frac{2}{3}\right),\left(\left[\frac{23}{u_6}\right],\frac{1}{3}\right)\right\}$$
$$\mu_{\Omega^*}\left(\left[\frac{s_k}{u_k}\right]\right)=\left\{\left(\left[\frac{29}{u_1}\right],1\right),\left(\left[\frac{30}{u_2}\right],1\right),\left(\left[\frac{35}{u_3}\right],\frac{1}{3}\right),\left(\left[\frac{20}{u_4}\right],\frac{1}{3}\right),\left(\left[\frac{30}{u_5}\right],1\right),\left(\left[\frac{23}{u_6}\right],\frac{1}{3}\right)\right\}.$$

5. Step: We find the final decision set by adding $\Omega_* = (\underline{\omega}_*, Z), \Omega = (\omega, Z)$ and $\Omega^* = (\overline{\omega}^*, Z)$ calculated as

$$\mu_{\Omega_*+\Omega+\Omega_*}\left(\left[\frac{s_k}{u_k}\right]\right)=\mu_{\Omega_*}\left(\left[\frac{s_k}{u_k}\right]\right)+\mu_{\Omega}\left(\left[\frac{s_k}{u_k}\right]\right)+\mu_{\Omega^*}\left(\left[\frac{s_k}{u_k}\right]\right)$$
$$-\left[\mu_{\Omega_*}\left(\left[\frac{s_k}{u_k}\right]\right)\times\mu_{\Omega}\left(\left[\frac{s_k}{u_k}\right]\right)\times\mu_{\Omega^*}\left(\left[\frac{s_k}{u_k}\right]\right)\right]$$

Then we can arrange the groups according to their decision values as:

$$\begin{array}{cccccc}\left[\frac{30}{u_2}\right] > & \left[\frac{30}{u_5}\right] > & \left[\frac{29}{u_1}\right] > & \left[\frac{20}{u_4}\right] > & \left[\frac{35}{u_3}\right] \approx & \left[\frac{23}{u_6}\right] \\ 2 & >1,667> & 1,556> & 0,953> & 0,66 & \approx 0,66\end{array}$$

6. Step: Finally the alternative having maximum decision value can be selected as optimal solution. Since $\left[\frac{30}{u_2}\right]$ is the group having maximum decision value. So, $\left[\frac{30}{u_2}\right]$ is selected by the psychiatrists which has more chances of depression.

Remark 6. The final optimal choices are same in above both decision making problem. According to data from the above example which use a new type of soft multi covering rough approximations gives more precise results. However Algorithm 2 in [20] gives little information to support the decision making process. The key difference is that in the Step 3 at Algorithm 2, we use soft multi covering rough approximations instead of soft multi rough approximations.

Remark 7. The closeness of the values obtained from Algorithm 2 given in [20] makes the decision-making task difficult. In this study, it is aimed to make the task of determining the most suitable choice easier by using new approximations. Soft multi covering upper approximation can be used to add the optimal objects possibly neglected by the experts in primary evaluation while soft multi covering lower approximation can be used to remove the objects that are asymmetrically selected as optimal. Since the boundaries are reduced by means of new approximations, the definability increases. Hence the use of a new type of soft multi covering approximations reduce the error to some extent caused by personal nature of analyst during group decision-making. Thus, the results become clear.

At the end of our discussion, we point out the most important advantage of this new hybrid mathematical tools is that they can reduce the possible error margin of decision-makers. It is worth noting that we cannot take out the best alternative directly from the primary evaluation results. It should be noted that the use of the soft multi covering rough technique in our new proposal also refines the primary evaluation results of the whole expert group and thus enables us to select the optimal object in a more reliable manner. This is also very meaningful since in the real world, many important decision is made by an expert group, instead of only a single.

4 Conclusions

In this work we defined the soft multi neighborhood of a object. Based on the soft multi neighborhoods, we have proposed a new kind of soft multi covering upper approximation operator and presented its basic properties. Then we gave counterexamples for unsatisfied properties. Also from the definitions of two types of soft multi covering upper approximation operations, we showed that our soft multi upper approximation operator is smaller than with other type of soft multi upper approximation operator. It is evident that the second type of soft multi covering upper approximation has the more exactitude than the other one. So our soft multi covering upper approximation is more accurate and have more properties. Nowadays, with the differentiation and diversification of human needs, it has become a necessity to bring more qualified mathematical models to the literature for the uncertainty problems encountered in many areas. This study can be seen as a source of motivation for many researchers since the decision-making processes managed by new type of soft multi covering rough approximation operators, especially for the uncertainty problems, in a more ideal way. Riaz et al. [19,20] gave methods for obtaining near-ideal results for decision-making. However, the similarity of the closeness coefficients obtained based on this approach may increase the possible error. The aim of this article is to overcome this inadequacy via the soft multi covering approximations and to develop the decision making network. To test the success of this novel approach, the example given in [20] have solved again with new approximations. With new approaches, clearer and more ideal results have obtained. By making the efficiency of decision-makers more active in the decision-making process, possible deviations in results have reduced. We think that will be useful to apply the new type of soft multi covering rough sets theory to actual practice. Therefore we have proposed a soft multi covering rough set based scheme for supporting multicriteria group decision making, illustrated by a concrete example. To extend this work, one can consider to apply new soft multi covering rough approximations to multicriteria group decision making problems.

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