Investigating Problems Posed by Pre-Service Mathematics Teachers for the Four Operations in Fractions

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Abstract: This study investigates 40 preservice middle school teachers' problems posed for two given fractions. Pre-service teachers were asked to pose three problems (farea, length and set model) for each of the operations of addition, subtraction, multiplication and division with these fractions. 12 problems posed by each pre-service teacher were examined in terms of their suitability for the given operation, the targeted model and real life relevance. Errors in the posed problems were also examined. As a result of the analysis, it was observed that the model in which pre-service teachers were the most unsuccessful in posing problems was the set model, and the model in which they were most successful was the length model. Many types of errors were encountered in the problems posed by the pre-service teachers. It has been observed that many errors exist together, especially in the problems posed for the set model.

Keywords: problem posing, mathematics teacher education, fraction operations, area models, length models, set models.

Introduction

Fractions are among the concepts that students have difficulty in learning (Van de Walle et al., 2019) and teachers have difficulty in teaching (Charalambous & Pitta-Pantazi, 2007). The reason fractions are difficult for most people is because they involve relationships between quantities (Lin, 2010) and they also have a variety of meanings (Toluk-Uçar, 2009). Because of the difficulties experienced with fractions, teaching the concept of fractions in mathematics lessons is very important (Alacacı, 2009). Teachers must have a deep knowledge of fractions, but they must also have the ability to express these concepts effectively (Ball, 2000; Ma,1999).

Students should gain experience with fractions in real-world contexts that make sense to them (Cramer&Whitney, 2010). During these experiences, students are required to use representations that are compatible with the real-word context. There are three different types of models used in teaching the concept of fraction. These are "area models", "length models",

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and “set models”. The use of different types of models is very important in teaching fractions because different models offer different learning opportunities to students. In addition, the use of all kinds of models expands and deepens students' understanding of the concept of fractions (Van de Walle et al., 2019). Providing rich learning environments where different types of fraction models are used together is necessary to improve both students’ operational and conceptual understanding (Birgin & Gürbüz, 2009).

Problem posing is defined as reshaping a given problem or creating new problems from a given situation (Silver, 1994). Problem posing has positive effects on the reasoning, problem solving and creativity skills of the person who created the problem (Kar & Işık, 2014). However, problem posing skill is important in terms of approaching problem solving from another aspect and understanding the relationships in the problem (Altun, 2012). Problems reflect the mathematical understanding of their creator (Toluk-Uçar, 2009). So, it can be used by teachers as an important assessment tool to identify students' conceptual understanding, mistakes and misconceptions (Işık & Kar, 2015; Toluk-Uçar, 2009).

Several studies have been conducted to examine the problem posing activities of pre-service teachers (Akbaba-Dağ et al., 2019; Kilic, 2015, Işık & Kar, 2012; Akçay & Ardıç, 2020) or students (Özer et al., 2020; Kar & Işık, 2014; Martinez & Blanco, 2021; Işık & Kar, 2015) about fractions. In these studies, no restrictions were imposed on the problem posing tasks related to fractions that required the students to match the context of the problem with a given model.

In this study, the aim was to examine the problems that pre-service teachers (PSTs) pose for each operation of addition, subtraction, multiplication and division with fractions and for each fraction model (area, length and set). It is an important skill in terms of mathematics teaching that PSTs, who are the teachers of the future, can pose realistic and appropriate problems for their students' levels and teaching purposes. In order for PSTs to create a rich students' understanding of fractions, they should be able to create contexts that will direct their students to use different models. Therefore, in the education of mathematics teachers, it is important to reveal the mistakes made by PSTs while posing problems, their understanding and tendencies about fractions, and thus to try to eliminate the deficiencies in this subject. So, the following questions are investigated in this study:

1. What is the suitability of the problems posed by the PSTs with respect to a given operation, model and real life?
2. What is the situation with the error-free problems and erroneous problems posed by the PSTs?

Methods

Research Design

An explanatory research design model was followed for this study. This research design allows researchers to use both quantitative and qualitative methods in the analysis of data, helping researchers understand certain phenomena more efficiently (Fraenkel et al., 2011). Therefore, this research design is suitable for the purpose of this study, which conducts in-depth analysis on problems posed by pre-service teachers. Both qualitative and quantitative methods were used in the analysis and reporting of the data of this study.

Participants and The Data Collection Procedure

The participants of this study were 40 third-year pre-service teachers (9 males and 31 females) enrolled in the middle school mathematics program of a Turkish university. The PSTs were attending a course on mathematics education called "Special Teaching Methods II" in the spring semester of 2020. Due to the outbreak of the pandemic 5 weeks after the start of the spring semester, the course continued with distance education. Within the scope of the course, models used for fractions were introduced and examples of contextual problems for the four operations in fractions were given. Hence, the aim was to examine the problems that PSTs posed for different models for the given operations by following a purposeful sampling method.

The data for the study was collected online through a given assignment. In the assignment, which students had one week to complete, the following operations were given to the PSTs and they were asked to pose three problems for each operation: (a) $1\frac{3}{4}+\frac{2}{3}$, (b) $1\frac{3}{4}-\frac{2}{3}$, (c) $1\frac{3}{4}\times\frac{2}{3}$, (d) $1\frac{3}{4}:\frac{2}{3}$. The criteria for the problems to be posed was "the solutions of the problems should be the given operations", "one of the three problems to be posed should lead the problem solver to the area model, one to the length model, and the other to the set model". Thus, each PST was expected to pose 12 problems.

The Data Analysis

After the PSTs' assignments were collected, the problems were recorded in a 12-page Excel file, with each problem group on one page. In the analysis of the data, the problems were examined in terms of suitability for the given operation (the solution to the problem is the given operation or not), for the given model (the model to be used to visualize the problem is the given model or not) and the real life. The first two criteria were explicitly requested from the PSTs in the assignment. The third criterion was added as a result of realizing that some problems include situations that will not occur in real life during the analysis of the data. Because a teacher will need to set up problems that can direct students' thinking and solve them with students in the problem environment. Therefore, the problems had to be compatible with real life.

The problems were coded as "suitable", "partially suitable" and "unsuitable" based on these three criteria. In addition, content analysis was used to analyze the errors in the problems posed by the PSTs.

Contexts dealing with measuring areas, measuring weights, measuring liquids, sharing pizza or cake, etc. evaluated suitable the area model. Contexts dealing with length or distance measurement etc. were evaluated suitable the length model. For the set model, contexts dealing with situations such as packages, boxes, bags containing a certain number of objects were evaluated in accordance with the model.

Problems were coded as "partially appropriate" in cases such as setting up the problem in a way that could be interpreted differently, missing details in the problem (missing unit or some other detail, no emphasis on "identical whole", etc) for the criteria of suitability for the operation. The "partially suitable" code for the model suitability criteria mostly appeared in set model problems. This code was used for problems in which whole is a set with a certain number, but which includes natural numbers instead of given fractions, and problems with unit-related error (e.g. "$\frac{2}{3}$ of candy" instead of "$\frac{2}{3}$ packets of candy"). The "partially suitable" code for the suitability real life criterion was given to problems where realistic or not depends on some missing details, and problems where there are slightly illogical situations in their fiction. The coding was done again at different times and brought to the final form. Sample coding will be explained in the results section.

Results

The results obtained by examining the problems posed by PST in terms of their suitability for the operations, models and real life.

The frequencies of the problems posed by PST in terms of their suitability for the operation, to the models and real life are included in separate tables for addition, subtraction, multiplication and division (respectively Table 1, Table 2, Table 3, Table 4).

Table1

The frequencies of the problems posed for the addition in terms of their suitability for the addition, to the models and real life

		Suitable			Partially Suitable			Unsuitable		
		Area	length	set	Area	length	set	Area	length	set
For Addition	f	38	34	18	1	3	4	1	3	18
For Model	f	40	39	20	-	-	8	-	1	12
For Real life	f	16	26	25	9	2	3	15	12	12

The frequencies (Table 1) of the problems in terms of their suitability for the addition indicate that the PST performed close to each other in the area (95%) and the length model (85%) while their performance was low in the set model (45%). A similar situation exists at the model suitability frequencies. While the PSTs are very successful in the area (100%) and length (97.5%) model in problem writing suitable to the model, the success in the set model is only 50%.

The frequencies in the last row of the “suitable” column are the frequencies of real-life fit, regardless of operation and model fit. Especially in the set model, while the model fit frequencies is 50%, the accuracy frequency is more than 50%. In order to make a more accurate evaluation in terms of the model, "suitable" coded frequencies were determined for model and real life criteria by filtering them as " suitable" in the excel file. So, the new frequencies in the last row and under the "suitable" column of Table 1 are 16, 26, and 18, respectively. Therefore, the updated frequencies show that the success of the PSTs in terms of real-life suitability is moderate in the length model (65%) and lower in the other models (area:40%, set:45%).

For the first two criteria, the success rate in the area model was the highest, while the success rate for the last criterion was the lowest. In the area model, especially “not paying attention to the part-whole relationship” and “unit-related errors” decreased the rate of conformity to real life.

The frequencies in the first row and under the "suitable" column of Table 2 indicate that PSTs are more successful in the length model (82.5%) in posing appropriate subtraction problems. Success in area and set model is 65% and 35%, respectively. The frequencies in the second row of Table 2 show that PSTs' successfulness in posing appropriate subtraction problems suitable for the model is 100% length model, 92.5% area model, 32.5% set model. When the last row of Table 2 is updated for suitability for both the model and real life, frequencies change as 19, 22, 11, respectively. Thus, the success in the length model is 55%, in the area model is 47.5%, and in the set model is 27.5%, respectively.

Table 2

The frequencies of the problems posed for the subtraction in terms of their suitability for the subtraction, to the models and real life.

		Suitable			**Partially Suitable**			**Not Suitable**		
		Area	length	set	Area	length	set	Area	length	set
For Subtraction	F	26	33	14	3	3	3	11	4	22
For Model	F	37	40	13	1	-	10	2	-	16
For Real life	F	20	22	21	2	5	6	18	13	12

Note: A student did not pose any problem for the set model.

PSTs seem to be more successful in posing subtraction problems in accordance with the length model compared to other models. The error that reduces the frequency of operation suitability in subtraction problems established for the area model (it is seen in 11 problems) is that it is posed in accordance with the $1\frac{3}{4} - 1\frac{3}{4} * \frac{2}{3}$ operation instead of the $1\frac{3}{4} - \frac{2}{3}$ operation. This error was seen in only one problem for the length model and in 14 problems for the set model.

Table 3

The frequencies of the problems posed for the multiplication in terms of their suitability for the multiplication, to the models and real life.

	Suitable			**Partially Suitable**			**Not Suitable**		
	Area	length	set	Area	length	set	Area	length	set

For multiplication	f	27	31	23	3	3	10	9	5	6
For Model	f	39	36	15	-	2	10	-	1	14
For Real life	f	7	15	18	5	1	7	27	23	14

Note: A student did not pose any problem for all models.

The frequency (Table 3) of the problems posed for multiplication in terms of suitability for multiplication shows that the order of success according to the model is length model (77.5%), the area model (67.5%) and the set model (57.5%). The frequencies in the second row of Table 3 show that the PSTs performed close to each other in the area and length model (97.5% and 90%, respectively) in posing a model-appropriate multiplication problem, but their performance was very low (37.5%) in the set model. When the last row of Table 3 is updated for suitability both for the model and real life, frequencies change as 7, 14,10 respectively. In this case, the success in the length model is 35%, in the set model is 25%, and in the area model is 17.5%, respectively.

In the problems posed for the multiplication, the success in terms of real-life compatibility is lower than the addition and subtraction.

Table 4

The frequencies of the problems posed for the division in terms of their suitability for the division, to the models and real life.

		Suitable			**Partially Suitable**			**Not Suitable**		
		Area	length	set	Area	length	set	Area	length	set
For division	f	34	31	22	2	1	11	4	8	6
For Model	f	38	38	19	1	1	1	1	1	19
For Real life	f	22	24	18	5	2	11	13	14	10

Note: A student did not pose any problem for the set model.

The frequencies of the problems posed for division in terms of their suitability for division (Table 4) indicate that the PST performed close to each other in the area and length model (respectively 85% and 77.5%), while their performance in the set model was lower (55%) than the other model. Table 4 shows that PSTs have the same success in the area and length model 95% and in the set model 47.5% in posing division problem in accordance with the model. When the last row of Table 4 is updated for suitability both for the model and the real life, frequencies change to 20, 24,11 respectively. In this case, the success in the length model is 60%, in the area model is 55% and in the set model is 27.5%, respectively.

In the following, the frequencies in Table 1, Table 2, Table 3 and Table 4 will be compared according to the operations for each model.

When the first rows of the columns named "suitable" in Table 1, Table 2, Table 3, Table 4 are compared (suitability to the operation), it is noted that the frequencies in the length model vary between 31-34 (77.5% - 85%). The frequencies in the area model are distributed over a wider range (26-39; that is, 65-97.5%). The range of frequencies in the set model is 13-23 (32.5%-57.5%).

Comparison the second rows (suitability to the model) of the columns named suitable in Table 1, Table 2, Table 3, Table 4 show that the frequencies in the area model are in the range of 36-40 (90%-100%), in the length model were in the range of 33-40 (82.5%-100%) and in the set model were in the range of 13-20 (32.5%-50%). That is PSTs performed close to each other in the area and length model, but their performance was low in the set model.

When the third rows of the columns named suitability in Table 1, Table 2, Table 3, Table 4 (suitability to real life) are evaluated together with the criterion of suitability for the model, the frequencies change as 16, 26, 18 for addition; 19, 22, 11 for subtraction; 7, 14,10 for multiplication and 20, 24,11 for division. In this case, it is seen that the lowest frequencies are in the multiplication.

The distribution of the frequencies of the problems (problems that meet the first two criteria together), which are suitable for both the operation and the model, according to the models and operations are given in Table 5.

Table 5

The frequencies of the problems which are suitable for both the operation and the model.

	Area model	**Length model**	**Set model**
Addition	38	34	9
Subtraction	24	33	4
Multiplication	27	28	8
Division	32	31	9

Table 5 indicates that the frequencies of problems which are suitable for both the operation and the model posed for area model are in the range of 24-38 (60%-95%), posed for length model are in the range of 28-34 (70%-85%) and posed for set model are in the range of 4-9 (10%-0.22.5%). Among the problems posed for the set model, the frequencies of the problems that are suitable for both the operation and the model are quite low. This shows that PSTs have insufficient performance posing problems for the set model.

The frequency of the problems that meet all three criteria is given in Table 6. Table 6 shows that PSTs have the worst performance in the set model and have a better performance in the length model in posing problems suitable for all three criteria. But even in the length model, the percentage of problems that meet all criteria does not exceed 60%. It is also noteworthy that the frequencies in multiplication and subtraction operations are lower than in other operations.

Table 6

The frequencies of the problems which are suitable for all three criteria.

	Area model	**Length model**	**Set model**
Addition	15	24	9
Subtraction	13	18	2
Multiplication	5	12	7
Division	18	21	7

The results obtained by examination of error-free problems and erroneous problems posed by PSTs

In the analysis of the problems posed by the PSTs, many and a wide variety of errors were encountered (Table 8). However, in only one of the twelve problem groups (posed for addition problem with the length model), the amount of error-free problems (Table 7) is 50%. In most other problem groups, very few problems are error-free.

Table 7:

Distribution of error-free problems by operations

		Models		
Operations		**Area**	**Length**	**Set**
Addition	f	*7**	*20*	*6*
subtraction	f	*12**	*18**	*2*
multiplication	f	*6***	*12**	*5*

division	f	*7*	*9*	*4*

Note. (*) One /(**) two of these problems have a "partially suitable" code in one of the criteria of fit for the model or fit for the real life. The others have "suitable" code for all three criteria.

The frequencies of error-free problems for each model (Tablo 7) show that the frequencies in the area model were between 6-12 (15%-30%), in the length model were between 9-20 (22.5%-50%) and in the set model were between 2-6 (5%-15%). This shows that PSTs are more successful in posing problems for the length model than other models. The length model is followed by the area model and finally the set model.

It is noteworthy that the number of error-free problems is much less in the area model, which students encounter most and are most familiar with, than in the length model. When the addition problems for these two models were examined, it was seen that half of the problems in the area model were problems that needed attention to the part-whole relationship (the size expressed by the fractions). In the length model, however, it was noticed that only five problems required attention to the part-whole relationship. Giving one of the fractions as an integer required extra attention to the part-whole relationship. However, at this point, PSTs acted carelessly.

The fractions given in all the error-free addition problems and subtraction problems posed for the area model and the length model, were used to express the measurement result. This reduced the possibility of making mistakes. Because thus, it is possible to pose a problem by using fractions as natural numbers. In addition, it is not necessary to pay attention to the part-whole relationship. On the other hand, the set model-addition problems and subtraction problems which were error-free need to attention part-whole relationship. One of the error-free subtraction-problem is in the following:

"$1\frac{3}{4}$ of the boxes containing 12 candies are full. If $\frac{2}{3}$ of box candies are added to $1\frac{3}{4}$ boxes of candies, how many boxes of candies will there be?"

In error-free multiplication problems (all in the area model, all but two of the length model), the first fraction is used as a number expressing the measurement result, and the second fraction was used as a ratio (for example, $\frac{2}{3}$ of $1\frac{3}{4}$ m^2). Only two length model multiplication problems have been created for the velocity-time problem. In the error-free set model multiplication problems, it was asked how many boxes $\frac{2}{3}$ of $1\frac{3}{4}$ box objects were. For example:

"Barış has $1\frac{3}{4}$ packs of marbles. If $\frac{2}{3}$ of the package is white, how many packages of white marbles are there? (There are 12 marbles in 1 pack)"

In the error-free area model division problems, fractions were used as numbers expressing the measurement result. It is about dividing the object in the big bowl into the small bowl. All but three for the error-free length model division problems were posed as velocity-time problems. In the other three asked the problem that required determining how many times the larger length was the smaller one. In the set model, problems related to allocation were created. For example:

"$1\frac{3}{4}$ of the boxes containing 24 balls are full. If $1\frac{3}{4}$ boxes of balls are emptied into smaller box with capacity $\frac{2}{3}$ box of balls, how many boxes are filled?"

The frequencies of the errors encountered in the problems posed by the PSTs and the distribution of these frequencies according to the operations and models are given in Table 7. Table 8 contains the examples of problems with errors in Table 7.

The distribution of errors in terms of operations shows that the number of errors in the division is the highest and in the addition is the least (Tablo 8). Subtraction is second after division and multiplication is third. When the numbers of errors for models were ordered from most to least, it was seen that the order of model for each operation was set, area and length.

Table 8 indicates that the most common mistake made by PSTs is the "unit-related errors". This code include some errors in which not to use standard units when it is necessary, to use standard units when it is unnecessary (using units in expressions that indicate ratios), the use of units in a way that does not have a real-life counterpart ($1\frac{3}{4}$ marble, $\frac{2}{3}$ m of the road traveled by car, $\frac{2}{3}$ m^2 of cultivated field size, etc.). It also includes incorrect selection of the whole of the fraction or not specifying the whole. For example in problem (a) in Table 9 unit should be "box of notebook" instead of "notebook". Table 8 shows that the unit related error code is present in all operations and all models.

Illogical fiction is the second most common mistake. In Table 9 Problem (a), (c), (f) and (j) have this code. Problem (a) has "illogical fiction" code as the number of unlined notebooks cannot be calculated. Problem (c) The fraction $\frac{2}{3}$ is considered as a natural number. In Problem (f), it is valid for a fully filled box expressed as $\frac{2}{3}$ of each box, but not valid for a box that is $\frac{3}{4}$ full. In this case, it does not correspond to the product of two fractions.

Table 8

The frequencies of errors in the problems posed for all the operations according to each model.

	Addition			**Subtraction**			**Multiplication**			**Division**			**Total**
	A	**L**	**S**	**A**	**L**	**S**	**A**	**L**	**S**	**A**	**L**	**S**	
Unit-related errors	15	10	10	9	15	10	13	8	13	14	11	17	**145**
Illogical fiction	1	6	6	10	6	6	9	8	7	3	8	6	**76**
Wrong model	-	1	10	2	-	16	-	2	15	1	1	15	**63**
Wrongly asked question	2	3	22	-	2	9	3	-	12	3	5	-	**61**
Assigning natural number meaning to fractions	5	-	4	3	-	11	8	8	10	2	1	4	**56**
Not paying attention to the part-whole relationship	6	3	10	4	-	4	10	8	4	4	-	-	**53**
Require the result to be rounded to a natural number	-	-	-	-	-	-	-	-	-	21	14	15	**50**
Treat a mixed fraction as an integer and a fraction separately	3	2	7	4	2	6	2	1	4	3	1	4	**39**
Suitable for a different operation than the given one	-	-	-	11	1	14	2	1	-	2	4	1	**36**
No emphasis on "identical whole"	8	-	2	4	1	2	1	-	-	1	-	-	**19**
Not specifying which whole the part belongs to	5	1	3	2	-	1	1	-	-	-	-	4	**17**
Missing detail	-	3	3	1	2	-	-	-	1	-	1	1	**12**
Posing the problem with natural numbers	-	-	3	-	-	3	-	-	2	1	-	2	**11**
Asking given value	-	-	-	-	1	-	6	1	-	-	1	1	**10**
Total for models	**45**	**29**	**80**	**50**	**30**	**82**	**55**	**37**	**68**	**55**	**47**	**70**	
Total for operations		**154**			**162**			**158**			**172**		

A: area, L: length, S: set

The “wrongly asked question” code was mostly occurred in the problems for set model (the operations other than division) problems. Interestingly, this error was not encountered in the division problems for the set model. On the other hand, the "Require the result to be rounded to a natural number" code, which does not affect the suitability for operation as much as the "wrongly asked question" code, has only been seen in division problems. In Problem (a) in Table 9 has this code because the problem asks for the number of notebooks. This can be calculated without the need for fractions. A similar situation exists in Problem (g) in Table 9.

The “assigning natural number meaning to fractions” code was mostly seen in the multiplication problems. For example, in Problem (c) in Table 9 the fraction $\frac{2}{3}$ was used as a whole number.

The "not paying attention to the part-whole relationship" code has not emerged in problems that use fractions to express the measurement result, since there is no need to pay attention to this relationship in these problems. The fact that one of the fractions given in the homework was an "mixed fraction" required special attention to the part-whole relationship. This code is mostly seen in multiplication problem (for example, Problem (f) in Table 9).

The "not specifying which whole the part belongs to" code most commonly seen in addition problems. In Problem (k), for the fraction $\frac{2}{3}$, it is unclear whether it is a whole pastry or $1\frac{3}{4}$ pastry, so it is coded with this code.

The code "Requiring the result to be rounded to a natural number" only appeared in division problems. The result of the division operation given in the assignment was not an whole number. Among the problems posed by the PSTs, there were problems that required rounding the result of the division operation to the natural number (for example, Problem (g) in Table (9)) .

The "Suitable for a different operation than the given one" code is most commonly seen in subtraction problems. 25 subtraction problems have this code (Table 7). Interestingly all these problems’ solution is $1\frac{3}{4}$-$1\frac{3}{4}$x$\frac{2}{3}$ instead of $1\frac{3}{4}$- $\frac{2}{3}$ (For example Table 8- Problem (b)).

The "No emphasis on identical whole" code has mostly been seen in problems posed for addition and subtraction. For example Problem (i) has this code. Although it is emphasized that the number of slices of the cakes mentioned in the problem is the same, there is no information about the size of the slices.

The other codes in Table 8 and examples of them (Table 9) in the following: "Treat a mixed fraction as an integer and a fraction separately" in Problem (i), "missing detail" in Problem (j), "posing the problem with natural numbers" in Problem (i) , "asking given value" in Problem (c), (d).

Table 9:
Example problems for addition and their codes

Problem	**Suitability**			**Errors**
	O	M	R	
(a) A stationer bought notebooks in boxes containing 24 notebooks each. $\mathbf{1\frac{3}{4}}$ notebooks are square, $\mathbf{\frac{2}{3}}$ notebook are lined, and the remaining notebooks are unlined. What is the sum of the lined and unlined notebooks? (Addition, set)	U	PS	U	“wrongly asked question”, “unit-related error”, “illogical fiction”
(b) If we pour $\mathbf{\frac{2}{3}}$ of the $\mathbf{1\frac{3}{4}}$ liters of olive oil we have on a salad, how many liters of olive oil will remain? (Subtraction, area)	U	S	U	“unit-related error” “suitable for a different operation than the given one”
(c) There are $\mathbf{\frac{2}{3}}$ of equal sized bottles filled with olive oil with $\mathbf{1\frac{3}{4}}$ liter capacity. How many of these bottles of olive oil are there? (Multiplication, area)	U	S	U	“illogical fiction” “Assigning natural number meaning to fractions” “asking given value”
(d) A gym is divided into sections for men and women. $\mathbf{\frac{2}{3}}$ m^2 of $\mathbf{1\frac{3}{4}}$ m^2 of this gym is reserved for women. According to this, how many square meters is the area reserved for women? (Multiplication, area)	U	S	U	“unit-related error” “asking given value”
(e) If $\mathbf{\frac{2}{3}}$ of $\mathbf{1\frac{3}{4}}$ of a road is traveled, how many of the roads have been traveled? (Multiplication, length)	S	S	U	Not paying attention to the part-whole relationship

(f) There are 12 eggs in a box. I have $1\frac{3}{4}$ boxes. $\frac{2}{3}$ of the eggs in each box are brown. How many of the eggs are brown? (multiply, set)	PS	S	S	"Illogical fiction" "wrongly asked question"
(g) Eray fills $1\frac{3}{4}$ liter water into $\frac{2}{3}$ liter bottles. How many bottles are needed? (division, area)	S	S	S	"Require the result to be rounded to a natural number"
(h) How many liters of orange juice are there in total in $\frac{2}{3}$ containers, each containing $1\frac{3}{4}$ liters of orange juice? (division, area)	U	S	U	"Suitable for a different operation than the given one" "Assigning natural number meaning to fractions"
(i) Asiye ate a whole 12-slice cake. Nesrin ate 9 slices of a 12-slice cake, and Sena ate 8 slices of a 12-slice cake. What is the ratio of the slices that Asiye and Nesrin ate to the slices that Sena ate? (Division, area)	U	S	S	"Treat a mixed fraction as an integer and a fraction separately" "No emphasis on identical whole" "Posing the problem with natural numbers"
(j) Ayla will make $\frac{2}{3}$ kg jams from $1\frac{3}{4}$ kg strawberries. How many jars are filled with jam? (Division, length)	U	U	U	"Missing detail" "illogical fiction" "wrong model"
(k) Meltem reserves 1+3/4 pastries to eat. However, she can only eat 2/3 of it. How many pastries does Meltem have left?	PS	S	S	"Not specifying which whole the part belongs to"

O: operation, M: model, R: reality, S: suitable, PS: partially suitable, U: unsuitable

Discussion and Conclusions

This study examined the problems posed by 40 PSTs in terms of suitability for given operation, given model and real life. Errors in the posed problems were also examined. There are few studies that deal with problem-posing studies for fractions operation. It was noteworthy that none of these studies dealt with multiplication and none of them had a model-oriented restriction.

Regarding the first research question, the findings showed that while the frequencies of the problems that meet the criteria of suitability for operation and for the model, separately or both, are close to each other in the area and length model, the frequencies are quite low in the set model. The frequency of the problems that meet all three criteria shows that PSTs have the worst performance in the set model and have a better performance in the length model in posing problems suitable for all three criteria. But even in the length model, the percentage of problems that meet all criteria does not exceed 60%. This percentage is lower than expected.

The findings obtained as a result of the examination of error-free problems show that few problems are error-free and that the number of error-free problems is the highest in the length model and the least in the set model. On the other hand in error-free problems posed for addition and subtraction (except set model), fractions are mostly used to indicate the measurement result. This use of fractions may have reduced the possibility of error, as it allowed it to be used as a natural number.

According to the distribution of the error numbers posed by the PSTs according to the operations, it was seen that the PSTs made the most mistakes in the division operation. The division operation is followed by subtraction, multiplication and addition, respectively. The distribution of the error numbers according to the models shows that the PSTs had the most difficulty in the set model, followed by the area and length model, respectively. In addition, this order is the same in every operation.

The analysis of the errors in the problems posed by PSTs showed that the most common errors were “unit-related errors”, “illogical fiction”, “wrong model” “incorrectly asked the question”, “assigning natural number meaning to fractions”, and “not paying attention to the part-whole relationship”. Although different errors emerged in this study, the most frequently encountered errors in Kar & Işık (2014)'s study are similar. Kar&Işık (2014) examined 7th-grade students' problem-posing studies on subtraction and reported that the most common errors were unit confusion, inability to establish a part-whole relationship, inability to reflect the operation to the question root, and attributing a natural number meaning to the result of the operation. Akbaba-Dağ et al. (2019), who examined 5th grade students' problem posing studies on addition and subtraction in fractions, reported that the most common errors were attributing the meaning of natural numbers to fractions and not establishing a part-whole relationship. In a similar study conducted with pre-service teachers and division (Işık & Kar, 2012), there are common errors such as unit confusion, not establishing a part-whole relationship, and attributing natural number meaning to fractions. In addition, the error of "using multiplication instead of division", which occurs in the division process, did not occur in this study.

Implications for Teaching and Suggestions for Future Studies

In this study, it was observed that PSTs were quite careless in using units. In particular, uses of standard units that have no equivalent in daily life will negatively affect future students' experiences with standard units. For this reason, awareness should be created for PSTs to be more careful in using standard units.

It should also be kept in mind that PSTs may have been careless or sloppy in constructing problems, but they may also have insufficient knowledge about fractions and all these errors may be due to their inadequate understanding of fractions.

The source of PSTs` errors in problem posing may be that they consider some details unimportant or they are not aware of their importance. On the other hand, the source of their deficiencies or errors may also be related to their language skills. It is recommended that the source of errors should be identified and necessary measures should be taken.

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