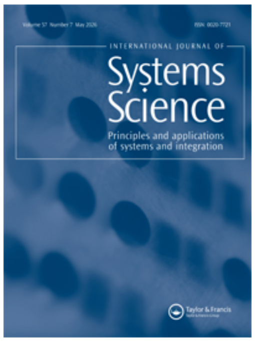

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Comparison models on soft, fuzzy, and intuitionistic fuzzy sets

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ABSTRACT

The main purpose of this study is to demonstrate the advantages and practicality of fuzzy set and intuitionistic fuzzy set operations, especially in comparison problems, and the parametric and ease of operation advantages of soft sets, which can be used mutually and transferred to the computer environment. This study is supported by many comparative analysis, both by solving our own problem with the methods presented in other studies and by solving the problem given in other studies with our own method. Inverse positive soft set (IPSS), inverse negative soft set (INSS), and some new useful notations are introduced. With the help of these concepts, a similarity measure was defined in which not only the alternatives but also the parameters were used in soft sets. Again, with the help of concepts such as IPSS and INSS, fuzzy and intuitionistic fuzzy sets generated by soft sets were obtained. Thus, the defined similarity measure was compared not only with the similarity measures defined on soft sets but also with the similarity measures on fuzzy and intuitionistic fuzzy sets. Finally, a similarity measure application is given for the comparison of products.

KEYWORDS
Soft set; fuzzy set; intuitionistic fuzzy set; similarity measure; distance measure; comparison problems

1. Introduction

In real life, we frequently encounter problems involving uncertainty. These uncertainties are often caused by the unpredictability of the parameters affecting the situation. Therefore, in order to cope with uncertainty in complex problems, classical mathematics have been insufficient, and new theories have been developed. The concept of fuzzy set was first proposed by Zadeh (1965), which assigns a membership degree to an object or element of a set in the unit interval $[0, 1]$, for dealing with uncertain events. Membership degree 0 indicates no affiliation, and membership degree 1 indicates full affiliation. Other values in $[0, 1]$ are used to indicate partial affiliation, depending on their degree of belonging to the set. Fuzzy set theory has been successfully applied in many fields. Trabia et al. (1999) applied fuzzy logic to a signal controller for an isolated intersection. Xie and Beni (1991) studied validity measures for fuzzy clustering. Zhan and Xu (2018) introduced two types of covering-based multigranulation rough fuzzy set models, and using this model, they presented an approach to the multiple criteria group decision-making problem. Also, Zhang and Zhan (2019) and Zhang and Zhan (2018) proposed and studied the concepts of fuzzy soft β-coverings, fuzzy soft β-neighbourhoods, fuzzy soft complement β-neighbourhoods, and Pedrycz (1990) studied pattern recognition with fuzzy sets. After Zadeh (1965), fuzzy sets were extended to other fuzzy concepts such as complex fuzzy sets, bipolar complex fuzzy sets, picture fuzzy sets, and Pythagorean fuzzy sets. Ashraf et al. (2020) studied fuzzy decision support modelling based on Pythagorean fuzzy sets. Mahmood and Ur Rehman, in their paper (Mahmood & Ur Rehman, 2021), introduced generalised similarity measures on bipolar complex fuzzy sets. In Atagün and Kamacı (2023a), Atagün and Kamacı introduced a new fuzzy concept called strait fuzzy set, in which membership degrees are represented as intervals that are partitions of $[0, 1]$ instead of exact values in $[0, 1]$.

In 1999, Molodtsov (1999) proposed a completely new approach called soft set theory for modelling vagueness and uncertainty. Since this approach is free

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from the problem of setting the membership function, this theory can be easily applied to many different fields, including the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory, and measurement theory. Some of these applications have already been indicated using soft set theory in Molodtsov (1999). At present, works on soft set theory are progressing rapidly in both theoretical and practical studies. In Aktaş and Çağman (2007) compared soft sets to fuzzy sets and rough sets (Pawlak, 1982), gave some basic concepts of soft set theory, and defined the concept of soft group. In Maji et al. (2002), soft sets were applied to a decision-making problem. Maji et al. (2003) defined new notations on soft sets. Ali et al. (2009) introduced new operations on soft sets such as restricted intersection, restricted union, and extended intersection. After Ali et al. (2009), other mathematicians introduced new operations on soft sets (Pei & Miao, 2005; Sezgin & Atagün, 2011). In Atagün and Aygün (2016) introduced two new operations on soft sets, called inverse production and characteristic production.

Since the paper (Aktaş & Çağman, 2007), many authors have studied the soft algebraic structures, namely Acar et al. soft rings Acar et al. (2010), Feng et al. (2008) soft semirings, Jun (2008) soft BCK/BCI-algebras, Jun and Park (2008) ideal theory of soft BCK/BCI-algebras, Jun et al. (2009) soft *p*-ideals of soft BCI-algebras, Kazancı et al. (2010) soft BCH-algebras, Sezgin et al. (2011) soft near-rings and idealistic soft near-rings, Sezgin Sezer et al. (2015) soft intersection semigroups. Soft matrix theory was introduced by Çağman and Enginoğlu (2010). They defined soft matrices and some of their operations, then constructed a soft max-min decision-making problem that contained uncertainties. The theory of soft sets and soft matrices was successfully applied to decision-making problems. Atagün (2018) studied reduced soft matrices and applied this concept to decision-making. Kamacı et al. (2018) studied cardinality inverse soft matrix theory and applied in multi criteria decision making (MCDM). In Petchimuthu and Garg (2020) generalised the products of two fuzzy soft matrices and showed that three or more fuzzy soft matrices with the different types can be multiplied. Atagün and Kamacı (2023b) introduced two new concepts, namely strait soft set and strait rough set, between the structures of rough sets and soft sets.

Atanasov (1986, 1989) introduced the concept of intuitionistic fuzzy set, which is a generalisation of the concept of fuzzy set. Intuitionistic fuzzy sets have gained a lot of attention since their introduction. These intuitionistic fuzzy sets reflect real-world problems more realistically and accurately. It has been widely studied and applied in various areas, such as logic programming (Atanasov & Gargov, 1990), decision making (Chen & Tan, 1994; Szmidt, 2000), pattern recognition (D. Li & Cheng, 2002; Y. Li et al., 2007; Liang & Shi, 2003) and medical diagnosis (De et al., 2001; Szmidt & Kacprzyk, 2005). In addition, different similarity measures have been defined and applied to intuitionistic fuzzy sets to be used in comparison problems (Chen, 1995; Y. Li et al., 2002; Mitchell, 2003).

Similarity measure is one of the important tools used in many application areas, such as ranking degrees of alternatives, pattern recognition, multiattribute group decision-making (MADM) problems, machine learning, market prediction, etc. Liu et al. (2019), Szmidt and Kacprzyk (2004), Ullah et al. (2018) and Wei et al. (2019). Also, similarity measures are defined in many different ways in modelling methods of uncertainty, such as soft sets, fuzzy sets, rough sets, and their hybrid structures. Similarity measures appear in notable studies in fuzzy set theory. A similarity measure between two fuzzy sets was proposed by P. Z. Wang (1982). A cosine similarity measure between fuzzy sets was introduced and applied to information retrieval of words by Salton and McGrill (1983). Geometric distance and Hausdorff metrics are used to obtain similarity measures of fuzzy sets by Zwick et al. (1987). Some similarity measures were proposed for fuzzy sets using the geometric model, the set theoretic approach, matching functions, and the operations union, intersection, difference, and sum in Chen et al. (1995) and Pappis and Karacapilidis (1993). Two similarity measures between fuzzy sets and between the elements of sets were given in W. J. Wang (1997). An axiomatic definition of similarity measure for fuzzy sets was provided in J. Fan and Xie (1999) and Liu (1992).

Similarity measures between soft sets have been defined in many different ways. Some are distance-based, weighted, matching function-based, and set-theoretic-based (using operations of soft sets and soft matrices). The similarity measure between two soft sets, which measure the similarity of both the parameter set and approximate value set, was defined by Majumdar and Samanta (2008). Then a new similarity measure improving the similarity measure was introduced in Majumdar and Samanta (2008) by Kharal (2010). But a counterexample illustrating that the similarity measure defined in Kharal (2010) contains an error and then redefined a similarity measure away from this error was given by Yang (2013). Some similarity measures for the soft matrices introduced in Gong et al. (2010) and Kamacı (2019). Generalizing the operation inverse product given by Atagün and Aygün (2016), a similarity measure proposed by Aygün and Kamacı (2019). Then, the axioms for uncertainty measurements among the soft sets were introduced, and then, novel categories of the measures of similarity and distance between two soft sets with comparison and performance analysis were presented by Aygün and Kamacı (2021).

Motivation: As is known, the results obtained when working with soft sets with parameters do not include a numerical value. On the other hand, although the alternatives take values in the range of [0,1] in fuzzy sets, parameters cannot be included. One of the main motivations of this study is to gather the advantages of these different structures under a single roof to produce effective solutions to problems with many uncertainties, especially comparison problems. In this study, the advantages offered by soft sets, especially the selection of parameters and corresponding alternatives according to the problem with the practicality of fuzzy and intuitionistic fuzzy sets in solving problems that require a large number of data entries, are used. Another motivation of this study is to introduce a similarity measure on soft sets in which parameters take precedence, not just through alternatives, unlike in general. In this way, it is aimed at solving the comparison problems with a broader approach.

Contributions: In this study, firstly, a new similarity measure for soft sets is proposed by using the number of parameters corresponding to the alternatives, and an application is presented that compares the products of a company with the products of competing companies by using it. Then, using the notations related to the parameter numbers corresponding to the alternatives of a soft set, three types of fuzzy sets generated by the given soft set, which we call characteristic, parametric, and general fuzzy sets, are defined. It is proven that the generated parametric fuzzy set of the relative complement of a soft set is the complement of the generated parametric fuzzy set of this soft set. Then, it is proved that the similarity measure for two soft sets and the similarity measure for parametric fuzzy sets generated by these soft sets using the (normalised) Hamming (Euclidean) distance give the same result for the comparison problems. It is obtained that the similarity measure of parametric fuzzy sets generated by two soft sets using the normalised Euclidean distance gives a result closer to the similarity measure for the two soft sets than the similarity measures obtained using Hamming, normalised, and Euclidean distances. Also, using the notations used in generating fuzzy sets, three types of intuitionistic fuzzy sets are defined, which we call characteristic, parametric, and general intuitionistic fuzzy sets. It is proven that for generated parametric and general intuitionistic fuzzy sets, the relative complement of a soft set is the complement of the generated parametric and general intuitionistic fuzzy sets of this soft set. Finally, some similarity measures of intuitionistic fuzzy sets are compared with the similarity measure on soft set defined in this paper. All the results obtained are supported by examples.

This study is organised as follows. In the second section of this study, some basic concepts about soft sets and fuzzy sets are reminded. In the third section, concepts such as inverse positive soft set and inverse negative soft set are introduced, and with the help of these concepts, a new similarity measure on soft sets is presented. In the fourth section, a comparison algorithm is given with the help of similarity measure. In the fifth section, fuzzy sets generated by soft sets are introduced through concepts such as inverse positive soft set and with the help of these new fuzzy sets, in the sixth part, some similarity measures on fuzzy sets are compared with the soft similarity measure. Finally, in the seventh part, intuitionistic fuzzy sets generated by soft sets are introduced.

2. Preliminaries

Definition 2.1 (Molodtsov, 1999): Let U be an initial universe set, E be a set of parameters, $P(U)$ be the power set of U and $A \subseteq E$. A soft set (F, A) or simply F_A on the universe U is defined by the ordered pairs

$$(F, A) = \{(x, F(x)) | x \in A, F(x) \in P(U)\},$$

where $F : A \to P(U)$.

In other words, a soft set over U is a parameterised family of subsets of the universe U.

Definition 2.2 (Pei & Miao, 2005): Let (F, A) and (G, B) be soft sets over U.

(a) If $A \subseteq B$ and $F(x) \subseteq G(x)$ for all $x \in A$, then, (F, A) is a soft subset of (G, B), denoted by $(F, A)\widetilde{\subseteq}(G, B)$.
(b) If $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$, then, (F, A) and (G, B) is said to be soft equal and denoted by $(F, A) = (G, B)$.

Definition 2.3 (Ali et al., 2009): The relative complement of a soft set (F, A) is denoted by $(F, A)^r$ and is defined by $(F, A)^r = (F^r, A)$, where $F^r : A \to P(U)$ is a mapping given by $F^r(x) = U - F(x)$, for all $x \in A$.

Definition 2.4 (Feng et al., 2008): Let (F, A) be soft set over U. Then, the set

$$supp(F, A) = \{x \in A | F(x) \neq \emptyset\}$$

is called the *support* of the soft set (F, A). The *null soft set* is a soft set with an empty support and we denote it by $\emptyset_A$. A soft set (F, A) is called *non-null* if $supp(F, A) \neq \emptyset$. Also, a soft set (F, A) is called a *universal soft set* if $F(e) = U$ for all $e \in A$ and it is denoted by U_A.

Definition 2.5 (Gong et al., 2010): Let (F, A) be soft set over U such that A is a nonempty parameter set. We say that (F, A) is a bijective soft set if

(a) $\bigcup_{e\in A} F(e) = U$ (i.e. (F, A) is a full soft set),
(b) $F(e_i) \cap F(e_j) = \emptyset$ for all $e_i, e_j \in A$ such that $e_i \neq e_j$.

Example 2.6: Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be the universe, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ and $A = \{e_1, e_2, e_4, e_6, e_7\} \subseteq E$. Suppose that corresponding soft set of A is

$$F_A = \{(e_1, \{u_1, u_2\}), (e_2, \{u_3, u_4, u_5\}), (e_4, \emptyset), (e_6, \{u_1, u_2, u_3, u_5\}), (e_7, U)\}.$$

Then, the complement set of F_A is

$$F^r_A = \{(e_1, U - \{u_1, u_2\}), (e_2, U - \{u_3, u_4, u_5\}), (e_4, U - \emptyset), (e_6, U - \{u_1, u_2, u_3, u_5\}), (e_7, U - U)\}.$$

F_A	$\{(e_1, \{u_1, u_2\}), (e_2, \{u_3, u_4, u_5\}), (e_4, \emptyset), (e_6, \{u_1, u_2, u_3, u_5\}), (e_7, U)\}$
F^r_A	$\{(e_1, \{u_3, u_4, u_5\}), (e_2, \{u_1, u_2\}), (e_4, U), (e_6, \{u_4\}), (e_7, \emptyset)\}$
$supp(F, A)$	$\{e_1, e_2, e_6, e_7\}$

Also, the elements of the support set of F_A are $x \in A$ such that $F(x) \neq \emptyset$ then, the following table shows the complement and support set of the soft set F_A.

Definition 2.7 (Zadeh, 1965): Let $A \neq \emptyset$ be a universal set, then, a fuzzy set μ on A is the set of pairs $\mu = \{(x, \mu(x)) | x \in A\}$, where $\mu : A \to [0, 1]$ is the membership function of the fuzzy set.

Definition 2.8 (Zadeh, 1965): Let $\mu, \nu \in \widetilde{F}(A)$. If $\mu(x) \leq \nu(x)$ for all $x \in A$, then μ is said to be contained in ν, and denoted by $\mu \subseteq \nu$.

The set of all fuzzy sets on A will be denoted by $\widetilde{F}(A)$. Some of the fuzzy set operations are as follows:

Definition 2.9 (Zadeh, 1965): Let $\mu, \nu \in \widetilde{F}(A)$. Then,

(a) The intersection of μ and ν, is defined as $(\mu \cap \nu)(x) = \mu(x) \wedge \nu(x) = \min\{\mu(x), \nu(x)\}$, for all $x \in A$.
(b) The union of μ and ν, is defined as $(\mu \cup \nu)(x) = \mu(x) \vee \nu(x) = \max\{\mu(x), \nu(x)\}$, for all $x \in A$.
(c) The complement of μ, is defined as $\mu^c(x) = 1 - \mu(x)$, for all $x \in A$.

Definition 2.10 (Pedrycz & Gomide, 2007): Let $\mu \in \widetilde{F}(A)$. Given a number $t \in [0, 1]$, a t-level set (or t-cut) of a fuzzy set μ is a crisp subset of A defined by $\mu_t = \{x \in A | \mu(x) \geq t\}$.

Definition 2.11 (Grzegorzewski, 2004): Let $U = \{x_1, x_2, \ldots, x_n\}$ be an n-element initial universe set and let μ_A and μ_B be membership functions of two fuzzy subsets A and B of U, respectively. Then, well-known distance measures are,

(a) The Hamming distance:
$d(A, B) = \sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|$.
(b) The Normalised Hamming distance:
$l(A, B) = \frac{1}{n} \sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|$.
(c) The Euclidean distance:

$e(A,B) = \sqrt{\sum_{i=1}^{n}(\mu_A(x_i) - \mu_B(x_i))^2}$.

(d) The Normalised Euclidean distance:
$q(A,B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\mu_A(x_i) - \mu_B(x_i))^2}$.

Definition 2.12 (Koczy & Domonkos, 2000): Let A and B be two fuzzy subsets of U. If $DM(A,B)$ is the distance measure of A and B, then $SM(A,B) = \frac{1}{1+DM(A,B)}$ is a similarity measure of them.

Definition 2.13 (Atanasov, 1986): Let $A \neq \emptyset$ be a universal set. Then, an intuitionistic fuzzy set (μ^T, μ^F) on A is the set of ordered pairs $(\mu^T, \mu^F) = \{(x, \mu^T(x), \mu^F(x)) | x \in A\}$, where $0 \leq \mu^T(x) + \mu^F(x) \leq 1$. $\mu^T : A \to [0,1]$ is called membership function and $\mu^F : A \to [0,1]$ is called nonmembership function of the intuitionistic fuzzy set.

Definition 2.14 (Atanasov, 1986): Let $A \neq \emptyset$ be a universal set and (μ^T, μ^F) be an intuitionistic fuzzy set on A. Then, complement of (μ^T, μ^F) is defined as (ν^T, ν^F) where $\mu^T = \nu^F$ and $\mu^F = \nu^T$.

3. Inverse positive soft sets and a new similarity measure

Definition 3.1: Let (F, A) be a soft set over U such that $A \neq \emptyset$. Then,

(a) $P_F : U \to F^{-1}(P(U)) \subseteq A$ where $P_F(u_i) = \{e_j | u_i \in F(e_j)\}$ is called an inverse positive soft set (IPSS) of U corresponding to (F, A).
(b) $N_F : U \to F^{-1}(P(U)) \subseteq A$ where $N_F(u_i) = \{e_j | u_i \notin F(e_j)\}$ is called an inverse negative soft set (INSS) of U corresponding to (F, A).
(c) For each $u_i \in U$, $Y_F(u_i) = F(e_k)$ such that $F(e_k)$ is the set with the highest cardinality that contains u_i. If there are two or more sets that satisfy this condition, only one of them is taken.
(d) For each $u_i \in U$, $Z_F(u_i) = F(e_k)$ such that $F(e_k)$ is the set with the highest cardinality that does not contains u_i. If there are two or more sets that satisfy this condition, only one of them is taken.
(e) $I_F : A \to F(A)$, where $I_F(e_i) = \{u_j \in U | u_j \in F(e_i)\}$.
(f) $U|_{F_A} = \{u_i \in U | u_i \in F(e_j), e_j \in A\}$ is called a useful alternative set (UAS) of (F, A).

From now on, let $U = \{x_1, x_2, \ldots, x_n\}$ be an n-element initial universe set, $E = \{e_1, e_2, \ldots, e_m\}$ be an m-element set of parameters and $A \subseteq E$.

The set of all soft sets over U is denoted by $\widetilde{SU}$.

If $(F, A) \in \widetilde{SU}$, then we have the following notations:

- P_{Fx}: the number of parameters e such that $x \in F(e)$, in other words $P_{Fx} = |P_F(x)|$.
- N_{Fx}: the number of parameters e such that $x \notin F(e)$, in other words $N_{Fx} = |N_F(x)|$.
- Y_{Fx}: biggest number $|F(e)|$ such that $x \in F(e)$, in other words $Y_{Fx} = |Y_F(x)|$.
- Z_{Fx}: biggest number $|F(e)|$ such that $x \notin F(e)$, in other words $Z_{Fx} = |Z_F(x)|$.
- I_{Fe}: the number of alternatives x such that $x \in F(e)$, where $e \in A$, in other words $I_{Fe} = |I_F(e)|$
- $\overline{I_{Fe}}$: the number of alternatives x such that $x \notin F(e)$, where $e \in A$, in other words $\overline{I_{Fe}} = |U - I_F(e)|$
- $U_{(F,A)}$: the number of different elements $x \in U$ such that $x \in F(e)$ for all $e \in A$, that is, $U_{(F,A)} = |U|_{F_A}|$.
- The set $supp(F, A)$ is denoted by $S_F A$ for the sake of brevity.

The following example is given to make the above notations and expressions in the Definition 3.1 more understandable.

Example 3.2: Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ be the universe, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ and $A = \{e_1, e_2, e_4, e_5, e_6, e_7\} \subseteq E$. Suppose that corresponding soft set of A is $F_A = \{(e_1, \{u_1, u_5, u_6\}), (e_2, \{u_2, u_7\}), (e_4, \{u_3, u_4, u_6, u_7\}), (e_5, \{u_2, u_4\}), (e_6, \{u_1, u_3, u_7\}), (e_7, \{u_3, u_5, u_7\})\}$. In this case, the sets inverse positive soft set (IPSS) and inverse negative soft set (INSS) of F_A are as follows.

On the other hand, $Y_F(u_i)$ and $Z_F(u_i)$ are given in the following table.

Here, according to the definition of $Y_F(u_i)$, $Y_F(u_1) = \{u_1, u_3, u_7\}$ could have been used instead of $Y_F(u_1) = \{u_1, u_5, u_6\}$. Also, the same situation applies to $Z_F(u_i)$.

Lemma 3.3: *Let $F_A, G_B \in \widetilde{SU}$. Then, the following statements hold,*

(a) $0 \leq P_{Fx} \leq |S_F A|$ *for all* $x \in U$.
(b) $0 \leq Y_{Fx} \leq U_{(F,A)}$ *for all* $x \in U$.
(c) $P_{Fx} = 0$ *if and only if* $Y_{Fx} = 0$ *for all* $x \in U$.

i	1	2	3	4	5	6	7
$P_F(u_i)$	$\{e_1, e_6\}$	$\{e_2, e_5\}$	$\{e_4, e_6, e_7\}$	$\{e_4, e_5\}$	$\{e_1, e_7\}$	$\{e_1, e_4\}$	$\{e_2, e_4, e_6, e_7\}$
$N_F(u_i)$	$\{e_2, e_4, e_5, e_7\}$	$\{e_1, e_4, e_6, e_7\}$	$\{e_1, e_2, e_5\}$	$\{e_1, e_2, e_6, e_7\}$	$\{e_2, e_4, e_5, e_6\}$	$\{e_2, e_5, e_6, e_7\}$	$\{e_1, e_5\}$

i	1	2	3	4	5	6	7
$Y_F(u_i)$	$\{u_1, u_5, u_6\}$	$\{u_2, u_4\}$	$\{u_3, u_4, u_6, u_7\}$	$\{u_3, u_4, u_6, u_7\}$	$\{u_3, u_5, u_7\}$	$\{u_3, u_4, u_6, u_7\}$	$\{u_3, u_4, u_6, u_7\}$
$Z_F(u_i)$	$\{u_3, u_4, u_6, u_7\}$	$\{u_3, u_4, u_6, u_7\}$	$\{u_1, u_5, u_6\}$	$\{u_3, u_5, u_7\}$	$\{u_3, u_4, u_6, u_7\}$	$\{u_1, u_3, u_7\}$	$\{u_1, u_5, u_6\}$

(d) *If* $(F, A)\widetilde{\subseteq}(G, B)$, *then* $P_{Fx} \leq P_{Gx}$ *and* $Y_{Fx} \leq Y_{Gx}$ *for all* $x \in U$.
(e) $F(e) = U$ *for any* $e \in A$ *if and only if* $Y_{Fx} = U_{(F,A)} = |U|$, *for all* $x \in U$.
(f) *If* (F, A) *is a full (or bijective) soft set, then* $U_{(F,A)} = |U|$.
(g) *If* (F, A) *is a bijective soft set, then* $P_{Fx} \leq Y_{Fx}$ *for all* $x \in U$.
(h) *If* (F, A) *is a bijective soft set such that* $A = S_F A$, *then* $P_{Fx} = 1$ *for all* $x \in U$.

Lemma 3.4: *Let* $F_A, G_B \in \widetilde{SU}$. *Then, the following statements hold,*

(a) $0 \leq Z_{Fx} \leq |U| - 1$ *for all* $x \in U$.
(b) *If* $(F, A)\widetilde{\subseteq}(G, B)$, *then* $Z_{Gx} \leq Z_{Fx}$ *for all* $x \in U$.
(c) (F, A) *is the null soft set if and only if* $Z_{Fx} = Y_{Fx} = 0$ *for all* $x \in U$.
(d) $P_{Fx} + N_{Fx} = |A|$ *for all* $x \in U$.
(e) *If* F_A^r *is the relative complement of* F_A, *then* $P_{Fx} = N_{F^r x}$ *and* $N_{Fx} = P_{F^r x}$ *for all* $x \in U$.

Definition 3.5: Let (F, A) and (G, B) be any soft sets over U. Then, a mapping S from $\widetilde{SU} \times \widetilde{SU}$ to $[0, 1]$ is called a similarity measure if its value $S((F, A), (G, B))$ satisfies the following axioms:

(s1) $0 \leq S((F, A), (G, B)) \leq 1$;
(s2) If $(F, A) = (G, B)$, then $S((F, A), (G, B)) = 1$;
(s3) $S((F, A), (G, B)) = S((G, B), (F, A))$;
(s4) If $(F, A)\widetilde{\subseteq}(G, B)\widetilde{\subseteq}(H, C)$, then $S((F, A), (H, C)) \leq S((F, A), (G, B))$ and $S((F, A), (H, C)) \leq S((G, B), (H, C))$.

Consider the function $\Im : \widetilde{SU} \times \widetilde{SU} \to [0, 1]$,

$$\Im(F_A, G_B) = 1 - \frac{1}{|U||E|}\sum_{i=1}^{|U|}[\max\{P_{Fu_i}, P_{Gu_i}\} - \min\{P_{Fu_i}, P_{Gu_i}\}] = 1 - \frac{1}{|U||E|}\sum_{i=1}^{|U|}|P_{Fu_i} - P_{Gu_i}|$$

where $F_A, G_B \in \widetilde{SU}$.

Theorem 3.6: $\Im$ *is a similarity measure on* $\widetilde{SU}$.

Proof: Let $F_A, G_B \in \widetilde{SU}$.

(s1) By Lemma 3.3, $0 \leq P_{Fx} \leq |S_F A| \leq |E|$ and $0 \leq P_{Gx} \leq |S_F B| \leq |E|$. Then, $0 \leq \frac{1}{|U||E|}\sum_{i=1}^{|U|}[\max\{P_{Fu_i}, P_{Gu_i}\} - \min\{P_{Fu_i}, P_{Gu_i}\}] \leq 1$. Therefore, $0 \leq 1 - \frac{1}{|U||E|}\sum_{i=1}^{|U|}[\max\{P_{Fu_i}, P_{Gu_i}\} - \min\{P_{Fu_i}, P_{Gu_i}\}] \leq 1$.
(s2) Let $(F, A) = (G, B)$. Then, $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$. By Lemma 3.3, $P_{Fx} \leq P_{Gx}$ and $P_{Gx} \leq P_{Fx}$, i.e. $P_{Fx} = P_{Gx}$ for all $x \in U$. Then, $[\max\{P_{Fu_i}, P_{Gu_i}\} - \min\{P_{Fu_i}, P_{Gu_i}\}] = P_{Fu_i} - P_{Fu_i} = 0$. Therefore, $1 - \frac{1}{|U||E|}\sum_{i=1}^{|U|}[\max\{P_{Fu_i}, P_{Gu_i}\} - \min\{P_{Fu_i}, P_{Gu_i}\}] = 1$.
(s3) Symmetry is obvious by definition of the function $\Im$.
(s4) Let $(F, A)\widetilde{\subseteq}(G, B)\widetilde{\subseteq}(H, C)$ and $P_{Fx} \leq P_{Gx} \leq P_{Hx}$ for all $x \in U$. Then, we have

$$\Im((G, B), (H, C)) = 1 - \frac{1}{|U||E|}\sum_{i=1}^{|U|}[\max\{P_{Hu_i}, P_{Gu_i}\} - \min\{P_{Hu_i}, P_{Gu_i}\}] = 1 - \frac{1}{|U||E|}\sum_{i=1}^{|U|}[P_{Hu_i} - P_{Gu_i}] = 1 - \frac{1}{|U||E|}\sum_{i=1}^{|U|}[P_{Hu_i} - P_{Fu_i} + P_{Fu_i} - P_{Gu_i}]$$

$$= 1 - \frac{1}{|U||E|}\sum_{i=1}^{|U|}[P_{Hu_i} - P_{Fu_i}] + 1$$
$$- \left[1 - \frac{1}{|U||E|}\sum_{i=1}^{|U|}[P_{Gu_i} - P_{Fu_i}]\right]$$
$$= \Im((F,A),(H,C)) + 1 - \Im((F,A),(G,B))$$

Then, $\Im((F,A),(G,B)) + \Im((G,B),(H,C)) = 1 + \Im((F,A),(H,C))$. Since $0 \le \Im((F,A),(G,B)) \le 1, 0 \le \Im((G,B),(H,C)) \le 1$ and $0 \le \Im((F,A),(H,C)) \le 1$, then $\Im((F,A),(H,C)) \le \Im((F,A),(G,B))$ and $\Im((F,A),(H,C)) \le \Im((G,B),(H,C))$ are satisfied.

■

4. Comparison algorithms and similarity measure application on comparison of products

The method is implemented with the following steps:

First Comparison Algorithm:

Step 1. A soft set (F,A) of the product A to be compared is obtained.

Step 2. Soft sets (F_1,A_1),(F_2,A_2),$\cdots$,(F_k,A_k) of other products $A_1, A_2, \ldots, A_k$ to be compared are obtained.

Step 3. The values $\Im((F,A),(F_1,A_1))$, $\Im((F,A),(F_2,A_2))$,$\cdots$,$\Im((F,A),(F_k,A_k))$ are calculated and sorted.

If two products are equally similar to product A after applying the first comparison algorithm, a new comparison tool will be needed to break the equality. The following definition is the secondary comparison value that will be used in breaking the equality.

Definition 4.1: Let (F,A) and (G,B) two soft sets, then, secondary similarity value between F_A and G_B defined as

$$\Gamma_{F_AG_B} = \sum_{i=1}^{|A\cup B|} \frac{||F(e_i)\cap G(e_i)| - |F(e_i)\triangle G(e_i)||}{|A\cup B||U|}.$$

For soft sets F_A, G_B and H_C, if $\Gamma_{F_AG_B} > \Gamma_{F_AH_C}$ then, $C > B$, which means that C is more similar to A than B.

Second Comparison Algorithm:

Step 1. Soft sets G_B and H_C such that $\Im((F,A),(G,B)) = \Im((F,A),(H,C))$ after the Step 3 of the first similarity algorithm are determined.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}
F_A	u_3	u_1	u_2	u_1	u_1	u_1	u_1	u_1	u_1	u_2	u_1
	u_8	u_5	u_4	u_2	u_3	u_3	u_2	u_3	u_2	u_4	u_3
	u_{10}	u_7	u_6	u_3	u_5	u_4	u_3	u_5	u_3	u_5	u_7
			u_9	u_4	u_6	u_6	u_7	u_9	u_7	u_6	u_9
				u_7	u_7	u_8	u_9	u_{10}	u_9	u_8	u_{10}
				u_8	u_9	u_{10}	u_{10}		u_{10}		
				u_9	u_{10}						
				u_{10}							

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}
G_B	u_2	u_3	u_1	u_2	u_1	u_1	u_2	u_2	u_1	u_2	u_1
	u_6	u_5	u_4	u_3	u_2	u_3	u_3	u_5	u_3	u_4	u_3
	u_8	u_7		u_4	u_3	u_4	u_6	u_6	u_4	u_5	u_6
		u_9		u_5	u_6	u_7	u_8	u_9	u_6	u_7	u_8
		u_{10}		u_7	u_8	u_{10}	u_{10}		u_8	u_{10}	u_9
				u_{10}	u_9						

Step 2. The values $\Gamma_{F_AG_B}$ and $\Gamma_{F_AH_C}$ are calculated and sorted.

If $\Gamma_{F_AG_B} = \Gamma_{F_AH_C}$ after the second comparison algorithm, then product B and product C are said to be equally similar to product A.

Example 4.2: An X company wants to compare 10 types of A products that it produces in the ready-made food sector with B, C, and D products of competing companies that produce the same products. In this case, their common universal set will be $U = \{u_1, u_2, \ldots, u_{10}\}$. Company X examines these products according to the parameters it has determined. e_1 :Price range appealing to low income, e_2 :Price range appealing to middle-income, e_3 :Price range appealing to high income, e_4 :Products preferred by female customers, e_5 :Products preferred by male customers, e_6 :Products preferred by young customers, e_7 :Products preferred by middle-aged customers, e_8 :Products preferred by elderly customers, e_9 :Products with high customer satisfaction, e_{10} :Products with low sales rates, e_{11} :Products with high sales rates. Then, the common parameter set is $E = \{e_1, e_2, \ldots, e_{11}\}$. The corresponding soft sets F_A, G_B, H_C, and K_D are obtained in the following tables.

Since, u_1 is contained by the sets $F(e_2)$, $F(e_4)$, $F(e_5)$, $F(e_6)$, $F(e_7)$, $F(e_8)$, $F(e_9)$ and, $F(e_{11})$, then, by Definition 3.1 $P_F(u_1) = \{e_2, e_4, e_5, e_6, e_7, e_8, e_9, e_{11}\}$ and $|\ P_F(u_1)\ | = P_{Fu_1} = 8$. Similarly P_{Fx}, P_{Gx}, P_{Hx}, and P_{Kx} for all $x \in U$ are obtained by the following table:

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}
H_C	u_2	u_1	u_4	u_2	u_1	u_1	u_2	u_2	u_1	u_1	u_3
	u_3	u_7		u_3	u_3	u_4	u_3	u_5	u_3	u_2	u_4
	u_5	u_8		u_4	u_4	u_6	u_6	u_7	u_4	u_5	u_6
	u_6	u_9		u_5	u_6	u_7	u_8	u_8	u_6	u_7	u_9
		u_{10}		u_6	u_7	u_9	u_9	u_9		u_8	
				u_8	u_8	u_{10}		u_{10}		u_{10}	
				u_{10}	u_9						

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}
K_D	u_2	u_3	u_1	u_2	u_1	u_2	u_1	u_2	u_1	u_1	u_3
	u_{10}	u_5	u_4	u_3	u_3	u_4	u_3	u_5	u_3	u_2	u_4
		u_6	u_7	u_5	u_4	u_5	u_4	u_7	u_4	u_5	u_6
			u_8	u_7	u_6	u_6	u_8	u_8	u_6	u_7	u_9
			u_9	u_8	u_9	u_9	u_{10}	u_9		u_8	
				u_9	u_{10}	u_{10}		u_{10}		u_{10}	

Table 1. Number of parameters corresponding to alternatives.

i	1	2	3	4	5	6	7	8	9	10
P_{Fu_i}	8	5	8	4	4	4	6	4	7	8
P_{Gu_i}	5	6	7	5	4	6	4	5	4	5
P_{Hu_i}	5	5	6	6	4	7	5	6	6	5
P_{Ku_i}	5	5	6	6	5	5	4	5	6	6

As is seen in Table 1, $\Im(F_A, G_B) = 0.84$, $\Im(F_A, H_C) = 0.84$, and $\Im(F_A, K_D) = 0.86$. In this case, the similarity comparison of product A with other products is $D > B = C$. Since $\Im(F_A, G_B) = \Im(F_A, H_C)$, secondary similarity values will be used to break the equality. Thus, $|F(e_i) \cap G(e_i)|$, $|F(e_i) \triangle G(e_i)|$, $|F(e_i) \cap H(e_i)|$, and $|F(e_i) \triangle H(e_i)|$ are obtained by the following table:

By using the values in the Table 2, secondary similarity values of B and C to A will be

$$\Gamma_{F_A G_B} = \sum_{i=1}^{|A \cup B|} \frac{||F(e_i) \cap G(e_i)| - |F(e_i) \triangle G(e_i)||}{|A \cup B||U|} = 19/110$$

and

$$\Gamma_{F_A H_C} = \sum_{i=1}^{|A \cup C|} \frac{||F(e_i) \cap H(e_i)| - |F(e_i) \triangle H(e_i)||}{|A \cup C||U|} = 20/110.$$

Therefore, with the secondary similarity algorithm, $\Gamma_{F_A G_B} < \Gamma_{F_A H_C}$, thus $B > C$. As a result of the first and second similarity algorithms, the order of similarities of products B, C, and D to product A is obtained as $D > B > C$ (Figures 1–2).

Table 2. Cardinality of intersections and symmetric differences of soft sets.

i	1	2	3	4	5	6	7	8	9	10	11
$\lvert F(e_i) \cap G(e_i)\rvert$	1	2	1	5	4	4	3	2	2	3	3
$\lvert F(e_i) \triangle G(e_i)\rvert$	4	4	4	4	5	3	5	5	7	4	4
$\lvert F(e_i) \cap H(e_i)\rvert$	1	2	1	5	5	4	3	3	2	3	2
$\lvert F(e_i) \triangle H(e_i)\rvert$	5	4	3	5	4	4	5	5	6	5	5

Table 3. Comparisons of the soft similarity measures.

Method	Ranking	The best one
Aygün and Kamacı (2019)	$B > C > D$	B
Aygün and Kamacı (2021)	$D = B > C$	D and B
Kamacı (2019)	$B > C > D$	B
Majumdar and Samanta (2008)	$B > C > D$	B
Yang(1) (Yang, 2013)	$B > C > D$	B
Yang(2) (Yang, 2013)	$B > C > D$	B
$\Im$	$D > B = C$	D
$\Im + \Gamma$	$D > B > C$	D

4.1. Comparative analysis of similarity measures on soft sets

Although a new similarity measure has been introduced on soft sets, it is important to compare this new similarity measure with the existing similarity measures in order to understand the functionality of the defined measure. The following table shows the comparison of the results of Example 4.2 obtained with the similarity measures defined in Aygün and Kamacı (2019, 2021), Kamacı (2019), Majumdar and Samanta (2008), Yang (2013) and the one proposed in this study (Table 3).

For the similarity measure $\Im$, the ranking is $D > B = C$; however, with the help of the secondary similarity value Γ, the rankings change to $D > B > C$. Also, the only similarity measure that gives close results (rankings-wise) with $\Im + \Gamma$ is the similarity measure in Aygün and Kamacı (2021). The main reason why the results obtained with the similarity measure proposed in this paper are different from the results given by the other similarity measures examined is that $\Im + \Gamma$ takes into account the parameters when determining similarity, while other similarity measures do not.

Even though $\Im + \Gamma$ and Aygün and Kamacı's similarity measures in Aygün and Kamacı (2021) give close results, the numerical values of these similarity measures differ, as can be seen from Figure 3.

On the other hand, if the soft similarity measure $\Im$ is applied to the financial diagnosis problem in Yang (2013), the result is the same as Yang's findings. That is to say, between the two companies ABC

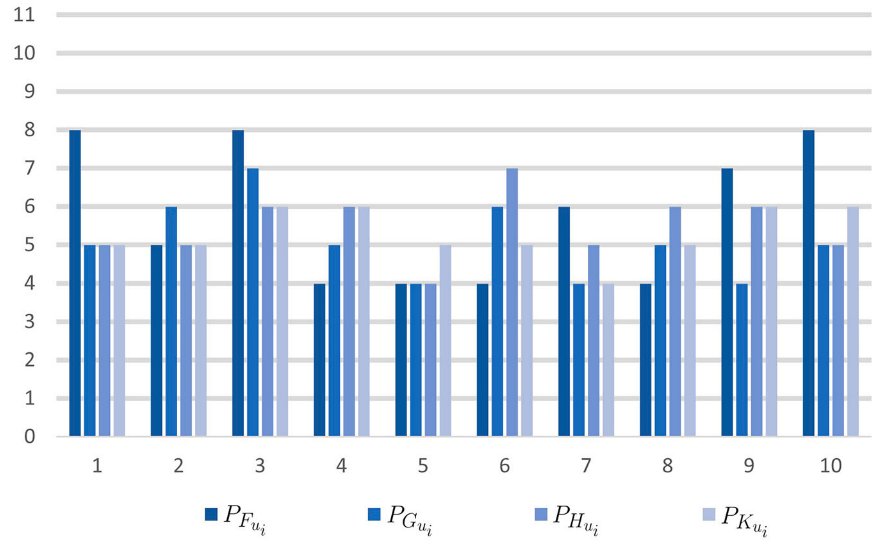

Figure 1. Numbers of parameters corresponding to alternatives.

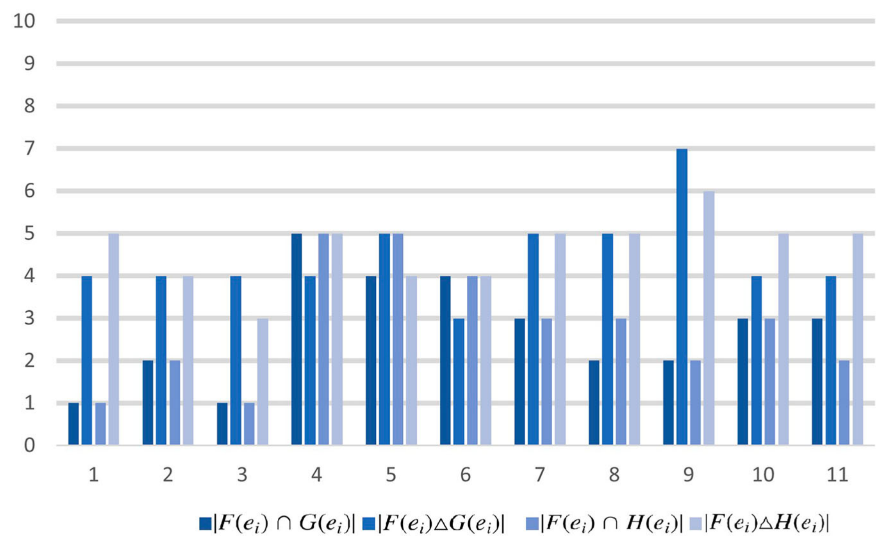

Figure 2. Cardinality of intersections and symmetric differences of soft sets.

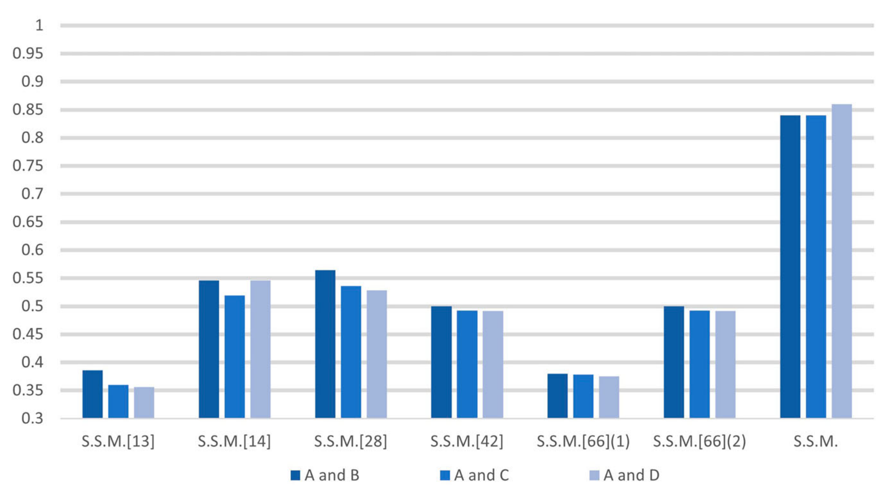

Figure 3. Comparison of the soft similarity measures. S.S.M.: Soft similarity measure.

and XYZ, XYZ is suffering from a liquidity problem, according to $\Im$. Let (F, A) and (G, B) be soft sets corresponding to companies ABC and XYZ, and the model soft set for a company suffering from liquidity problems be (H, C), then $\Im((F, A), (H, C)) = 0.94$ and $\Im((G, B), (H, C)) = 0.98$, therefore $B > C$. This result shows that the proposed similarity measure is a reliable tool that can be used for comparison problems.

5. Fuzzy sets generated by soft sets

Definition 5.1: Let $A \subseteq E$ and (F, A) be a soft set over U. For the soft set (F, A),

$$\mu : U \to [0,1], \quad \mu(x) = \frac{P_{Fx} Y_{Fx}}{U_{(F,A)} |S_F A|}$$

is called a *characteristic fuzzy set* generated by the soft set (F, A) and denoted by μ_{F_A}.

$$\nu : U \to [0,1], \quad \nu(x) = \frac{P_{Fx}}{|A|}$$

is called a *parametric fuzzy set* generated by the soft set (F, A) and denoted by ν_{F_A}.

$$g : U \to [0,1], \quad g(x) = \frac{P_{Fx}}{|E||U|}$$

is called a *general fuzzy set* generated by the soft set (F, A) and denoted by g_{F_A}.

Theorem 5.2: *If (F, A) is a soft set over U, then μ_{F_A}, ν_{F_A} and g_{F_A} are fuzzy sets.*

Proof: Since (F, A) is a soft set over U, then $F : A \to P(U)$ is a function and hence $0 \leq P_{Fx} \leq |S_F A|$ and $0 \leq Y_{Fx} \leq U_{(F,A)}$. Therefore, $0 \leq \mu_{F_A}(x) = \frac{P_{Fx} Y_{Fx}}{U_{(F,A)}|S_F A|} \leq 1$ for all $x \in U$. Since $0 \leq P_{Fx} \leq |S_F A| \leq |A|$, then $0 \leq \nu_{F_A}(x) = \frac{P_{Fx}}{|A|} \leq 1$ for all $x \in U$. Finally, since $0 \leq P_{Fx} \leq |A| \leq |E|$, then $0 \leq g_{F_A}(x) = \frac{P_{Fx}}{|E||U|} \leq 1$ for all $x \in U$. ■

Example 5.3: Let F_A, G_B, H_C and K_D be the soft sets in Example 4.2. Then, $A = B = C = D = E = \{e_1, e_2, \ldots, e_{11}\}$, $U = \{u_1, u_2, \ldots, u_{10}\}$ and for all $u_i \in U$ $P_{Fu_i}, P_{Gu_i}, P_{Hu_i}$ and P_{Ku_i} are obtained in Example 4.2. Then, the characteristic fuzzy sets generated by the soft sets F_A, G_B, H_C and, K_D are

Similarly, the following tables shows the parametric fuzzy sets and general fuzzy sets generated by soft sets, respectively.

Theorem 5.4: *Let (F, A) be a soft set over U. Then, $\nu_{(F_r,A)} = \nu^c_{F_A}$, which means that generated parametric fuzzy set of the relative complement of the soft set (F, A) is the complement of generated parametric fuzzy set of (F, A). However, this property doesn't hold for neither generated characteristic nor general fuzzy sets.*

Proof: Let $x \in U$and $P_{F^r x}$ be the number of parameters e such that $x \in F^r(e)$. By Definition 2.3, $F^r(e) = U - F(e)$ for all $e \in A$. Then, $P_{F^r x} = |A| - P_{Fx}$, since P_{Fx} is the number of parameters e such that $x \in F(e)$. Therefore, $\nu^c_{F_A}(x) = 1 - \nu_{F_A}(x) = 1 - \frac{P_{Fx}}{|A|} = \frac{|A| - P_{Fx}}{|A|} = \frac{P_{F^r x}}{|A|}$. For the remain of the proof, we have Example 5.5. ■

Example 5.5: Let $U = \{u_1, u_2, \ldots, u_8\}$ be the universe, $E = \{e_1, e_2, \ldots, e_{12}\}$ be the parameter set and $A = \{e_1, e_2, e_3, e_5, e_6, e_9, e_{11}\} \subseteq E$. Suppose that corresponding soft set of A is

$F_A = \{(e_1, \{u_1, u_2, u_7\}), (e_2, \emptyset), (e_3, \{u_6, u_7\}), (e_5, \{u_1, u_2\}), (e_6, \emptyset), (e_9, \{u_7\}), (e_{11}, \{u_8\})\}$.

Then, the support set of F_A is

$$S_F A = supp(F, A) = \{e_1, e_3, e_5, e_9, e_{11}\}.$$

It is seen that $|A| = 7$, $|S_F A| = 5$, $U_{(F,A)} = 5$, $P_{Fu_1} = 2$, $P_{Fu_2} = 2$, $P_{Fu_3} = 0$, $P_{Fu_4} = 0$, $P_{Fu_5} = 0$, $P_{Fu_6} = 1$, $P_{Fu_7} = 3$, and $P_{Fu_8} = 1$. $Y_{Fu_1} = 3$, $Y_{Fu_2} = 3$, $Y_{Fu_3} = 0$, $Y_{Fu_4} = 0$, $Y_{Fu_5} = 0$, $Y_{Fu_6} = 2$, $Y_{Fu_7} = 3$, and $Y_{Fu_8} = 1$.

Then, the characteristic, parametric and general fuzzy sets generated by the soft set F_A are

This table gives us several pieces of data about the soft set, such as the fact that the alternatives u_1 and u_2 have the same effect value; alternatives u_3, u_4, and u_5 do not provide any parameters; and alternative u_7 either corresponds to the most parameters or has common features with other alternatives.

To complete the proof of Theorem 5.4, firstly we need to obtain the relative complement of the soft set (F, A).

$(F^r, A) = \{(e_1, \{u_3, u_4, u_5, u_6, u_8\}), (e_2, U), (e_3, \{u_1, u_2, u_3, u_4, u_5, u_8\}), (e_5, \{u_3, u_4, u_5, u_6, u_7, u_8\}), (e_6, U), (e_9, \{u_1, u_2, u_3, u_4, u_5, u_6, u_8\}), (e_{11}, \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\})\}$.

Then, $|S_{F^r} A| = |supp(F^r, A)| = 7$, $U_{(F^r,A)} = 8$, $P_{F^r u_1} = 5 = |A| - P_{Fu_1}$, $P_{F^r u_2} = 5$, $P_{F^r u_3} = 7$, $P_{F^r u_4} = 7$, $P_{F^r u_5} = 7$, $P_{F^r u_6} = 6$, $P_{F^r u_7} = 4$, $P_{F^r u_8} = 6$, and $Y_{F^r u_1} = Y_{F^r u_2} = \cdots = Y_{F^r u_8} = 8$. Therefore, the characteristic, parametric and general fuzzy sets generated by the soft set (F^r, A) are

x	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
$\mu_{F_A}(x)$	64/110	40/110	64/110	32/110	28/110	28/110	48/110	32/110	56/110	64/110
$\mu_{G_B}(x)$	30/110	36/110	42/110	30/110	24/110	36/110	24/110	30/110	24/110	30/110
$\mu_{H_C}(x)$	35/110	35/110	42/110	42/110	28/110	49/110	35/110	42/110	42/110	35/110
$\mu_{K_D}(x)$	30/110	30/110	36/110	36/110	30/110	30/110	24/110	30/110	36/110	36/110

x	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
$\nu_{F_A}(x)$	8/11	5/11	8/11	4/11	4/11	4/11	6/11	4/11	7/11	8/11
$\nu_{G_B}(x)$	5/11	6/11	7/11	5/11	4/11	6/11	4/11	5/11	4/11	5/11
$\nu_{H_C}(x)$	5/11	5/11	6/11	6/11	4/11	7/11	5/11	6/11	6/11	5/11
$\nu_{K_D}(x)$	5/11	5/11	6/11	6/11	5/11	5/11	4/11	5/11	6/11	6/11

Therefore, it is seen that $\nu_{(F_r,A)} = \nu^c_{F_A}$, but $\mu_{(F_r,A)} \neq \mu^c_{F_A}$ and $g_{(F_r,A)} \neq g^c_{F_A}$.

6. Comparison of similarity measure in soft sets and similarity measure of fuzzy sets generated by them

With the following theorem, it is proved that the similarity measure between soft sets and the similarity measure obtained by using the (normalised) Hamming distance of the parametric (general) fuzzy sets generated by these soft sets are very close in terms of usefulness in comparison problems.

Theorem 6.1: *Let (F,A) and (G,B) be two soft sets over U such that $A = B = E$.*

(i) *The similarity measure for soft sets $\Im(F_A, G_B)$ and the similarity measure for parametric fuzzy sets generated by these soft sets $S_l(\nu_F, \nu_G)$, where $l(\nu_F,\nu_G) = \frac{1}{|U|}\sum_{i=1}^{|U|}|\nu_F(x_i) - \nu_G(x_i)|$ is the normalised Hamming distance and $S_l(\nu_F,\nu_G) = \frac{1}{1+l(\nu_F,\nu_G)}$, give the same result for comparison problems.*

(ii) *$d(g_F, g_G) = l(\nu_F,\nu_G)$ therefore, $S_d(g_F,g_G) = S_l(\nu_F,\nu_G)$, where g_F is the general fuzzy set generated by the soft set (F,A) and $d(g_F,g_G)$ is the Hamming distance of the fuzzy sets g_F and g_G.*

(iii) *If $S_F A = A$, $S_G B = B$ and $F(e_i) = U$ for any $e_i \in A$, $G(e_j) = U$ for any $e_j \in B$, then, $d(g_F,g_G) = l(\nu_F,\nu_G) = l(\mu_F,\mu_G)$.*

Proof: **(i)** Since $\nu_{Fu_i} = \frac{P_{Fu_i}}{|A|} = \frac{P_{Fu_i}}{|E|}$ and $\nu_{Gu_i} = \frac{P_{Gu_i}}{|B|} = \frac{P_{Gu_i}}{|E|}$ then,

$$\begin{aligned}\Im(F_A,G_B) &= 1 - \frac{1}{|U||E|}\sum_{i=1}^{|U|}[\max\{P_{Fu_i},P_{Gu_i}\} - \min\{P_{Fu_i},P_{Gu_i}\}] \\ &= 1 - \frac{1}{|U||E|}\sum_{i=1}^{|U|}|P_{Fu_i} - P_{Gu_i}| \\ &= 1 - l(\nu_F,\nu_G)\end{aligned}$$

Shortly, we show this relation by $\Im_l = 1 - l$ and $S_l = \frac{1}{1+l}$, where $0 \le l \le 1$. Now, it is easily observed that, if $l_1 \le l_2$, then, $S_{l_2} \le S_{l_1}$ and $\Im_{l_2} \le \Im_{l_1}$. Therefore, $\Im(F_A,G_B)$ and $S_l(\nu_F,\nu_G)$ give the same result for the comparison problems.

(ii) Since $g_{Fu_i} = \frac{P_{Fu_i}}{|U||E|}$ and $g_{Gu_i} = \frac{P_{Gu_i}}{|U||E|}$, then,

$$\begin{aligned}d(g_F,g_G) &= \frac{1}{|U||E|}\sum_{i=1}^{|U|}|P_{Fu_i} - P_{Gu_i}| \\ &= \frac{1}{|U|}\sum_{i=1}^{|U|}\left|\frac{P_{Fu_i}}{|E|} - \frac{P_{Gu_i}}{|E|}\right| \\ &= l(\nu_F,\nu_G)\end{aligned}$$

(iii) If $F(e_i) = U$ for any $e_i \in A$ and $G(e_j) = U$ for any $e_j \in B$, then, $Y_{Fx} = U_{(F,A)} = |U|$, $Y_{Gx} = U_{(G,B)} = |U|$ for all $x \in U$ by Lemma 3.3. Since $S_F A = A$ and $S_G B =$

x	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
$g_{F_A}(x)$	8/110	5/110	8/110	4/110	4/110	4/110	6/110	4/110	7/110	8/110
$g_{G_B}(x)$	5/110	6/110	7/110	5/110	4/110	6/110	4/110	5/110	4/110	5/110
$g_{H_C}(x)$	5/110	5/110	6/110	6/110	4/110	7/110	5/110	6/110	6/110	5/110
$g_{K_D}(x)$	5/110	5/110	6/110	6/110	5/110	5/110	4/110	5/110	6/110	6/110

x	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
$\mu_{F_A}(x)$	6/25	6/25	0	0	0	2/25	9/25	1/25
$\nu_{F_A}(x)$	2/7	2/7	0	0	0	1/7	3/7	1/7
$g_{F_A}(x)$	2/96	2/96	0	0	0	1/96	3/96	1/96

x	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
$\mu_{(F',A)}(x)$	40/56	40/56	1	1	1	48/56	32/56	48/56
$\nu_{(F',A)}(x)$	5/7	5/7	1	1	1	6/7	4/7	6/7
$g_{(F',A)}(x)$	5/96	5/96	7/96	7/96	7/96	6/96	4/96	6/96

B, then, $\mu_F(x) = \frac{P_{Fx}Y_{Fx}}{U_{(F,A)}|S_F A|} = \frac{P_{Fx}}{|A|} = \nu_F(x)$ and similar way $\mu_G(x) = \frac{P_{Gx}}{|B|} = \nu_G(x)$ for all $x \in U$. The rest of proof is seen from (ii). ■

Example 6.2: Let F_A, G_B, H_C and K_D be the soft sets in Example 4.2. Then, $A = B = C = D = E = \{e_1, e_2, \ldots, e_{11}\}$, $U = \{u_1, u_2, \ldots, u_{10}\}$ and for all $u_i \in U$ $P_{Fu_i}, P_{Gu_i}, P_{Hu_i}$ and P_{Ku_i} are obtained in Example 4.2. The generated parametric fuzzy sets of the soft sets F_A, G_B, H_C, and K_D are obtained by the following table:

The similarity measures of these fuzzy sets are given by the following table:

Depending on these results, the comparison of the similarity in the soft sets and the similarity measures obtained with the Hamming distance measure indicates that $D > B = C$. It is easily seen from Figure 4 that the results of the soft similarity measure and Hamming similarity measure on fuzzy sets obtained with Hamming distance measure are extremely close.

The following theorem gives that the expressions given in Theorem 6.1(i), which is applied to the normalised Hamming distance, are also valid for the Hamming distance, Euclidean distance, and normalised Euclidean distance:

Theorem 6.3: *Let (F, A) and (G, B) be two soft sets over U such that $A = B = E$.*

(i) *The similarity measure for soft sets $\Im(F_A, G_B)$ and the similarity measure for parametric fuzzy sets generated by these soft sets $S_d(\nu_F, \nu_G)$, where $d(\nu_F, \nu_G) = \sum_{i=1}^{|U|} |\nu_F(x_i) - \nu_G(x_i)|$ is the Hamming distance and $S_d(\nu_F, \nu_G) = \frac{1}{1+d(\nu_F,\nu_G)}$, give the same result for comparison problems.*

(ii) *The similarity measure for soft sets $\Im(F_A, G_B)$ and the similarity measure for parametric fuzzy sets generated by these soft sets $S_e(\nu_F, \nu_G)$, where $e(\nu_F, \nu_G) = \sqrt{\sum_{i=1}^{|U|} (\nu_F(x_i) - \nu_G(x_i))^2}$ is the Euclidean distance and $S_e(\nu_F, \nu_G) = \frac{1}{1+e(\nu_F,\nu_G)}$, give the same result for comparison problems.*

(iii) *The similarity measure for soft sets $\Im(F_A, G_B)$ and the similarity measure for parametric fuzzy sets generated by these soft sets $S_q(\nu_F, \nu_G)$, where $q(\nu_F, \nu_G) = \sqrt{\frac{1}{|U|}\sum_{i=1}^{|U|} (\nu_F(x_i) - \nu_G(x_i))^2}$ is the normalised Euclidean distance and $S_q(\nu_F, \nu_G) = \frac{1}{1+q(\nu_F,\nu_G)}$, give the same result for comparison problems.*

(iv) *The similarity measure $S_q(\nu_F, \nu_G)$ for parametric fuzzy sets generated by (F, A) and (G, B) using the normalised Euclidean distance gives a result closer to number $\Im(F_A, G_B)$ than $S_d(\nu_F, \nu_G)$, $S_l(\nu_F, \nu_G)$ and $S_e(\nu_F, \nu_G)$.*

Proof: (i), (ii) and (iii) are proved similar to the proof of Theorem 6.1(i). To prove (iv), $|S_d(\nu_F, \nu_G) - \Im(F_A, G_B)| \geq |S_e(\nu_F, \nu_G) - \Im(F_A, G_B)| \geq |S_l(\nu_F, \nu_G) - \Im(F_A, G_B)| \geq |S_q(\nu_F, \nu_G) - \Im(F_A, G_B)|$ is obtained by mathematical calculation. ■

Example 6.4: Let the soft sets and generated fuzzy sets be the ones in Example 6.2. Then, using the distance measures on fuzzy sets and the similarity measures obtained by these measures, we have:

Figure 5 shows the numerical difference between Hamming, Normalised Hamming, Euclidean, and Normalised Euclidean distance measures on parametric fuzzy sets.

Figure 6 shows the numerical difference between similarity measures obtained by Hamming, Normalised Hamming, Euclidean, and Normalised Euclidean distance measures on parametric fuzzy sets.

Depending on these results, the comparison of the similarity in the soft sets and the similarity measures acquired with these distance measurements was obtained with the table below:

Depend on ν;

and the closeness of these similarity measures to $\Im(F_A, G_B)$, $\Im(F_A, H_C)$ and $\Im(F_A, K_D)$ are given, respectively, in the following Table 4:

For all these cases, since the order of differences is

$$|S_d - \Im| \geq |S_e - \Im| \geq |S_l - \Im| \geq |S_q - \Im|,$$

then the similarity measure S_q for parametric fuzzy sets generated by given soft sets using the normalised

x	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
$\nu_{F_A}(x)$	8/11	5/11	8/11	4/11	4/11	4/11	6/11	4/11	7/11	8/11
$\nu_{G_B}(x)$	5/11	6/11	7/11	5/11	4/11	6/11	4/11	5/11	4/11	5/11
$\nu_{H_C}(x)$	5/11	5/11	6/11	6/11	4/11	7/11	5/11	6/11	6/11	5/11
$\nu_{K_D}(x)$	5/11	5/11	6/11	6/11	5/11	5/11	4/11	5/11	6/11	6/11

A and B with ν	Distance Measure	Similarity Measure	$\Im(F_A, G_B)$
	$l(\nu_F, \nu_G) = 0,154$	$S_l(\nu_F, \nu_G) = 0.866$	0.846
A and C with ν	Distance Measure	Similarity Measure	$\Im(F_A, H_C)$
	$l(\nu_F, \nu_H) = 0,154$	$S_l(\nu_F, \nu_H) = 0.866$	0.846
A and D with ν	Distance Measure	Similarity Measure	$\Im(F_A, K_D)$
	$l(\nu_F, \nu_K) = 0,136$	$S_l(\nu_F, \nu_K) = 0.880$	0.864

	Distance Measure	Similarity Measure	$\Im$
	$d(\nu_F, \nu_G) = 1.545$	$S_d(\nu_F, \nu_G) = 0.392$	0.846
A and B with ν	$l(\nu_F, \nu_G) = 0,154$	$S_l(\nu_F, \nu_G) = 0.866$	
	$e(\nu_F, \nu_G) = 0.567$	$S_e(\nu_F, \nu_G) = 0.638$	
	$q(\nu_F, \nu_G) = 0.179$	$S_q(\nu_F, \nu_G) = 0.848$	
	$d(\nu_F, \nu_H) = 1.545$	$S_d(\nu_F, \nu_H) = 0.392$	0.846
A and C with ν	$l(\nu_F, \nu_H) = 0,154$	$S_l(\nu_F, \nu_H) = 0.866$	
	$e(\nu_F, \nu_H) = 0.582$	$S_e(\nu_F, \nu_H) = 0.632$	
	$q(\nu_F, \nu_H) = 0.184$	$S_q(\nu_F, \nu_H) = 0.844$	
	$d(\nu_F, \nu_K) = 1.363$	$S_d(\nu_F, \nu_K) = 0.423$	0.864
A and D with ν	$l(\nu_F, \nu_K) = 0,136$	$S_l(\nu_F, \nu_K) = 0.88$	
	$e(\nu_F, \nu_K) = 0.488$	$S_e(\nu_F, \nu_K) = 0.672$	
	$q(\nu_F, \nu_K) = 0.151$	$S_q(\nu_F, \nu_K) = 0.868$	

S_d	S_l	S_e	S_q	$\Im$
$D > B = C$	$D > B = C$	$D > B > C$	$D > B > C$	$D > B = C$

Table 4. Distance between soft similarity measure and fuzzy similarity measures.

For A and B	$\lvert S_d - \Im\rvert$	$\lvert S_e - \Im\rvert$	$\lvert S_l - \Im\rvert$	$\lvert S_q - \Im\rvert$
	0.454	0.208	0.02	0.002
For A and C	$\lvert S_d - \Im\rvert$	$\lvert S_e - \Im\rvert$	$\lvert S_l - \Im\rvert$	$\lvert S_q - \Im\rvert$
	0.454	0.214	0.02	0.002
For A and D	$\lvert S_d - \Im\rvert$	$\lvert S_e - \Im\rvert$	$\lvert S_l - \Im\rvert$	$\lvert S_q - \Im\rvert$
	0.441	0.192	0.016	0.004

Euclidean distance gives a result closer to the similarity measures of these soft sets than the others. Figure 7 shows the magnitude of the difference between $|S_d - \Im|$, $|S_e - \Im|$, $|S_l - \Im|$ and $|S_q - \Im|$.

7. Intuitionistic fuzzy sets generated by soft sets

Definition 7.1: Let $A \subseteq E$ and (F, A) be a soft set over U. For the soft set (F, A),

$$\mu^T : U \to [0, 1], \quad \mu^T(x) = \frac{P_{Fx} Y_{Fx}}{|U||A|}$$

and

$$\mu^F : U \to [0, 1], \quad \mu^F(x) = \frac{N_{Fx} Z_{Fx}}{|U||A|}$$

are called a *characteristic intuitionistic fuzzy set* generated by the soft set (F, A) and denoted by $(\mu^T_{F_A}, \mu^F_{F_A})$.

$$\nu^T : U \to [0, 1], \quad \nu^T(x) = \frac{P_{Fx}}{|A|}$$

and

$$\nu^F : U \to [0, 1], \quad \nu^F(x) = \frac{N_{Fx}}{|A|}$$

are called a *parametric intuitionistic fuzzy set* generated by the soft set (F, A) and denoted by $(\nu^T_{F_A}, \nu^F_{F_A})$.

$$g^T : U \to [0, 1], \quad g^T(x) = \frac{P_{Fx}}{|E||U|}$$

and

$$g^F : U \to [0, 1], \quad g^F(x) = \frac{N_{Fx}}{|E||U|}$$

are called a *general intuitionistic fuzzy set* generated by the soft set (F, A) and denoted by $(g^T_{F_A}, g^F_{F_A})$.

Theorem 7.2: *Let (F, A) be a soft set over U. Then, $(\mu^T_{F_A}, \mu^F_{F_A})$, $(\nu^T_{F_A}, \nu^F_{F_A})$ and $(g^T_{F_A}, g^F_{F_A})$ are intuitionistic fuzzy sets.*

Proof: (a) Since (F, A) is a soft set over U, by Lemma 3.3 $1 \le Y_{Fx} \le |U|$ and by Lemma 3.4 $1 \le Z_{Fx} \le |U| - 1$. Then, $\mu^T_{F_A}$ and $\mu^F_{F_A}$ gets their maximum values when $Y_{Fx} = |U|$ and $Z_{Fx} = |U| - 1$. Also, since $P_{Fx} + N_{Fx} = |A|$, then

$$\begin{aligned}
\mu^T_{F_A} + \mu^F_{F_A} &= \frac{P_{Fx} Y_{Fx}}{|U||A|} + \frac{N_{Fx} Z_{Fx}}{|U||A|} \\
&\le \frac{P_{Fx}|U|}{|U||A|} + \frac{N_{Fx}(|U| - 1)}{|U||A|} \\
&= \frac{|U|(P_{Fx} + N_{Fx})}{|U||A|} - \frac{N_{Fx}}{|U||A|} \\
&= \frac{|U||A|}{|U||A|} - \frac{N_{Fx}}{|U||A|}
\end{aligned}$$

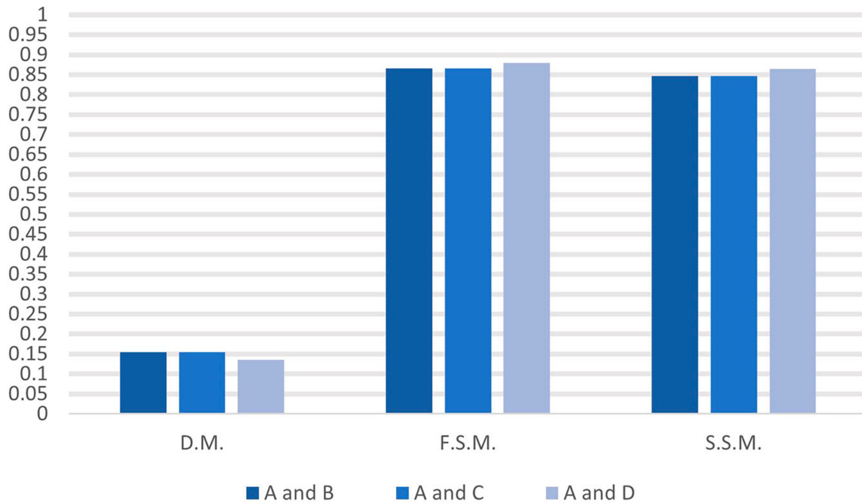

Figure 4. Comparison of distance and similarity measures of parametric fuzzy sets with the soft similarity measure. D.M.: Distance Measure. F.S.M.: Fuzzy similarity measure. S.S.M.:Soft similarity measure.

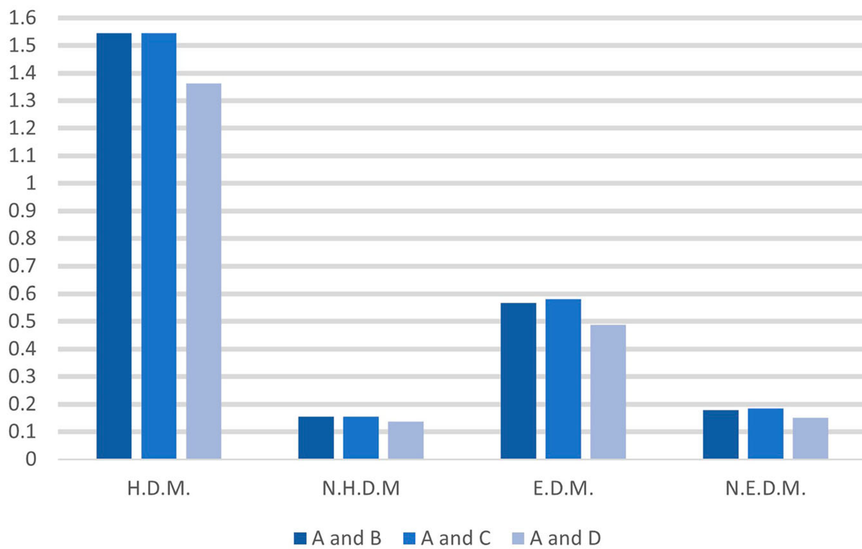

Figure 5. Distance measures of parametric fuzzy sets. H.D.M.: Hamming distance measure. N.H.D.M.: Normalised Hamming distance measure. E.D.M.: Euclidean distance measure. N.E.D.M.: Normalised Euclidean distance measure.

$$= 1 - \frac{N_{Fx}}{|U||A|} \leq 1$$

Therefore, $(\mu^T_{F_A}, \mu^F_{F_A})$ is an intuitionistic fuzzy set.

(b) Since (F, A) is a soft set over U, then by Lemma 3.4 $P_{Fx} + N_{Fx} = |A|$ and $\frac{P_{Fx}}{|A|} + \frac{N_{Fx}}{|A|} = 1$. Hence, $0 \leq \nu^T_{F_A}(x) + \nu^F_{F_A}(x) \leq 1$ for all $x \in U$.

(c) By Lemma 3.4 $P_{Fx} + N_{Fx} = |A| \leq |E|$, then $0 \leq \frac{P_{Fx}}{|E||U|} + \frac{N_{Fx}}{|E||U|} \leq 1$ for all $x \in U$. Thus, $(g^T_{F_A}, g^F_{F_A})$ is an intuitionistic fuzzy set on U. ■

Example 7.3: Let (F, A) be the soft set in Example 5.5. Then, its corresponding P_{Fx}, N_{Fx}, Y_{Fx} and Z_{Fx} for all $x \in U$ are obtained by the following table:

i	1	2	3	4	5	6	7	8
P_{Fu_i}	2	2	0	0	0	1	3	1
N_{Fu_i}	5	5	7	7	7	6	4	6
Y_{Fu_i}	3	3	0	0	0	2	3	1
Z_{Fu_i}	2	2	3	3	3	3	2	3

Then, the characteristic intuitionistic fuzzy sets generated by the soft set F_A are

x	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
$\mu^T_{F_A}(x)$	6/56	6/56	0	0	0	2/56	9/56	1/56
$\mu^F_{F_A}(x)$	10/56	10/56	21/56	21/56	21/56	18/56	8/56	18/56

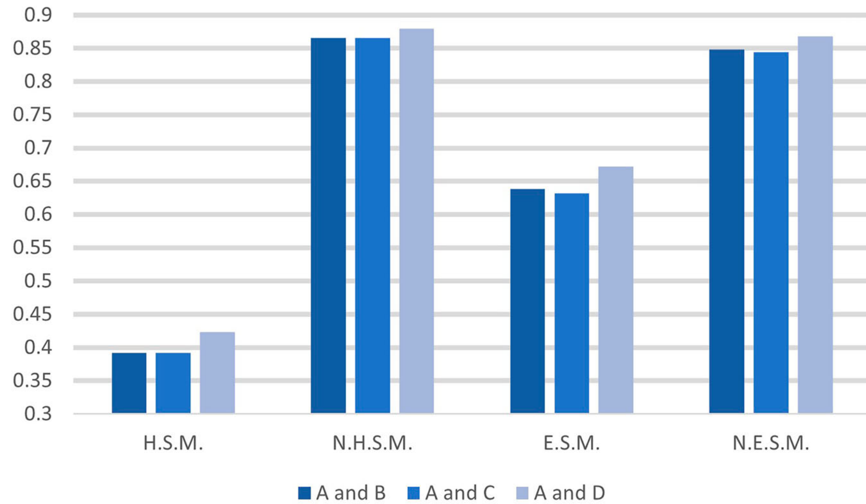

Figure 6. Similarity measures of parametric fuzzy sets obtained from distance measures. H.S.M.: Hamming similarity measure. N.H.S.M.: Normalised Hamming similarity measure. E.S.M.: Euclidean similarity measure. N.E.S.M.: Normalised Euclidean similarity measure.

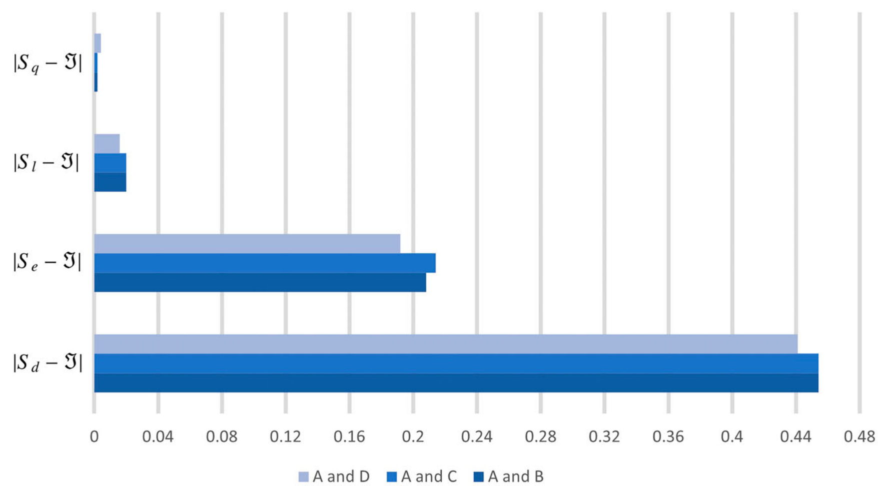

Figure 7. Distance between soft similarity measure and fuzzy similarity measures.

the parametric intuitionistic fuzzy sets generated by the soft set F_A are

x	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
$\nu^T_{F_A}(x)$	2/7	2/7	0	0	0	1/7	3/7	1/7
$\nu^F_{F_A}(x)$	5/7	5/7	1	1	1	6/7	4/7	6/7

and the general intuitionistic fuzzy sets generated by the soft set F_A are

x	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
$g^T_{F_A}(x)$	2/96	2/96	0	0	0	1/96	3/96	1/96
$g^F_{F_A}(x)$	5/96	5/96	7/96	7/96	7/96	6/96	4/96	6/96

Intuitionistic fuzzy sets (characteristic, parametric, general) generated by the soft set F_A were obtained above. In Example 5.5, fuzzy sets (characteristic, parametric, general) generated by the same soft set were given. When we consider these two structures, since only the membership function is present in fuzzy sets, a solution can be obtained from the positive side in decision-making problems or the problem can be handled only from the negative side with the help of the complement of the fuzzy set. On the other hand, both the membership function and the non-membership function can be used in the decision-making process with intuitionistic fuzzy sets. For this reason, a problem can be solved both positively and negatively with the help of intuitionistic fuzzy sets. This shows that intuitionistic fuzzy sets are more suitable for solving complex uncertainties. In addition, if we look at the

i	1	2	3	4	5	6	7	8
$P_{F^r u_i}$	5	5	7	7	7	6	4	6
$N_{F^r u_i}$	2	2	0	0	0	1	3	1
$Y_{F^r u_i}$	8	8	8	8	8	8	8	8
$Z_{F^r u_i}$	6	6	0	0	0	6	7	7

definitions of characteristic (parametric) intuitionistic fuzzy sets and characteristic (parametric) fuzzy sets generated by soft sets, it is clear that both have the same membership function. This shows that intuitionistic fuzzy sets are relatively more comprehensive than fuzzy sets.

Theorem 7.4: *Let (F, A) be a soft set over U. Then, $(\nu^T_{(F_r,A)}, \nu^F_{(F_r,A)}) = (\nu^T_{F_A}, \nu^F_{F_A})^c$ and $(g^T_{(F_r,A)}, g^F_{(F_r,A)}) = (g^T_{F_A}, g^F_{F_A})^c$. This means that the generated parametric (general) intuitionistic fuzzy set of the relative complement of the soft set (F, A) is the complement of the generated parametric (general) intuitionistic fuzzy set of (F, A). Nevertheless, this property doesn't hold for generated characteristic intuitionistic fuzzy set.*

Proof: Let (F, A) be a soft set over U and (F^r, A) be relative complement of (F, A). Then, by Lemma 3.4 $P_{Fx} = N_{F^r x}$ and $N_{Fx} = P_{F^r x}$ for all $x \in U$. By Definition 2.14

$$\begin{aligned}(\nu^T_{(F_r,A)}, \nu^F_{(F_r,A)}) &= \left(\frac{P_{F^r x}}{|A|}, \frac{N_{F^r x}}{|A|}\right)\\ &= \left(\frac{N_{Fx}}{|A|}, \frac{P_{Fx}}{|A|}\right)\\ &= (\nu^F_{(F,A)}, \nu^T_{(F,A)})\\ &= (\nu^T_{(F,A)}, \nu^F_{(F,A)})^c\end{aligned}$$

and

$$\begin{aligned}(g^T_{(F_r,A)}, g^F_{(F_r,A)}) &= \left(\frac{P_{F^r x}}{|E||U|}, \frac{N_{F^r x}}{|E||U|}\right)\\ &= \left(\frac{N_{Fx}}{|E||U|}, \frac{P_{Fx}}{|E||U|}\right)\\ &= (g^F_{(F,A)}, g^T_{(F,A)})\\ &= (g^T_{(F,A)}, g^F_{(F,A)})^c\end{aligned}$$

To complete the proof of Theorem 7.4, we use the soft set (F, A) and its relative complement in the Example 5.5. For (F^r, A) the values $P_{F^r x}$, $N_{F^r x}$, $Y_{F^r x}$ and $Z_{F^r x}$ for all $x \in U$ are obtained by the following table:

Then, the characteristic intuitionistic fuzzy sets generated by the soft set F^r_A are

x	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
$\mu^T_{F^r_A}(x)$	40/56	40/56	1	1	1	48/56	32/56	48/56
$\mu^F_{F^r_A}(x)$	12/56	12/56	0	0	0	6/56	21/56	7/56

the parametric intuitionistic fuzzy sets generated by the soft set F^r_A are

x	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
$\nu^T_{F^r_A}(x)$	5/7	5/7	1	1	1	6/7	4/7	6/7
$\nu^F_{F^r_A}(x)$	2/7	2/7	0	0	0	1/7	3/7	1/7

and the general intuitionistic fuzzy sets generated by the soft set F^r_A are

x	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
$g^T_{F^r_A}(x)$	5/96	5/96	7/96	7/96	7/96	6/96	4/96	6/96
$g^F_{F^r_A}(x)$	2/96	2/96	0	0	0	1/96	3/96	1/96

Therefore, by Example 7.3 $(\mu^T_{F^r_A}, \mu^F_{F^r_A}) \neq (\mu^T_{F_A}, \mu^F_{F_A})^c$. ■

7.1. Comparative analysis of similarity measures on intuitionistic fuzzy sets with the soft similarity measure

Example 7.5: Let F_A, G_B, H_C and K_D be the soft sets in the Example 4.2. Then, Figure 8 shows the result of Example 4.2 with the parametric intuitionistic fuzzy sets.

Also, these similarity measures on intuitionistic fuzzy sets can be applied to general intuitionistic fuzzy sets. Therefore, the results and rankings of these similarity measures on intuitionistic fuzzy sets can be compared based on two different intuitionistic fuzzy sets generated by the same soft set. Figure 9 shows the result of these intuitionistic fuzzy similarity measures on the general intuitionistic fuzzy set.

Although the numerical values in Figures 8 and 9 are different, it is observed that the rankings are the same. Based on these results, Table 5 gives the rankings of the products by using intuitionistic fuzzy similarity measures on generated intuitionistic fuzzy sets.

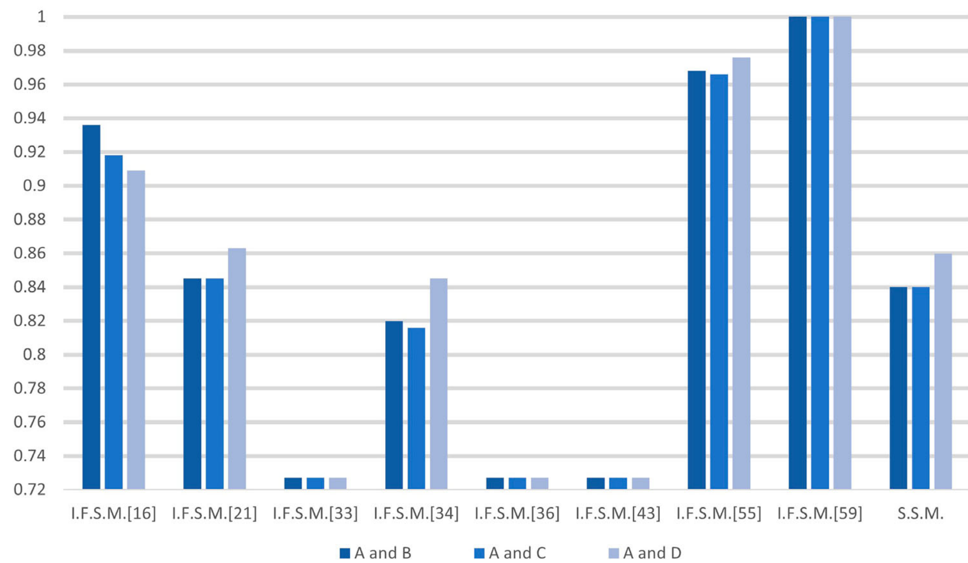

Figure 8. Comparison of the similarity measures on parametric intuitionistic fuzzy sets with the soft similarity measure. I.F.S.M.: Intuitionistic fuzzy similarity measure.

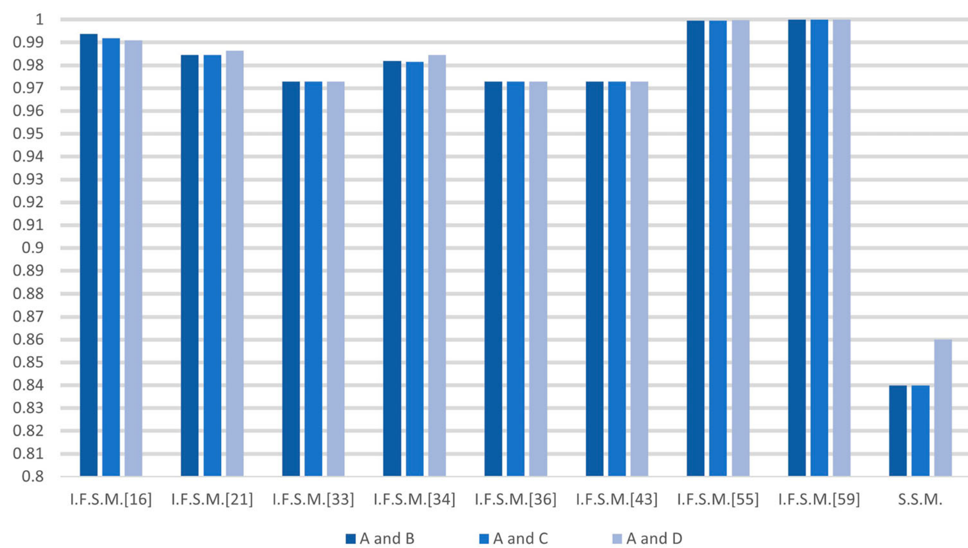

Figure 9. Comparison of the similarity measures on general intuitionistic fuzzy sets with the soft similarity measure. I.F.S.M.: Intuitionistic fuzzy similarity measure.

Table 5. Comparison of the similarity measures on parametric and general intuitionistic fuzzy sets with the soft similarity measure.

Method	Ranking	The best one
Chen (1995)	$B > C > D$	B
L. Fan and Zhangyan (2001)	$D > B = C$	D
D. Li and Cheng (2002)	$D = B = C$	–
Y. Li et al. (2002)	$D > B > C$	D
Liang and Shi (2003)	$D = B = C$	–
Mitchell (2003)	$D = B = C$	–
Singh and Sharma (2021)	$D > B > C$	D
Szmidt et al. (2022)	$D = B = C$	–
$\Im$	$D > B = C$	D
$\Im + \Gamma$	$D > B > C$	D

8. Conclusion and future studies

In this study, we proposed the concepts of the inverse positive soft set and the inverse negative soft set. Also, we proposed a new similarity measure on the soft sets whose parameters are taken into account by means of the inverse positive soft set. In addition, the parametric structure of soft sets is combined with the numerical value results of fuzzy and intuitionistic fuzzy sets by generating fuzzy and intuitionistic fuzzy sets with the help of inverse positive soft sets and inverse negative soft sets. Also, the proposed similarity measure on soft sets and the existing similarity measures on

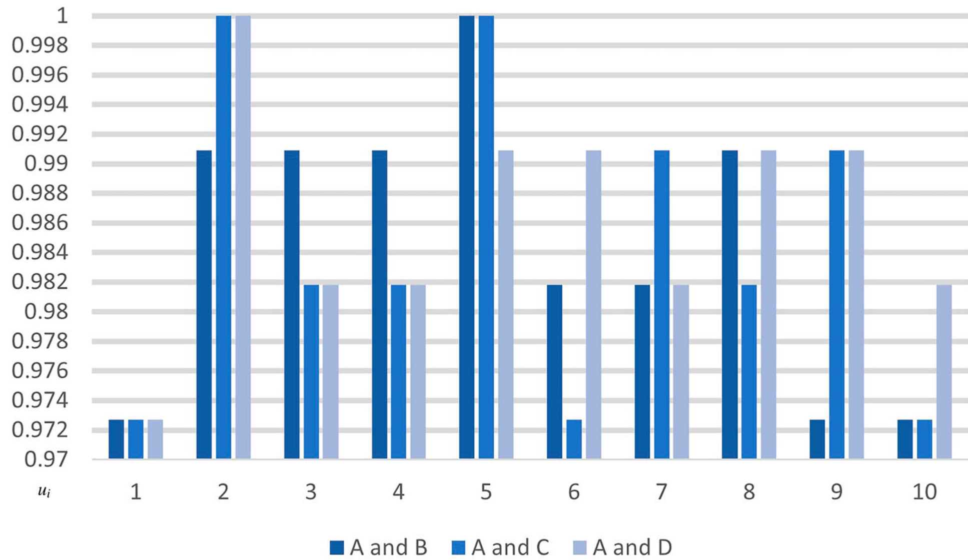

Figure 10. Comparison of the product types.

Table 6. Comparison of product types.

Types	$\Im_{u_i}(F_A, G_B)$	$\Im_{u_i}(F_A, H_C)$	$\Im_{u_i}(F_A, K_D)$	Rankings
u_1	0.9727	0.9727	0.9727	$B = C = D$
u_2	0.9909	1	1	$C = D > B$
u_3	0.9909	0.9818	0.9818	$B > C = D$
u_4	0.9909	0.9818	0.9818	$B > C = D$
u_5	1	1	0.9909	$B = C > D$
u_6	0.9818	0.9727	0.9909	$D > B > C$
u_7	0.9818	0.9909	0.9818	$C > B = D$
u_8	0.9909	0.9818	0.9909	$B = D > C$
u_9	0.9727	0.9909	0.9909	$D = C > B$
u_{10}	0.9727	0.9727	0.9818	$D > B = C$

soft sets were compared. Similarity measures defined on soft sets were compared not only with similarity measures defined on soft sets but also with similarity measures defined on fuzzy and intuitionistic fuzzy sets. This comparison is performed using fuzzy and intuitionistic fuzzy sets generated by soft sets.

One of the advantages of the similarity measure proposed in this paper is that it can be used to compare different products or to compare a specific type of product. To illustrate this statement, Figure 10 shows the results of the comparison of the products B, C, and D to A by using

$$\Im_{u_i}(F_A, G_B) = 1 - \frac{|P_{Fu_i} - P_{Gu_i}|}{|U||E|}$$

for all product types u_i's in Example 4.2.

Based on Figure 10, the similarity of B, C, and D to A based on u_4 is $B > C = D$, and based on u_9, it is $D = C > B$. Table 6 shows the result and the ranking of the products based on types of products.

On the other hand, three different fuzzy sets generated by soft sets were defined. Moreover, when distance-based similarity is used, it has been found that the similarity measure between soft sets and the similarity measure between fuzzy sets give the same result in solving comparison problems, which supports the purpose of the study. The results obtained were supported by an application. Also, three different intuitionistic fuzzy sets generated by soft sets were defined. By using these intuitionistic fuzzy sets, similarity measures on intuitionistic fuzzy sets and similarity measures on soft sets are compared. Thus, with the help of fuzzy and intuitionistic fuzzy sets generated by soft sets, similarity measures on two different structures were compared.

Inspired by this study, it is thought that useful results will be obtained for many different types of problems, such as decision-making. Also, the notations of fuzzy and intuitionistic fuzzy sets generated by soft sets can be extended to other fuzzy concepts, such as complex fuzzy sets, bipolar complex fuzzy sets, picture fuzzy sets, and Pythagorean fuzzy sets, by a similar approach.

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Data availability statement

Enquiries about data availability may be directed to the authors.

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