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On absolute matrix summability factors of infinite series and Fourier series

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ABSTRACT

Quite recently, Bor (2021) [22] has proved two main theorems dealing with absolute Riesz summability factors of infinite series and Fourier series by using an almost increasing sequence. In this paper, we have generalized these theorems to the $|A, p_n|_k$ summability method.

1. Introduction

A positive sequence (b_n) is said to be almost increasing if there exists a positive increasing sequence (z_n) and two positive constants A and B such that $Az_n \leq b_n \leq Bz_n$ (see [1]). For any sequence (λ_n) we write that

$$\Delta^2\lambda_n = \Delta\lambda_n - \Delta\lambda_{n+1} \quad \text{and} \quad \Delta\lambda_n = \lambda_n - \lambda_{n+1}.$$

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A sequence (λ_n) is said to be of bounded variation, denoted by $(\lambda_n) \in \mathcal{BV}$, if

$$\sum_{n=1}^{\infty} |\Delta\lambda_n| < \infty.$$

Let $\sum a_n$ be a given infinite series with partial sums (s_n). By t_n^α we denote the nth Cesàro means of order α, with $\alpha > -1$, of the sequence (na_n), that is (see [23])

$$t_n^\alpha = \frac{1}{A_n^\alpha} \sum_{v=1}^{n} A_{n-v}^{\alpha-1} v a_v, \quad (t_n^1 = t_n)$$

where

$$A_n^\alpha = \frac{(\alpha+1)(\alpha+2)...(\alpha+n)}{n!} = O(n^\alpha), \qquad A_{-n}^\alpha = 0 \quad \text{for} \quad n > 0.$$

The series $\sum a_n$ is said to be summable $|C,\alpha|_k$, $k \geq 1$, if (see [25])

$$\sum_{n=1}^{\infty} \frac{1}{n} |t_n^\alpha|^k < \infty.$$

If we take $\alpha = 1$, then $|C,\alpha|_k$ summability reduces to $|C,1|_k$ summability.

Let (p_n) be a sequence of positive real numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty \quad as \quad n \to \infty, \quad (P_{-i} = p_{-i} = 0, \quad i \geq 1).$$

The sequence-to-sequence transformation $(s_n) \to (v_n)$ with

$$v_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v, \quad P_n \neq 0$$

defines the sequence (v_n) of the Riesz mean or simply the $(\bar{N}, p_n)$ mean of the sequence (s_n) generated by the sequence of coefficients (p_n) (see [26]).

The series $\sum a_n$ is said to be summable $|\bar{N}, p_n|_k$, $k \geq 1$, if (see [2])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} \mid v_n - v_{n-1} \mid^k < \infty.$$

When $p_n = 1$ for all values of n, then we get $|C,1|_k$ summability.

Let $A = (a_{nv})$ be a normal matrix, i.e., a lower triangular matrix with nonzero diagonal entries. Then A defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $As = (A_n(s))$, where

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v, \quad n = 0, 1, ...$$

The series $\sum a_n$ is said to be summable $|A, p_n|_k$, $k \geq 1$, if (see [29])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} |A_n(s) - A_{n-1}(s)|^k < \infty.$$

If we take $p_n = 1$ for all n, then $|A, p_n|_k$ summability reduces to $|A|_k$ summability (see [28]). If we take $a_{nv} = \frac{p_v}{P_n}$, then $|A, p_n|_k$ summability reduces to $\left|\bar{N}, p_n\right|_k$ summability. If we take $a_{nv} = \frac{p_v}{P_n}$ and $k = 1$, then $|A, p_n|_k$ summability reduces to $\left|\bar{N}, p_n\right|$ summability (see [30]). Also if we take $a_{nv} = \frac{p_v}{P_n}$ and $p_n = 1$ for all n, then $|A, p_n|_k$ summability is the same as $|C, 1|_k$ summability.

2. Known result

Many works dealing with absolute matrix summability and the absolute summability factors of infinite series and Fourier series have been done in (see [3–22], [27], [31–36]). Among them, in [22], the following theorem has been proved.

Theorem 1. *Let (X_n) be an almost increasing sequence. If the sequences (X_n), (λ_n), and (p_n) satisfy the conditions*

$$|\lambda_m| X_m = O(1) \quad as \quad n \to \infty, \tag{1}$$

$$\sum_{n=1}^{m} n X_n |\Delta^2 \lambda_n| = O(1) \quad as \quad m \to \infty, \tag{2}$$

$$\sum_{n=1}^{m} \frac{|t_n|^k}{n X_n^{k-1}} = O(X_m) \quad as \quad m \to \infty, \tag{3}$$

$$\sum_{n=1}^{m} \frac{p_n}{P_n} \frac{|t_n|^k}{X_n^{k-1}} = O(X_m) \quad as \quad m \to \infty, \tag{4}$$

and

$$\sum_{n=1}^{m} \frac{P_n}{n} = O(P_m) \quad as \quad m \to \infty, \tag{5}$$

then the series $\sum a_n \lambda_n$ is summable $|\bar{N}, p_n|_k$, $k \geq 1$.

3. Main result

Given a normal matrix $A = (a_{nv})$, we associate two lower semimatrices $\bar{A} = (\bar{a}_{nv})$ and $\hat{A} = (\hat{a}_{nv})$ as follows:

$$\bar{a}_{nv} = \sum_{i=v}^{n} a_{ni}, \quad n, v = 0, 1, \tag{6}$$

and

$$\hat{a}_{00} = \bar{a}_{00} = a_{00}, \quad \hat{a}_{nv} = \bar{a}_{nv} - \bar{a}_{n-1,v}, \quad n = 1, 2, ... \tag{7}$$

It may be noted that $\bar{A}$ and $\hat{A}$ are the well-known matrices of series-to-sequence and series-to-series transformations, respectively. Then, we have

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v = \sum_{v=0}^{n} \bar{a}_{nv} a_v \tag{8}$$

and

$$\bar{\Delta} A_n(s) = \sum_{v=0}^{n} \hat{a}_{nv} a_v. \tag{9}$$

The aim of this paper is to generalize Theorem 1 for $|A, p_n|_k$ summability method by using the above notations. Now, we shall prove the following general theorem.

Theorem 2. *Let (X_n) be an almost increasing sequence and $A = (a_{nv})$ be a positive normal matrix such that*

$$\overline{a}_{n0} = 1, \; n = 0, 1, ..., \tag{10}$$

$$a_{n-1,v} \geq a_{nv}, \; \text{for} \quad n \geq v + 1, \tag{11}$$

$$a_{nn} = O\left(\frac{p_n}{P_n}\right), \tag{12}$$

$$\sum_{v=1}^{n-1} \frac{|\hat{a}_{n,v+1}|}{v} = O(a_{nn}), \tag{13}$$

and

$$\sum_{n=1}^{m} a_{nn} \frac{|t_n|^k}{X_n^{k-1}} = O(X_m) \quad as \quad m \to \infty. \tag{14}$$

If the conditions (1)-(3) of Theorem 1 are satisfied, then the series $\sum a_n \lambda_n$ is summable $|A, p_n|_k$, $k \geq 1$.

Remark. It should be noted that we apply to Theorem 2 to the weighted mean in which $A = (a_{nv})$ is defined as $a_{nv} = \frac{p_v}{P_n}$ when $0 \leq v \leq n$, where $P_n = p_0 + p_1 + ... + p_n$. In this case, the conditions (10)-(12) are obvious, the condition (13) reduces to (5) and the condition (14) reduces to (4). So, Theorem 2 returns to Theorem 1.

We need the following lemma for the proof of Theorem 2.

Lemma *([7]). Under the conditions of Theorem 1 we have the following*

$$nX_n|\Delta\lambda_n| = O(1) \quad as \quad n\to\infty,$$
$$\sum_{n=1}^{\infty} X_n|\Delta\lambda_n| < \infty.$$

Proof of Theorem 2. Let (W_n) denote the A-transform of the series $\sum a_n\lambda_n$. Then, by (8) and (9), we have

$$\bar{\Delta}W_n = \sum_{v=0}^{n} \hat{a}_{nv}a_v\lambda_v.$$

Applying Abel's transformation to this sum, we have that

$$\begin{aligned}\bar{\Delta}W_n &= \sum_{v=1}^{n} \frac{\hat{a}_{nv}\lambda_v}{v} va_v = \sum_{v=1}^{n-1} \Delta(\frac{\hat{a}_{nv}\lambda_v}{v}) \sum_{r=1}^{v} ra_r + \frac{\hat{a}_{nn}\lambda_n}{n} \sum_{v=1}^{n} va_v \\ &= \sum_{v=1}^{n-1} \Delta(\frac{\hat{a}_{nv}\lambda_v}{v})(v+1)t_v + \hat{a}_{nn}\lambda_n \frac{n+1}{n} t_n \\ &= \sum_{v=1}^{n-1} \Delta_v(\hat{a}_{nv})\lambda_v t_v \frac{v+1}{v} + \sum_{v=1}^{n-1} \hat{a}_{n,v+1}\Delta\lambda_v t_v \frac{v+1}{v} \\ &\quad + \sum_{v=1}^{n-1} \hat{a}_{n,v+1}\lambda_{v+1}\frac{t_v}{v} + a_{nn}\lambda_n t_n \frac{n+1}{n} \\ &= W_{n,1} + W_{n,2} + W_{n,3} + W_{n,4}.\end{aligned}$$

To complete the proof of Theorem 2, by Minkowski's inequality, it is sufficient to show that

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} \mid W_{n,r} \mid^k < \infty, \quad \text{for} \quad r = 1, 2, 3, 4. \tag{15}$$

Firstly, by using the conditions (6)-(7) and (10)-(11), we have $\sum_{v=1}^{n-1} |\Delta_v(\hat{a}_{nv})| \le a_{nn}$, and $\sum_{n=v+1}^{m+1} |\Delta_v(\hat{a}_{nv})| \le a_{vv}$. Now, applying Hölder's inequality with indices k and k', where $k > 1$ and $\frac{1}{k} + \frac{1}{k'} = 1$, we obtain that

$$\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} \mid W_{n,1} \mid^k \le \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} \left\{ \sum_{v=1}^{n-1} |\frac{v+1}{v}| \, |\Delta_v(\hat{a}_{nv})| \, |\lambda_v||t_v| \right\}^k$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\left\{\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\,|\lambda_v|^k|t_v|^k\right\}\times\left\{\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})|\right\}^{k-1}$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}a_{nn}^{k-1}\left\{\sum_{v=1}^{n-1}|\Delta_v(\hat{a}_{nv})||\lambda_v|^k|t_v|^k\right\}$$

$$= O(1)\sum_{v=1}^{m}|\lambda_v|^{k-1}|\lambda_v||t_v|^k\sum_{n=v+1}^{m+1}|\Delta_v(\hat{a}_{nv})|$$

$$= O(1)\sum_{v=1}^{m}a_{vv}\frac{|t_v|^k}{X_v^{k-1}}|\lambda_v|$$

$$= O(1)\sum_{v=1}^{m-1}\Delta|\lambda_v|\sum_{r=1}^{v}a_{rr}\frac{|t_r|^k}{X_r^{k-1}} + O(1)|\lambda_m|\sum_{v=1}^{m}a_{vv}\frac{|t_v|^k}{X_v^{k-1}}$$

$$= O(1)\sum_{v=1}^{m-1}|\Delta\lambda_v|X_v + O(1)|\lambda_m|X_m$$

$$= O(1) \quad \text{as} \quad m\to\infty,$$

by virtue of the hypotheses of Theorem 2 and Lemma. By using Hölder's inequality, and also the fact that $\sum_{n=v+1}^{m+1}|\hat{a}_{n,v+1}|\leq 1$, we obtain

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}|\,W_{n,2}\,|^k\leq\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\left\{\sum_{v=1}^{n-1}|\frac{v+1}{v}||\hat{a}_{n,v+1}||\Delta\lambda_v||t_v|\right\}^k$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\Delta\lambda_v||t_v|\right\}^k$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}|(v|\Delta\lambda_v|)^k\frac{|t_v|^k}{v}\right\}\times\left\{\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}\right\}^{k-1}$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}a_{nn}^{k-1}\sum_{v=1}^{n-1}(v|\Delta\lambda_v|)^{k-1}v|\Delta\lambda_v||\hat{a}_{n,v+1}|\frac{|t_v|^k}{v}$$

$$= O(1)\sum_{v=1}^{m}\frac{|t_v|^k}{vX_v^{k-1}}v|\Delta\lambda_v|\sum_{n=v+1}^{m+1}|\hat{a}_{n,v+1}|$$

$$= O(1)\sum_{v=1}^{m}\frac{|t_v|^k}{vX_v^{k-1}}v|\Delta\lambda_v|$$

$$= O(1)\sum_{v=1}^{m-1}\Delta(v|\Delta\lambda_v|)\sum_{r=1}^{v}\frac{|t_r|^k}{rX_r^{k-1}} + O(1)m|\Delta\lambda_m|\sum_{v=1}^{m}\frac{|t_v|^k}{vX_v^{k-1}}$$

$$= O(1)\sum_{v=1}^{m-1}|\Delta(v|\Delta\lambda_v|)|X_v + O(1)m|\Delta\lambda_m|X_m$$

$$= O(1)\sum_{v=1}^{m-1} vX_v|\Delta^2\lambda_v| + O(1)\sum_{v=1}^{m-1} X_v|\Delta\lambda_v| + O(1)m|\Delta\lambda_m|X_m$$

$$= O(1) \quad \text{as} \quad m\to\infty,$$

by virtue of the hypotheses of Theorem 2 and Lemma. Again, we have that

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1} | W_{n,3} |^k = \sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\left|\sum_{v=1}^{n-1}\hat{a}_{n,v+1}\lambda_{v+1}\frac{t_v}{v}\right|^k$$

$$\leq \sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\lambda_{v+1}|\frac{|t_v|}{v}\right\}^k$$

$$\leq \sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\lambda_{v+1}|^k\frac{|t_v|^k}{v}\right\}\times\left\{\sum_{v=1}^{n-1}\frac{|\hat{a}_{n,v+1}|}{v}\right\}^{k-1}$$

$$= O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1} a_{nn}^{k-1}\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\lambda_{v+1}|^k\frac{|t_v|^k}{v}$$

$$= O(1)\sum_{v=1}^{m}|\lambda_{v+1}|^{k-1}|\lambda_{v+1}|\frac{|t_v|^k}{v}\sum_{n=v+1}^{m+1}|\hat{a}_{n,v+1}|$$

$$= O(1)\sum_{v=1}^{m}|\lambda_{v+1}|\frac{|t_v|^k}{vX_v^{k-1}}$$

$$= O(1)\sum_{v=1}^{m-1}\Delta|\lambda_{v+1}|\sum_{r=1}^{v}\frac{|t_r|^k}{rX_r^{k-1}} + O(1)|\lambda_{m+1}|\sum_{v=1}^{m}\frac{|t_v|^k}{vX_v^{k-1}}$$

$$= O(1)\sum_{v=1}^{m-1}|\Delta\lambda_{v+1}|X_{v+1} + O(1)|\lambda_{m+1}|X_{m+1}$$

$$= O(1)\sum_{v=2}^{m-1}|\Delta\lambda_v|X_v + O(1)|\lambda_{m+1}|X_{m+1}$$

$$= O(1)\sum_{v=1}^{m-1}|\Delta\lambda_v|X_v + O(1)|\lambda_{m+1}|X_{m+1}$$

$$= O(1) \quad \text{as} \quad m\to\infty,$$

by virtue of the hypotheses of Theorem 2 and Lemma. Finally, as in $W_{n,1}$, we have that

$$\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{k-1}|W_{n,4}|^k = O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{k-1} a_{nn}^k|\lambda_n|^{k-1}|\lambda_n||t_n|^k$$

$$= O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{k-1} a_{nn}^{k-1}a_{nn}|\lambda_n|^{k-1}|\lambda_n||t_n|^k$$

$$= O(1)\sum_{n=1}^{m} a_{nn}|\lambda_n|\frac{|t_n|^k}{X_n^{k-1}} = O(1) \quad \text{as} \quad m\to\infty,$$

by virtue of hypotheses of Theorem 2 and Lemma. This completes the proof of Theorem 2. □

If we take $a_{nv} = \frac{p_v}{P_n}$ in Theorem 2, then we obtain Theorem 1 dealing with $\left|\bar{N}, p_n\right|_k$ summability method. If we take $p_n = 1$ for all n in Theorem 2, then we have a new result for $|A|_k$ summability method. Also if we take $p_n = 1$ for all n and $a_{nv} = \frac{p_v}{P_n}$ in Theorem 2, then we have a new theorem on $|C,1|_k$ summability method. Finally, if we take $a_{nv} = \frac{p_v}{P_n}$ and $k = 1$ in Theorem 2, then we have another new result for $\left|\bar{N}, p_n\right|$ summability method.

4. An application of absolute matrix summability to trigonometric Fourier series

Let f be a periodic function with period 2π and integrable (L) over $(-\pi,\pi)$. The trigonometric Fourier series of f is defined as

$$f(x) \sim \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n\cos nx + b_n \sin nx) = \sum_{n=0}^{\infty} C_n(x).$$

Set

$$\phi(t) = \frac{1}{2}\left\{f(x+t)+f(x-t)\right\},$$

$$\phi_\alpha(t) = \frac{\alpha}{t^\alpha}\int_0^t (t-u)^{\alpha-1}\phi(u)\,du, \quad (\alpha>0).$$

It is well known that if $\phi_1(t)\in\mathcal{BV}(0,\pi)$, then $t_n(x) = O(1)$, where $t_n(x)$ is the $(C,1)$ mean of the sequence $(nC_n(x))$ (see [24]).

The following theorem is known dealing with $|\bar{N}, p_n|_k$ summability factors of Fourier series.

Theorem 3 *([22]). Let (X_n) be an almost increasing sequence. If $\phi_1(t)\in\mathcal{BV}(0,\pi)$ and the sequences (p_n), (λ_n), and (X_n) satisfy the conditions of Theorem 1, then the series $\sum C_n(x)\lambda_n$ is summable $|\bar{N}, p_n|_k$, $k\geq 1$.*

Now, we generalize Theorem 3 for $|A, p_n|_k$ summability method in the following form.

Theorem 4. *Let (X_n) be an almost increasing sequence, and A be a positive normal matrix as in Theorem 2. If $\phi_1(t)\in\mathcal{BV}(0,\pi)$, and the sequences (p_n), (λ_n), and (X_n)*

satisfy the conditions of Theorem 2, then the series $\sum C_n(x)\lambda_n$ *is summable* $|A, p_n|_k$*,* $k \geq 1$*.*

Applications

1. If we take $a_{nv} = \frac{p_v}{P_n}$ in Theorem 4, then we have Theorem 3 for the factored trigonometric Fourier series.
2. If we take $p_n = 1$ for all values of n in Theorem 4, then we have a new result on $|A|_k$ summability method for the factored trigonometric Fourier series.
3. If we take $a_{nv} = \frac{p_v}{P_n}$ and $p_n = 1$ for all values of n in Theorem 4, then we have a new result concerning $|C, 1|_k$ summability method for the factored trigonometric Fourier series.
4. If we take $a_{nv} = \frac{p_v}{P_n}$ and $k = 1$ in Theorem 4, then we have another new result for $|\bar{N}, p_n|$ summability method for the factored trigonometric Fourier series.

Declaration of competing interest

The author declares that she has no competing interests.

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