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# On the Absolute Matrix Summability Factors of Fourier Series*

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**Abstract**—In this paper, two known theorems on $\left|\bar{N}, p_n\right|_k$ summability methods of Fourier series have been generalized for $|A, p_n|_k$ summability factors of Fourier series by using different matrix transformations. New results have been obtained dealing with some other summability methods.

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## 1. INTRODUCTION

Let $\sum a_n$ be a given infinite series with the partial sums $(s_n)$, and let $(p_n)$ be a sequence of positive numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty \qquad \text{as} \quad n \to \infty \quad (P_{-i} = p_{-i} = 0, \quad i \geq 1). \tag{1.1}$$

The sequence-to-sequence transformation

$$w_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v \tag{1.2}$$

defines the sequence $(w_n)$ of the Riesz mean or simply the $(\bar{N}, p_n)$ mean of the sequence $(s_n)$ generated by the sequence of coefficients $(p_n)$ (see [15]).

The series $\sum a_n$ is said to be summable $\left|\bar{N}, p_n\right|_k$, $k \geq 1$, if (see [1])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} \mid w_n - w_{n-1} \mid^k < \infty. \tag{1.3}$$

In the special case when $p_n = 1$ for all values of $n$ (resp. $k = 1$), $\left|\bar{N}, p_n\right|_k$ summability is the same as $|C, 1|_k$ (resp. $\left|\bar{N}, p_n\right|$) summability.

Let $A = (a_{nv})$ be a normal matrix. i.e., a lower triangular matrix with nonzero diagonal entries. Given a normal matrix $A = (a_{nv})$, we associate two lower semimatrices $\bar{A} = (\bar{a}_{nv})$ and $\hat{A} = (\hat{a}_{nv})$ as follows:

$$\bar{a}_{nv} = \sum_{i=v}^{n} a_{ni}, \qquad n, v = 0, 1, \ldots \tag{1.4}$$

$$\hat{a}_{00} = \bar{a}_{00} = a_{00}, \qquad \hat{a}_{nv} = \bar{a}_{nv} - \bar{a}_{n-1,v}, \quad n = 1, 2, \ldots \tag{1.5}$$

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In this case, $A$ defines the sequence-to-sequence transformation, mapping the sequence $s=(s_n)$ to $As=(A_n(s))$, where

$$A_n(s)=\sum_{v=0}^{n} a_{nv}s_v, \qquad n=0,1,\dots. \tag{1.6}$$

It may be noted that $\bar{A}$ and $\hat{A}$ are the well-known matrices of series-to-sequence and series-to-series transformations, respectively. Then we have

$$A_n(s)=\sum_{v=0}^{n} a_{nv}s_v=\sum_{v=0}^{n} a_{nv}\sum_{i=0}^{v} a_i=\sum_{i=0}^{n} a_i\sum_{v=i}^{n} a_{nv}=\sum_{i=0}^{n} a_i\bar{a}_{ni}=\sum_{v=0}^{n} \bar{a}_{nv}a_v. \tag{1.7}$$

Since $\bar{a}_{n-1,n}=0$, we have

$$\begin{aligned}\bar{\Delta}A_n(s)=A_n(s)-A_{n-1}(s)&=\sum_{v=0}^{n}\bar{a}_{nv}a_v-\sum_{v=0}^{n-1}\bar{a}_{n-1,v}a_v\\ &=\sum_{v=0}^{n}(\bar{a}_{nv}-\bar{a}_{n-1,v})a_v+\bar{a}_{n-1,n}a_n=\sum_{v=0}^{n}\hat{a}_{nv}a_v.\end{aligned} \tag{1.8}$$

The series $\sum a_n$ is said to be summable $|A,p_n|_k$, $k\ge 1$, if (see [19])

$$\sum_{n=1}^{\infty}\left(\frac{P_n}{p_n}\right)^{k-1}\left|\bar{\Delta}A_n(s)\right|^k<\infty, \tag{1.9}$$

where $\Delta A_n(s)=A_n(s)-A_{n+1}(s)$ and $\bar{\Delta}A_n(s)=A_n(s)-A_{n-1}(s)$.

If we take $p_n=1$ for all $n$, $|A,p_n|_k$ summability is the same as $|A|_k$ summability (see [20]) and if we take $a_{nv}=p_v/P_n$, then $|A,p_n|_k$ summability is the same as $\left|\bar{N},p_n\right|_k$ summability. Also, if we take $a_{nv}=p_v/P_n$ and $p_n=1$ for all $n$, then $|A,p_n|_k$ summability is the same as $|C,1|_k$ summability (see [16]).

Let $f$ be a $2\pi$-periodic function integrable $(L)$ over $(-\pi,\pi)$. Without any loss of generality, we may assume that the constant term in the Fourier series of $f(t)$ is zero, so that

$$\int_{-\pi}^{\pi} f(t)\,dt=0, \qquad f(t)\sim\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt)=\sum_{n=1}^{\infty}C_n(t). \tag{1.10}$$

We write

$$\varphi(t)=\frac{1}{2}\{f(x+t)+f(x-t)\}\sim\sum_{n=1}^{\infty}C_n(x)\cos nt, \qquad \varphi_1(t)=\frac{1}{t}\int_0^t\varphi(u)\,du\sim\sum_{n=1}^{\infty}C_n(x)\frac{\sin nt}{nt}. \tag{1.11}$$

## 2. KNOWN RESULTS

Many works dealing with absolute summability methods of infinite and Fourier series have been published (see [2]–[14]). Among them, in [2], two important theorems have been proved in the following forms.

**Theorem 1.** *Let* $(p_n)$ *be a sequence such that*

$$P_n=O(np_n), \tag{2.1}$$

$$P_n\Delta p_n=O(p_np_{n+1}). \tag{2.2}$$

*If $\varphi_1(t)$ is of bounded variation in $(0,\pi)$ for any $x \in (-\pi,\pi)$ and $(\lambda_n)$ is a sequence such that*

$$\sum_{n=1}^{\infty} \frac{1}{n}|\lambda_n|^k < \infty, \tag{2.3}$$

$$\sum_{n=1}^{\infty} |\Delta\lambda_n| < \infty, \tag{2.4}$$

*then the series $\sum C_n(t)P_n\lambda_n/np_n$ is summable $\left|\bar{N},p_n\right|_k$, $k \geq 1$.*

**Theorem 2.** *If the sequences $(p_n)$ and $(\lambda_n)$ satisfy conditions (2.1)–(2.4) of Theorem 1 and*

$$B_n \equiv \sum_{v=1}^{n} va_v = O(n), \qquad n \to \infty, \tag{2.5}$$

*then the series $\sum a_nP_n\lambda_n/np_n$ is summable $\left|\bar{N},p_n\right|_k$, $k \geq 1$.*

## 3. MAIN RESULTS

Many studies have been carried out in order to obtain matrix generalizations of Fourier series (see [18], [21]). The aim of this paper is to generalize Theorem 1 and Theorem 2, under suitable and different conditions, by using general summability factors for $|A,p_n|_k$ summability methods dealing with Fourier series.

**Theorem 3.** *If $A = (a_{nv})$ is a positive normal matrix such that*

$$\overline{a}_{no} = 1, \qquad n = 0,1,\dots, \tag{3.1}$$

$$a_{n-1,v} \geq a_{nv} \qquad \text{for} \quad n \geq v+1, \tag{3.2}$$

$$a_{nn} = O\left(\frac{p_n}{P_n}\right), \tag{3.3}$$

$$\hat{a}_{n,v+1} = O(v|\Delta_v(\hat{a}_{nv})|), \tag{3.4}$$

*and all the conditions of Theorem 1 are satisfied, then the series $\sum C_n(t)P_n\lambda_n/np_n$ is summable $|A,p_n|_k$, $k \geq 1$.*

**Theorem 4.** *If conditions (2.1)–(2.5) of Theorems 1 and 2 and also conditions (3.1)–(3.4) of Theorem 3 are satisfied, then the series $\sum a_nP_n\lambda_n/np_n$ is summable $|A,p_n|_k$, $k \geq 1$.*

**Remark 1.** It should be noted that if we take $a_{nv} = p_v/P_n$ in Theorem 3 and Theorem 4, then we obtain Theorem 1 and Theorem 2, respectively.

We need the following lemmas for the proof of our theorems.

**Lemma 1.** *If $\varphi_1(t)$ is of bounded variation in $(0,\pi)$ for any $x \in (-\pi,\pi)$, then*

$$\sum vC_v(x) = O(n) \qquad \text{as} \quad n \to \infty, \tag{3.5}$$

(*see* [17]).

**Lemma 2.** *If the sequence $(p_n)$ satisfies conditions (2.1) and (2.2) of Theorem 1, then*

$$\Delta\left\{\frac{P_n}{p_nn^2}\right\} = O\left(\frac{1}{n^2}\right), \tag{3.6}$$

(*see* [2]).

## 4. PROOF OF THEOREM 4

**Proof.** Let $(T_n)$ denote the A-transform of the series $\sum a_n P_n \lambda_n (np_n)^{-1}$. Then, by (1.7) and (1.8),

$$T_n = \sum_{v=0}^{n} \bar{a}_{nv} a_v P_v \lambda_v (vp_v)^{-1}, \qquad \bar{\Delta} T_n = T_n - T_{n-1} = \sum_{v=0}^{n} \hat{a}_{nv} a_v P_v \lambda_v (vp_v)^{-1}$$

and since $\hat{a}_{n0} = \bar{a}_{n0} - \bar{a}_{n-1,0} = 0$, we have

$$\bar{\Delta} T_n = \sum_{v=1}^{n} \hat{a}_{nv} a_v P_v \lambda_v (vp_v)^{-1}.$$

Applying Abel's transformation to this sum and the relation

$$B_n \equiv \sum_{v=1}^{n} v a_v = O(n),$$

which holds by condition (2.5), we obtain

$$\begin{aligned}
\bar{\Delta} T_n &= \sum_{v=1}^{n} \hat{a}_{nv} a_v P_v \lambda_v (vp_v)^{-1} = \sum_{v=1}^{n-1} \Delta_v \left( \frac{\hat{a}_{nv} P_v \lambda_v}{v^2 p_v} \right) \sum_{r=1}^{v} r a_r + \frac{\hat{a}_{nn} P_n \lambda_n}{n^2 p_n} \sum_{r=1}^{n} r a_r \\
&= \left\{ \sum_{v=1}^{n-1} \frac{\Delta_v (\hat{a}_{nv}) P_v \lambda_v}{v^2 p_v} + \sum_{v=1}^{n-1} \frac{\hat{a}_{n,v+1} P_v}{v^2 p_v} \Delta \lambda_v + \sum_{v=1}^{n-1} \hat{a}_{n,v+1} \lambda_{v+1} \Delta_v \left( \frac{P_v}{v^2 p_v} \right) \right\} \sum_{r=1}^{v} r a_r \\
&\quad + \frac{\hat{a}_{nn} P_n \lambda_n}{n^2 p_n} \sum_{r=1}^{n} r a_r \\
&= \frac{\hat{a}_{nn} P_n \lambda_n}{n^2 p_n} B_n + \sum_{v=1}^{n-1} \frac{\Delta_v (\hat{a}_{nv}) P_v \lambda_v}{v^2 p_v} B_v + \sum_{v=1}^{n-1} \hat{a}_{n,v+1} \lambda_{v+1} \Delta_v \left( \frac{P_v}{v^2 p_v} \right) B_v + \sum_{v=1}^{n-1} \frac{\hat{a}_{n,v+1} P_v}{v^2 p_v} \Delta \lambda_v B_v \\
&= T_{n,1} + T_{n,2} + T_{n,3} + T_{n,4}.
\end{aligned}$$

To complete the proof of Theorem 4, by Minkowski's inequality, it is sufficient to show that

$$\sum_{n=1}^{\infty} \left( \frac{P_n}{p_n} \right)^{k-1} |T_{n,r}|^k < \infty, \qquad \text{for} \quad r = 1, 2, 3, 4. \tag{4.1}$$

First, since $\hat{a}_{nn} = a_{nn}$, and $a_{nn} = O(p_n/P_n)$, we have

$$\begin{aligned}
\sum_{n=1}^{m} \left( \frac{P_n}{p_n} \right)^{k-1} |T_{n,1}|^k &= \sum_{n=1}^{m} \left( \frac{P_n}{p_n} \right)^{k-1} \left| \frac{a_{nn} \lambda_n P_n}{n^2 p_n} B_n \right|^k \\
&= O(1) \sum_{n=1}^{m} \left( \frac{P_n}{p_n} \right)^{k-1} |\lambda_n|^k |B_n|^k \frac{1}{n^{2k}} = O(1) \sum_{n=1}^{m} \left( \frac{P_n}{p_n} \right)^{k-1} \frac{|\lambda_n|^k}{n^k} \\
&= O(1) \sum_{n=1}^{m} \frac{|\lambda_n|^k}{n} = O(1) \qquad \text{as} \quad m \to \infty,
\end{aligned}$$

by condition (2.3) and by virtue of the assumptions of Theorem 4. Now, applying Hölder's inequality, we obtain

$$\begin{aligned}
\sum_{n=2}^{m+1} \left( \frac{P_n}{p_n} \right)^{k-1} |T_{n,2}|^k &= \sum_{n=2}^{m+1} \left( \frac{P_n}{p_n} \right)^{k-1} \left| \sum_{v=1}^{n-1} \Delta_v (\hat{a}_{nv}) \frac{P_v \lambda_v}{v^2 p_v} B_v \right|^k \\
&\leq \sum_{n=2}^{m+1} \left( \frac{P_n}{p_n} \right)^{k-1} \left\{ \sum_{v=1}^{n-1} |\Delta_v (\hat{a}_{nv})| \left( \frac{P_v}{v^2 p_v} \right)^k |\lambda_v|^k |B_v|^k \right\} \left\{ \sum_{v=1}^{n-1} |\Delta_v (\hat{a}_{nv})| \right\}^{k-1}.
\end{aligned}$$

By the definitions of $\bar{A}$ and $\hat{A}$ matrices of series-to-sequence and series-to-series transformations,

$$\begin{aligned}\Delta_v(\hat{a}_{nv}) &= \hat{a}_{nv} - \hat{a}_{n,v+1} = \bar{a}_{nv} - \bar{a}_{n-1,v} - \bar{a}_{n,v+1} + \bar{a}_{n-1,v+1} \\ &= \sum_{i=v}^{n} a_{ni} - \sum_{i=v}^{n-1} a_{n-1,i} - \sum_{i=v+1}^{n} a_{ni} + \sum_{i=v+1}^{n-1} a_{n-1,i} = a_{nv} - a_{n-1,v}, \qquad (4.2)\end{aligned}$$

and since $a_{n-1,v} \geq a_{nv}$, and $\bar{a}_{n0} = 1$, we have

$$\sum_{v=1}^{n-1} |\Delta_v(\hat{a}_{nv})| = \sum_{v=1}^{n-1} (a_{n-1,v} - a_{nv}) = 1 - a_{n-1,0} - 1 + a_{n0} + a_{nn} \leq a_{nn}. \qquad (4.3)$$

By using conditions (4.2), (4.3) and the assumptions of Theorem 4, we can write

$$\begin{aligned}\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} |T_{n,2}|^k &= O(1) \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} a_{nn}^{k-1} \sum_{v=1}^{n-1} |\Delta_v(\hat{a}_{nv})| \left(\frac{P_v}{v^2 p_v}\right)^k |\lambda_v|^k |B_v|^k \\ &= \sum_{v=1}^{m} \left(\frac{P_v}{v^2 p_v}\right)^k |\lambda_v|^k |B_v|^k \sum_{n=v+1}^{m+1} |\Delta_v(\hat{a}_{nv})|.\end{aligned}$$

Using the equality $\Delta_v(\hat{a}_{nv}) = a_{nv} - a_{n-1,v}$ and condition (3.2), we obtain

$$\sum_{n=v+1}^{m+1} |\Delta_v(\hat{a}_{nv})| = \sum_{n=v+1}^{m+1} (a_{n-1,v} - a_{nv}) = a_{vv} - a_{m+1,v} \leq a_{vv}, \qquad (4.4)$$

$$\begin{aligned}\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} |T_{n,2}|^k &= O(1) \sum_{v=1}^{m} \left(\frac{P_v}{v^2 p_v}\right)^k |\lambda_v|^k |B_v|^k \frac{p_v}{P_v} \\ &= O(1) \sum_{v=1}^{m} \left(\frac{P_v}{p_v}\right)^{k-1} |\lambda_v|^k \frac{1}{v^k} = O(1) \sum_{v=1}^{m} \frac{|\lambda_v|^k}{v} = O(1) \qquad \text{as} \quad m \to \infty.\end{aligned}$$

On the other hand, since $\Delta\left\{P_v/v^2 p_v\right\} = O\left(1/v^2\right)$ by Lemma 2, we have

$$\begin{aligned}\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} |T_{n,3}|^k &= \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} \left|\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}| |\lambda_{v+1}| \Delta_v \left(\frac{P_v}{v^2 p_v}\right) B_v\right|^k \\ &\leq \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} \left\{\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}| |\lambda_{v+1}|^k \frac{1}{v}\right\} \left\{\sum_{v=1}^{n-1} \frac{|\hat{a}_{n,v+1}|}{v}\right\}^{k-1}.\end{aligned}$$

Using (3.4) and (4.3), we obtain

$$\begin{aligned}\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} |T_{n,3}|^k &= O(1) \sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} \left\{\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}| |\lambda_{v+1}|^k \frac{1}{v}\right\} \times \left\{\sum_{v=1}^{n-1} |\Delta_v(\hat{a}_{nv})|\right\}^{k-1} \\ &= O(1) \sum_{v=1}^{m} |\lambda_{v+1}|^k \frac{1}{v} \sum_{n=v+1}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} a_{nn}^{k-1} |\hat{a}_{n,v+1}| \\ &= O(1) \sum_{v=1}^{m} |\lambda_{v+1}|^k \frac{1}{v} \sum_{n=v+1}^{m+1} |\hat{a}_{n,v+1}|.\end{aligned}$$

Since

$$\hat{a}_{n,v+1} = \sum_{i=v+1}^{n} a_{ni} - \sum_{i=v+1}^{n-1} a_{n-1,i} = \sum_{i=0}^{v} (a_{n-1,i} - a_{ni}) \geq 0,$$

it follows that

$$\sum_{n=v+1}^{m+1}|\hat{a}_{n,v+1}|=\sum_{n=v+1}^{m+1}\sum_{i=0}^{v}(a_{n-1,i}-a_{ni})=\sum_{i=0}^{v}\sum_{n=v+1}^{m+1}(a_{n-1,i}-a_{ni})$$
$$=\sum_{i=0}^{v}(a_{vi}-a_{m+1,i})\leq\sum_{i=0}^{v}a_{vi}=1 \qquad (4.5)$$

and, by virtue of the assumptions of Theorem 4 and Lemma 2, we can write

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}|T_{n,3}|^k=O(1)\sum_{v=1}^{m}|\lambda_{v+1}|^k\frac{1}{v+1}\left(1+\frac{1}{v}\right)=O(1) \qquad \text{as} \quad m\to\infty.$$

Finally, we also have

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}|T_{n,4}|^k=\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\left|\sum_{v=1}^{n-1}\frac{\hat{a}_{n,v+1}P_v}{v^2p_v}\Delta\lambda_vB_v\right|^k$$
$$=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\left|\sum_{v=1}^{n-1}\frac{\hat{a}_{n,v+1}}{v}\Delta\lambda_vB_v\right|^k$$
$$=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\Delta\lambda_v|\frac{|B_v|^k}{v^k}\right\}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\Delta\lambda_v|\right\}^{k-1};$$

taking into account Eqs. (3.1) and (3.2), for $1\leq v\leq n-1$, we obtain

$$\hat{a}_{n,v+1}=\sum_{i=0}^{v}(a_{n-1,i}-a_{ni})\leq\sum_{i=0}^{n-1}(a_{n-1,i}-a_{ni})=\bar{a}_{n-1,0}-\bar{a}_{n,0}+a_{nn}=a_{nn},$$

which implies

$$\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\Delta\lambda_v|\leq a_{nn}\sum_{v=1}^{n-1}|\Delta\lambda_v|=O(a_{nn})$$

by condition (2.4). Therefore,

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}|T_{n,4}|^k=O(1)\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}a_{nn}^{k-1}\left\{\sum_{v=1}^{n-1}|\hat{a}_{n,v+1}||\Delta\lambda_v|\frac{|B_v|^k}{v^k}\right\},$$

as in $T_{n,3}$, and we have

$$\sum_{n=2}^{m+1}\left(\frac{P_n}{p_n}\right)^{k-1}|T_{n,4}|^k=O(1)\sum_{v=1}^{m}|\Delta\lambda_v|\frac{|B_v|^k}{v^k}\sum_{n=v+1}^{m+1}|\hat{a}_{n,v+1}|$$
$$=O(1)\sum_{v=1}^{m}|\Delta\lambda_v|=O(1) \qquad \text{as} \quad m\to\infty.$$

This completes the proof of Theorem 4. □

**Proof of Theorem 3.** Theorem 3 is a direct consequence of Theorem 4 and Lemma 1.

## 5. CONCLUSIONS

First, if we take $p_n=1$ for all values of $n$ in Theorem 3 and Theorem 4, then we obtain a result concerning $|A|_k$ summability. And if we take $a_{nv}=p_v/P_n$ in Theorem 3 and Theorem 4, then we obtain Theorem 1 and Theorem 2, respectively. Finally, if we take $a_{nv}=p_v/P_n$ and $p_n=1$ for all values of $n$ in Theorem 3 and Theorem 4, then we obtain a result for $|C,1|_k$ summability.

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