

# Electronic and phonon properties of the full-Heusler alloys $X_2YAl$ ( $X = Co, Fe$ and $Y = Cr, Sc$ ): a density functional theory study

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**Abstract** First-principle calculations have been carried out on the structural, electronic, elastic, and phonon properties of the full-Heusler alloys  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ). The calculations predict that the  $Fe_2CrAl$  and  $Co_2CrAl$  are half-metallic ferromagnets at the equilibrium lattice constant with a minority-spin energy gap of 0.2912 and 0.668 eV, respectively.  $Fe_2ScAl$  exhibit a gap in the majority density of states, with a few states at the Fermi level and about 0.217 states  $eV^{-1}$ , unlike the other Heusler compounds; due to this, it is considered a false half metal, and  $Co_2ScAl$  is considered a non-magnetic compound. The elastic constants were derived from the slopes of the acoustic branches in the phonon-dispersion curve. The calculated lattice constants, bulk modulus, and first-order pressure derivative of the bulk modulus are reported for the  $L2_1$  structure and compared with previous values. Phonon-dispersion curves were obtained using the first-principle linear-response approach of the density-functional perturbation theory. The specific heat capacity at a

constant volume  $C_V$  of  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) alloys is calculated and discussed.

## Introduction

Heusler alloys have been of interest since their discovery in 1903 by Friedrich Heusler. It has been reported that it was possible to make ferromagnetic alloys from non-ferromagnetic constituents [1]. These ternary alloys are of the form  $X_2YZ$ , where  $Y$  and  $X$  are usually transition metals, and  $Z$  is a main group element with s, p valence electrons (Al, In, Sb, Si, Ge, and Sn). The majority of these alloys form an  $L2_1$  crystal structure with a face-centered cubic lattice (fcc). Further investigations indicated that the magnetic properties of these alloys are in relation with their chemical,  $L2_1$  structure, and the ordering of the  $Y$  atoms on an fcc sublattice.

Recently, half-metallic ferromagnetic (HMFs) have attracted attention, as they showed great potential for spintronics, owing to tunneling magnetoresistance (TMR) and giant magnetoresistance (GMR) [2], and electro-mechanical [3] applications. Ever since De Groot et al., many more materials have been obtained both theoretically and experimentally to have HMFs properties. Apart from Heusler and half-Heusler alloys [4], half-metallic behavior has been found in perovskite structures [5], dilute magnetic semiconductors [6], some oxides [7], and materials possessing a zinc-blend structure [8, 9]. Many of the Heusler alloys of the  $X_2YZ$  stoichiometric composition are good candidates for devices based on a spin injection such as the GMR and the huge TMR in magneto electronic devices. They can also be used as perfect spin filters and spin-injection devices as an alternative to ferromagnetic 3d metals.

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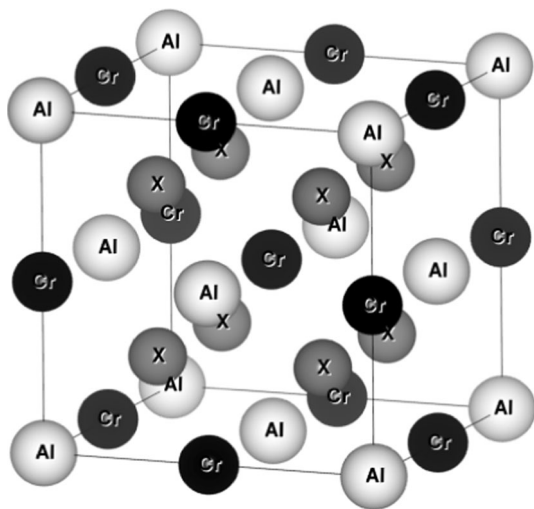
Despite the fact that about one hundred of full-Heusler alloys have been synthesized and investigated till date, the majority of all possible chemical compositions still waits for this realization. Iron- and cobalt-based  $\text{Fe}_2\text{CrAl}$  and  $\text{Co}_2\text{CrAl}$  alloys have been extensively studied using different theoretical and experimental methods [10–35].  $\text{Fe}_2\text{CrAl}$  and  $\text{Co}_2\text{CrAl}$  alloys are more much suitable for wide technological applications because of their high Curie temperature, low coercivities, high-spin polarization, low saturation magnetization, and crystal structure compatibility. The electronic band structure, magnetism and transport properties, and formation energies of the Heusler  $\text{X}_2\text{YAl}$  ( $\text{X} = \text{Fe}, \text{Co}, \text{Ni}$ ;  $\text{Y} = \text{Ti}, \text{Cr}$ ) alloys using the full potential linearized augmented plane-wave (FP-LAPW) method have been studied [1, 17]. It has been shown that Cr sites carry large magnetic moments, and the moments at the Fe and Co sites are usually small, when compared with Ti substitution. All the densities of states are marked by a pseudogap that is to the left of the Fermi level, except for  $\text{Fe}_2\text{AlTi}$ , where the pseudogap is to the right of  $E_F$ . Among the selected materials, the  $\text{Fe}_2\text{AlCr}$  and  $\text{Co}_2\text{AlCr}$  alloys present a pronounced half metallicity character. The experimental results of the optical, electrical, magnetic properties, electronic structure, and the calculated optical spectrum of  $\text{Fe}_2\text{TiAl}$ ,  $\text{Fe}_2\text{VAl}$ , and  $\text{Fe}_2\text{CrAl}$  alloy have been also presented by Shreder et al. [12]. They found a drastic transformation of the band spectrum, especially near the Fermi level, when one Y element ( $\text{Y} = \text{Ti}, \text{V}, \text{and Cr}$ ) replaces another. These changes are closely related to the magnetic properties of the  $\text{Fe}_2\text{MeAl}$  alloys and accompanied by a drastic change in the electrical and optical properties of these materials. The structural and magnetic properties of the  $\text{Fe}_2\text{CrAl}$  alloy have been measured using Mössbauer spectroscopy by Paduani et al. [14] and Lakshmi et al. [15], and using X-ray powder diffraction (XRD), and conducting quantum interface device (SQUID) magnetometer and a vibrating sample magnetometer (VSM) by Umetsu et al. [16]. The Mossbauer spectra showed the co-existence of a paramagnetic part with a magnetic hyperfine portion at all recorded temperatures. The confirmed crystal structures of the  $\text{Fe}_2\text{CrAl}$  alloy were single phases of A2, B2, and  $L_{21}$ , depending on the quenched temperature. The values of the spontaneous magnetic moment, ( $I_S$ ), and the curie temperature ( $T_C$ ) of the A2 phase were  $2.2 \mu_B/\text{f.u.}$  and 316 K, and those of the B2 phase were  $2.1 \mu_B/\text{f.u.}$  and 265 K, respectively, which were larger than those of the  $L_{21}$ -type phase of  $1.6 \mu_B/\text{f.u.}$  and 191 K.

For the  $\text{Co}_2\text{CrAl}$  alloy, electronic and magnetic properties were studied by several groups [18–22]. It is found to be perfectly half-metallic (HM), and a transition pressure of 75 GPa is obtained from HM to metal. The substitution of Cr for Co leads to a high-antibonding peak above  $E_F$  in

the majority-spin band, while the energy gap in the minority-spin band is retained. With an increase in Cr content, the total spin moment decreases linearly from  $3 \mu_B/\text{f.u.}$  to zero, which obeys the Slater–Pauling curve quite well. The structural, microstructural, and magnetic properties of the  $\text{Co}_2\text{CrAl}$  thin film were investigated by Dubowik et al. [23] using selective-area microdiffraction of a transmission microscope (TEM) and a SQUID magnetometer. Their measurements show that the films with the best  $B2/L_{21}$  structure exhibit a ferromagnetic order below the curie temperature  $T_C = 330\text{--}340$  K and below 200 K, they exhibited magnetic characteristics suggesting the presence of antiferromagnetic ordering. The calculated electronic band structure and transport properties of the  $\text{Co}_2\text{CrAl}$  alloy, with a full-Heusler ( $L_{21}$ ) structure, were studied and it is suggested to be a half-metallic ferromagnet (HMF) with a magnetic moment of  $3 \mu_B$  per formula unit and a spin-flip gap of 0.181 eV [24, 25, 27].

It is very interesting to have a look at those compounds that are missing from the series of  $\text{X}_2\text{ScAl}$  Heusler compounds. Neither the compound  $\text{Co}_2\text{ScAl}$  nor  $\text{Fe}_2\text{ScAl}$  is reported in any experiment. Electronic band structure calculations for  $\text{X}_2\text{YZ}$  Heusler compounds are presented [30]. Among the interesting aspects of the electronic structure of the materials are the contributions from both X and Y atoms to the total magnetic moment. It is found that the total magnetic moment also depends on the kind of Z atoms, although they do not directly contribute to it.  $\text{Co}_2\text{ScAl}$  has 24 valence electrons, and the magnetic moment vanishes at all sites; however, in  $\text{Co}_2\text{ScSi}$ , the  $\text{Co}_2$  sub-lattice starts ordering ferromagnetically, whereas the Sc atoms do not contribute to the magnetic moment. In addition, the magnetic moment of Co is changing with the exchange of Al by Si. In order to find the stable magnetic configuration and the optimized lattice constant of  $\text{Co}_2\text{YAl}$  ( $\text{Y} = \text{Sc}, \text{Ti}, \text{V}, \text{Cr}, \text{Mn}, \text{and Fe}$ ) Heusler alloys, Rai et al. [36] performed the total-energy calculations of  $\text{Co}_2\text{YAl}$  ( $\text{Y} = \text{Sc}, \text{Ti}, \text{V}, \text{Cr}, \text{Mn}, \text{and Fe}$ ) Heusler alloys based on the generalized gradient approximation (GGA) within the FP-LAPW method. A study of the thermo-chemical stability and magnetism of 810 possible full-Heusler alloys using a rigorous computational search based on the density-functional theory have been performed [37]. It is found that some of them are candidates for itinerant ferromagnetism with 100 % spin polarization, and about 56 % of the calculated phases appear to be ferromagnetic with a magnetic moment larger than  $0.5 \mu_B$ . Although the saturation moments may reach a maximum value of approximately  $7.5 \mu_B$ , a value of  $1.91 \mu_B$  for the magnetic moment of  $\text{Fe}_2\text{ScAl}$  alloys is found.

Although considerable progress has been made in theoretically describing the structural and electronic properties of  $\text{Fe}_2\text{CrAl}$  and  $\text{Co}_2\text{CrAl}$  alloys, many of their dynamical



**Fig. 1** Crystal structure of the full-Heusler alloys  $X_2YZ$

properties are still not well established. To the best of our knowledge, the structural, elastic, electronic, and phonon properties of  $Co_2ScAl$  and  $Fe_2ScAl$  in the  $L2_1$  phase have not been studied using the density-functional theory. The full phonon-dispersion curves are necessary for a microscopic understanding of the lattice dynamics. Knowledge of the phonon spectrum plays a significant role in determining many material properties such as phase transition, thermodynamic stability, transport and thermal properties, and electron–phonon interactions. Thus, the aim of this work is to investigate the structural, elastic, and electronic properties with interchanging elements at the X and Y positions of  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) Heusler alloys by employing the plane-wave pseudopotential method. These results are used, within a linear-response approach, to calculate the phonon-dispersion curves and the density of states (DOS). This article is organized as follows. In Sec. II, we describe our calculational methods, and Sec. III is devoted to the discussion of the results of our calculations for the structural and vibrational properties of  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) Heusler alloys, including a comparison with previous available works. The electronic structure of these alloys is examined. A conclusion is provided in Sec. IV.

### Structure and method of calculation

Most of the full-Heusler alloys  $X_2YZ$  often crystallize in the cubic  $L2_1$  structure (space group  $Fm-3m$ ). Generally, X and Y atoms are transition metals, and Z is a main group element. In some cases, Y is replaced by a rare earth element. The X atoms are placed on 8a ( $1/4, 1/4, 1/4$ ), and the Y and Z atoms are placed on 4a ( $0, 0, 0$ ) and 4b ( $1/2, 1/2,$

$1/2$ ) Wyckoff positions, respectively. The cubic  $L2_1$  structure consists of four inter-penetrating fcc sub-lattices, two of which are equally occupied by X. The two X fcc sub-lattices combine and form a simple cubic sub-lattice. The Y and Z atoms occupy, alternately the center of the simple cubic  $X_2$  sub-lattice, resulting in a CsCl-type super structure (see Fig. 1).

The calculations were performed using a plane-wave pseudopotential scheme within the density-functional theory (DFT) as implemented in the Quantum-ESPRESSO package [38]. The electronic exchange–correlation potential was calculated by the GGA as parametrized by Perdew–Burke–Ernzerhof (PBE) [39]. The wave functions were expanded in a plane-wave basis set with a kinetic energy cut-off of 40 Ry. Brillouin-zone integrations were performed using a  $10 \times 10 \times 10$  k-point mesh. Integration up to the Fermi surface was performed using the smearing technique [40] with a smearing parameter  $\sigma = 0.02$  Ry. Having obtained self-consistent solutions of Kohn–Sham equations, the lattice-dynamical properties were calculated within the framework of the self-consistent density-functional perturbation theory [41, 42]. To find complete phonon dispersions and DOS, eight dynamical matrices were calculated on a  $4 \times 4 \times 4$  q-point mesh. The dynamical matrices at arbitrary wave vectors were evaluated using Fourier deconvolution on this mesh. Specific heat at constant volume is calculated at various temperatures from the phonon frequencies obtained through the quasi-harmonic Debye model (QHA) [43].

Elastic constants were obtained from the slopes of the acoustic branches in the phonon-dispersion curves [44]. The sound velocities correspond to the small-wavelength behavior of the acoustic phonons. These velocities are associated with  $C_{11}$ ,  $C_{44}$ , and  $C_{12}$  by [44]

$$v_{LA}^{[001]} = \sqrt{C_{11}/\rho} \quad (1)$$

$$v_{TA}^{[001]} = \sqrt{C_{44}/\rho} \quad (2)$$

$$v_{LA}^{[110]} = \sqrt{(C_{11} + C_{12} + 2C_{44})C_{11}/\rho}, \quad (3)$$

where  $\rho$  is the mass density of the  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) Heusler alloy.

### Results and discussion

Ground-state properties and determination of elastic constants

Spin-polarized total energies were calculated as a function of lattice constant for  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) Heusler alloys and were fitted to the Murnaghan's equation

**Table 1** Calculated lattice constant  $a$  (Å), bulk modulus  $B$  (GPa), the total magnetic moment  $M_t$  ( $\mu_B$ ), and elastic constants  $C_{ij}$  (GPa), for  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) full-Heusler alloys, compared with the available experimental and theoretical data

	$a$	$M_t$	$B$	$C_{11}$	$C_{12}$	$C_{44}$	$B/G$
<b>Fe<sub>2</sub>CrAl</b>							
This work	5.65	1.01	177.46	229.56	151.41	275.94	0.979
CASTEP (GGA) [10]	–	–	201.08	289.75	156.75	153.07	1.886
CASTEP (LDA) [10]	–	–	177.09	258.24	134.78	139.14	1.785
Exp. [12]	5.811						
Exp. [13]	5.800						
Exp. [27]	5.800						
Exp. [15]	5.807						
Exp. [14]	5.811	1.51					
Exp. [16]	5.789						
FP-LAPW nm [17]	5.654	–	225.00				
FP-LAPW fm [17]	5.654	–	229.00				
FSKKR [33]	–	0.91					
<b>Co<sub>2</sub>CrAl</b>							
This work	5.71	3.00	328.40	355.82	314.69	417.86	1.268
FP-LAPW nm [17]	5.693	–	208.00				
FP-LAPW fm [17]	5.714	–	204.00				
FSKKR [33]	–	2.95					
KKR [34]	–	2.99					
Exp. [18]	5.5726						
Exp. [30]	5.727	1.55					
FLAPW [19]	5.70	3.00	207.23				
FLAPW [30]		3.00					
Exp. [26]	5.75						
<b>Fe<sub>2</sub>ScAl</b>							
This work	6.012	1.92	127.40	280.14	51.03	120.03	1.081
[50]	6.006	1.91					
<b>Co<sub>2</sub>ScAl</b>							
This work	5.961	0	168.23	180.16	162.26	150.86	1.787
[36]	6.320						
[30]	5.960						
[50]	5.957	0					

of state [45] to obtain the basic ground-state properties. It was also well-proved that the ferromagnetic configuration is lower in energy than the non-magnetic case for the studied compounds except for the Co<sub>2</sub>ScAl alloy, which is found to be a non-magnetic compound. Measurements of the temperature and field dependence of the magnetization and the  $ac$  susceptibility show that Fe<sub>2</sub>CrAl is a ferromagnet with  $T_C = 246$  K [12]. The calculated lattice constants and bulk modulus of  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) alloys are presented in Table 1. Comparing the experiments and theoretical data [10, 12–18, 26, 27, 30, 33, 34], the maximum deviation in the lattice constant is 2 % for Fe<sub>2</sub>CrAl, and the minimum deviation is around 0.3 % for the Co<sub>2</sub>CrAl alloy. The optimized lattice parameters were in excellent agreement with other theoretical studies [30, 36] but less those that found in [36] for Co<sub>2</sub>ScAl. It is observed that the lattice constant decreases

by about 0.8 %, when the Fe atom is replaced by the Co atom in  $X_2ScAl$  alloys and increases by 1 % in  $X_2CrAl$  alloys. It should be mentioned that the bulk modulus of Co<sub>2</sub>CrAl is the largest and is about 328.40 GPa; as a result, it is predicted to be the hardest one in the studied compounds. On the other hand, the bulk modulus increases by 85 % when Fe is replaced by Co in  $X_2CrAl$  alloys and by –32 % in  $X_2ScAl$  alloys. However, when we replace Cr by Sc in Co<sub>2</sub>YAl, it decreases by 49 %.

Our calculations show that Fe<sub>2</sub>CrAl and Co<sub>2</sub>CrAl have total magnetic moments of 1.01 and 3  $\mu_B$  per formula unit, respectively, which is equal to an integer number of  $\mu_B$  expected for an HMF. This is in excellent agreement with the Slater–Pauling rule and is confirmed by several studies [17, 24, 25, 27, 30, 46, 47]. It is found that Co<sub>2</sub>CrAl has local magnetic moments of about 0.7–0.85  $\mu_B$  on Co and in a range of 1.44–1.54  $\mu_B$  on Cr. The exchange interactions

between the Co(Fe) and Cr atoms are found to be ferromagnetic, leading to a total calculated moment of 3.0 (1.0)  $\mu_B$  [11, 31]. In general, the Y atoms, and in some cases, also the X atoms carry the magnetic moments. However, in the compounds that contain Cr atoms, it has been shown that Cr sites carry large magnetic moments, and the moments at the Al sites are usually small [17]. The direction of the spin polarization of Al is antiparallel to the others. The replacement of Fe by Co in the studied alloys leads to a substantial increase in the magnetic moment from 1 to 3  $\mu_B$ . In the present study, the  $\text{Co}_2\text{ScAl}$  Heusler alloy does not show any magnetic moment. This is explained by the fact that  $\text{Co}_2\text{ScAl}$  has 24 valence electrons, and the magnetic moment vanishes at all sites. The result is a vanishing total magnetic moment as expected from the Slater–Pauling rule. It has also been found that the total magnetic moment depends on the type of Z element, although they do not directly contribute to it. In  $\text{Co}_2$  compounds, the replacement of Al by Si or an exchange between other members of the 3A and 4A groups plays an important role in the distribution of electrons in the various symmetry distinguished states ( $t_{2g}$  and  $e_g$ ) at Co as well as at B sites [30]. However, the other compound  $\text{Fe}_2\text{ScAl}$  has a moderate magnetic moment of about 1.92  $\mu_B$  per formula unit, which is far less than the predicted one of 2  $\mu_B$  using the Slater–Pauling rule. This is an indication that  $\text{Fe}_2\text{ScAl}$  is not a perfectly half-metallic compound, because the total spin moment is not an integer number. We find that the total magnetic moment per formula unit increases when we move from  $\text{Co}_2\text{ScAl}$  to  $\text{Fe}_2\text{ScAl}$ .

Elastic constants are known as numbers that quantify the response of a particular material to elastic or non-elastic deformation when a stress load is applied to that material. The elastic constants of solids provide valuable information on their dynamical and mechanical properties. In particular, they provide information on the stiffness and stability of materials. Various experimental techniques are available for the measurement of elastic constants such as neutron scattering, ultrasonic wave propagation, and Brillouin scattering.

An efficient way of calculating elastic constants is important, because these constants are directly employed in practical uses of materials. Calculation of the elastic constants at given temperatures can also serve as a measure of the reliability of the interatomic potential at those temperatures. The elastic constants of the full-Heusler alloys  $\text{X}_2\text{YAl}$  ( $\text{X} = \text{Co}, \text{Fe}$  and  $\text{Y} = \text{Cr}, \text{Sc}$ ) are calculated using the approximation reported in [48]. They are listed in Table 1, along with the data available from other calculations for a comparison [10]. Our calculated elastic constants  $C_{12}$  and  $C_{11}$  of  $\text{Fe}_2\text{CrAl}$  are in excellent agreements with those calculated with the same method incorporated in Castep code [10]. Unfortunately, the experimental and

theoretical values of the elastic constants for  $\text{Co}_2\text{CrAl}$ ,  $\text{Co}_2\text{ScAl}$ , and  $\text{Fe}_2\text{ScAl}$  materials are not available in the literature. Thus, further experimental studies are needed to compare with these computed results for  $\text{Co}_2\text{CrAl}$ ,  $\text{Co}_2\text{ScAl}$ , and  $\text{Fe}_2\text{ScAl}$ . In addition, it should be mentioned that the elastic constants increase as we replace Fe with Co in these Heusler alloys except for  $C_{11}$  which decreases when we progress from  $\text{Fe}_2\text{ScAl}$  to  $\text{Co}_2\text{ScAl}$ . The mechanical stability of these compounds has been analyzed in terms of their elastic constants. For cubic crystals, the conditions for mechanical stability are given by [49]

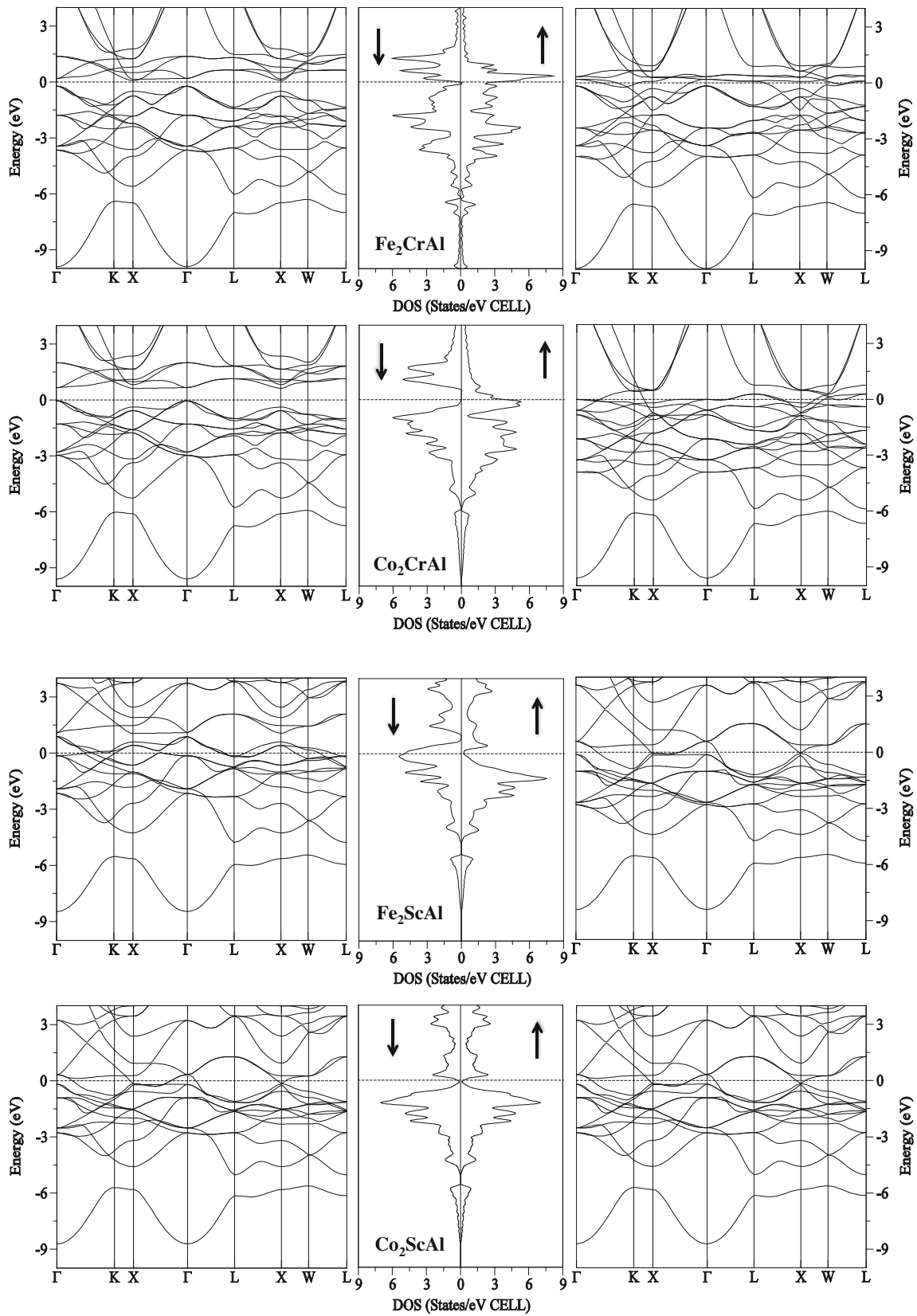
$$C_{44} > 0, (C_{11} - C_{12})/2 > 0 \text{ and} \\ B = (C_{11} + 2C_{12})/3 > 0.$$

These criteria are verified (see Table 1), so it is concluded that all compounds are stable in the  $L2_1$  phase. According to the ratio  $B/G$ ,  $\text{Fe}_2\text{CrAl}$ ,  $\text{Co}_2\text{CrAl}$ , and  $\text{Fe}_2\text{ScAl}$  are brittle, whereas  $\text{Co}_2\text{ScAl}$  is ductile.

#### Density of states and electronic band structure

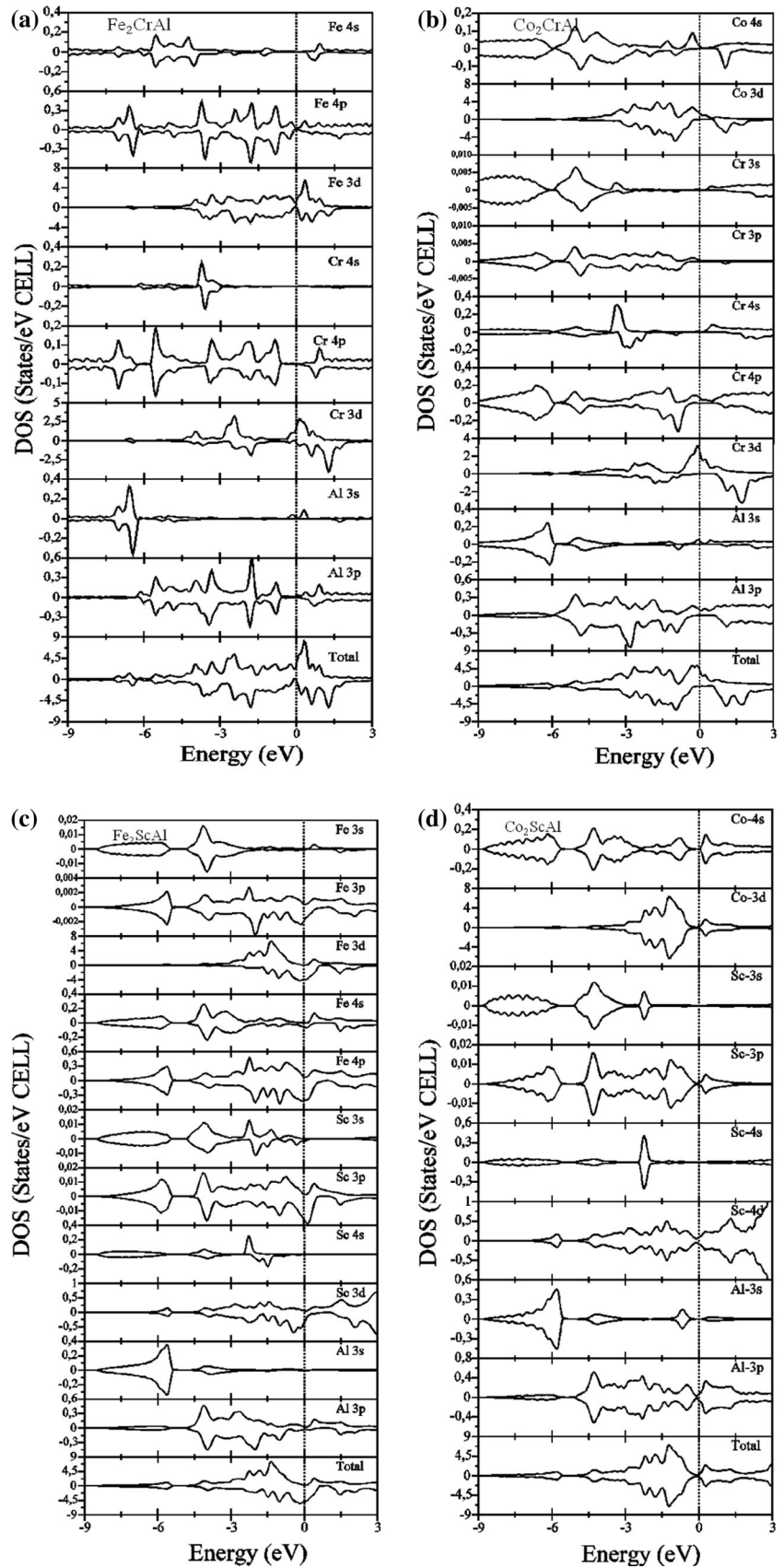
We present the spin-dependent energy bands of  $\text{X}_2\text{YAl}$  ( $\text{X} = \text{Co}, \text{Fe}$  and  $\text{Y} = \text{Cr}, \text{Sc}$ ) along high-symmetry directions in the Brillouin zone, as presented in Fig. 2. The Fermi level  $E_F$  shown by a broken horizontal line intersects the  $E(k)$  curves in the majority (up)-spin state but is located at the energy gap in the minority (down)-spin state for  $\text{Fe}_2\text{CrAl}$  and  $\text{Co}_2\text{CrAl}$ . It is obvious that the majority-spin electrons are metallic, whereas there is an energy gap of about 0.2912 and 0.668 eV around the Fermi level in the bands for the minority-spin electrons for  $\text{Fe}_2\text{CrAl}$  and  $\text{Co}_2\text{CrAl}$  alloys, respectively. This suggests that  $\text{Fe}_2\text{CrAl}$  and  $\text{Co}_2\text{CrAl}$  are half metals (HM), and electrons at the Fermi level are 100 % spin polarized. The spin-flip gaps are 0.2188 and 0.0424 eV for  $\text{Fe}_2\text{CrAl}$  and  $\text{Co}_2\text{CrAl}$  alloys, respectively. The non-zero spin-flip gap implies that  $\text{Fe}_2\text{CrAl}$  and  $\text{Co}_2\text{CrAl}$  are true HMF. The size of the gap increases if Fe is replaced by Co; however, the spin-flip gap decreases. We find that  $\text{Fe}_2\text{ScAl}$  exhibit a gap in the majority DOS, with a few states at the Fermi level being about 0.217 states  $\text{eV}^{-1}$  unlike the other Heusler compounds. The reason is that  $\text{Fe}_2\text{ScAl}$  has only 22 valence electrons, that is, less than 24; therefore, we can consider it a false half metal. In addition, on replacing Fe by Co, in the compound  $\text{X}_2\text{ScAl}$ ,  $\text{Co}_2\text{ScAl}$  becomes metallic.

The band gap in the minority bands basically arises from the covalent hybridization between the d states of the Co(Fe) and Cr atoms, leading to the formation of bonding and antibonding bands with a gap in between. This covalent hybridization is the reason for the formation of bonding and antibonding states and determines the position of  $E_F$ . The majority DOS at  $E_F$  is lower in  $\text{Fe}_2\text{CrAl}$  (2.98791 states  $\text{eV}^{-1}$ ) as compared with the  $\text{Co}_2\text{CrAl}$



**Fig. 2** Calculated minority spin, majority spin-band structure, and total DOS for  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) Heusler alloys

**Fig. 3** The total and partial DOS of **a**  $\text{Fe}_2\text{CrAl}$  **b**  $\text{Co}_2\text{CrAl}$  **c**  $\text{Fe}_2\text{ScAl}$  **d**  $\text{Co}_2\text{ScAl}$  **e** and **f** total DOS of  $\text{X}_2\text{CrAl}$  and  $\text{X}_2\text{ScAl}$  ( $\text{X} = \text{Co}, \text{Fe}$ ) Heusler alloys



compound (4.166 states  $\text{eV}^{-1}$ ). The main characters of our results are similar with previous investigations of electronic structure [10–12, 18–22].

In order to further study the electronic structure of these alloys, in Fig. 3a, b, we present the spin-projected partial DOS of  $\text{Fe}_2\text{CrAl}$  and  $\text{Co}_2\text{CrAl}$ . The lowest valence bands in the energy region that is lower than  $-6$  eV in both the majority and minority spin states are mainly due to the “s” electrons of the Al atom. In addition, the band structure is almost identical for both spin directions. The low-energy part around  $-5$  eV mainly consists of the p states of Al, which hybridize with the p and d states of the Cr and Fe(Co) atoms, and the energy region between  $-3$  and  $2$  eV mainly forms the d electrons of the Fe(Co) and Cr atoms. The states of the 3d atoms extend from about  $-5$  to  $+2$  eV and hybridize with each other. The d states are wide on an energy scale, which may result from the strong hybridization between the Fe(Co) and Cr atoms. The upper dispersed bands are due to the strong hybridization of Cr d and Co d electrons, including a contribution from Al p states in the occupied valence states. The detailed analysis shows important changes of DOS (see Fig. 3e, f) in the vicinity of the Fermi level while moving from  $\text{Fe}_2\text{CrAl}$  to  $\text{Co}_2\text{CrAl}$  and in the minority states of  $\text{X}_2\text{ScAl}$  alloys. The addition of an extra electron, by the change from Fe to Co in  $\text{X}_2\text{CrAl}$ , again leads to a shift of the Fermi level to lower energies, but it is still in the gap for the spin minority DOS and to a partial occupation of the conduction band. The character of the band states for  $\text{Co}_2\text{ScAl}$  has been identified by calculating its total and partial DOS (in Fig. 3d). It is shown that there is no gap at the Fermi level and the total DOS, which is  $0.329$  states  $\text{eV}^{-1}$  cell for  $\text{Co}_2\text{ScAl}$ , indicates that the system is metallic and non-magnetic. The results indicate that the predominant contributions of the DOS at the Fermi level come from the Co  $3d$ , Sc  $4d$ , and Al  $3p$  states for  $\text{Co}_2\text{ScAl}$ . In addition, from the calculated total DOS of  $\text{Co}_2\text{ScAl}$  alloys, it can be seen that there are some peaks below and above the Fermi level. Those peaks are mainly dominated by the Co  $3d$  state. The majority DOS at  $E_F$  in  $\text{Fe}_2\text{ScAl}$  is ( $0.217$  states  $\text{eV}^{-1}$ ), and the minority one is ( $4.962$  eV states  $\text{eV}^{-1}$ ); the main contribution around the Fermi level comes from the d-Fe states.

### Phonon properties and specific heats

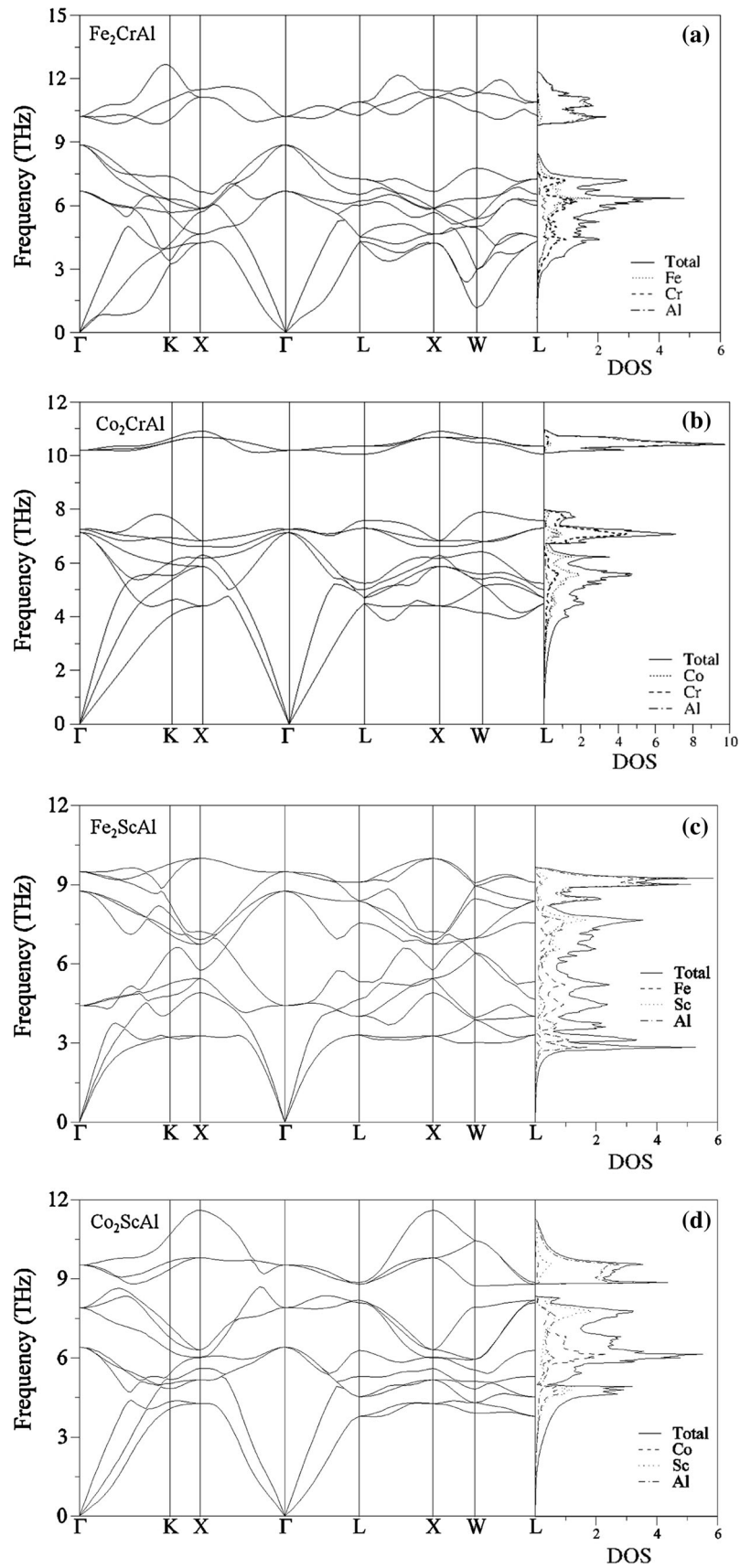
Phonon-dispersion curves calculated for the  $\text{L}_{21}$  phase Heusler alloys  $\text{X}_2\text{YAl}$  ( $\text{X} = \text{Co}, \text{Fe}$  and  $\text{Y} = \text{Cr}, \text{Sc}$ ) are shown in Fig. 4. Since primitive cells of  $\text{X}_2\text{YAl}$  ( $\text{X} = \text{Co}, \text{Fe}$  and  $\text{Y} = \text{Cr}, \text{Sc}$ ) contain four atoms, the corresponding number of vibration modes is twelve, as seen in Fig. 4, of which three are acoustic branches and the remaining nine are optical modes. We have not observed any phonon anomaly in the phonon DOS for  $\text{Fe}_2\text{ScAl}$  and  $\text{Co}_2\text{ScAl}$

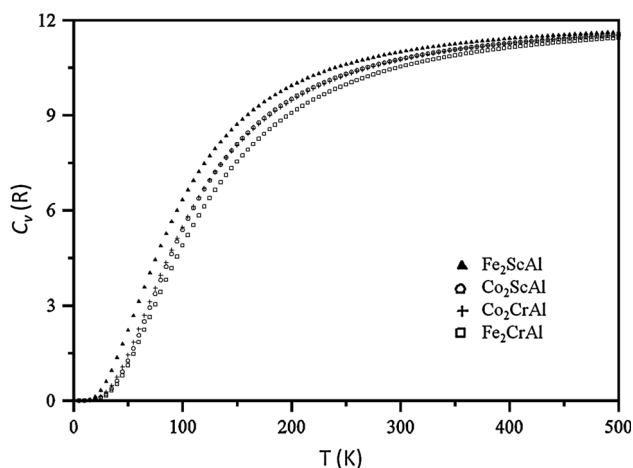
alloys. We do not find any imaginary phonon frequency in the whole Brillouin zone for  $\text{X}_2\text{YAl}$  ( $\text{X} = \text{Co}, \text{Fe}$  and  $\text{Y} = \text{Cr}, \text{Sc}$ ) alloys. This supports the dynamical stability of  $\text{X}_2\text{YAl}$  ( $\text{X} = \text{Co}, \text{Fe}$  and  $\text{Y} = \text{Cr}, \text{Sc}$ ) in the Heusler structure. The contribution of the heavier atoms to the total DOS is dominated at low frequencies, and that of the lighter atoms is important at high frequencies (see Fig. 4, right panel). The three highest optical phonon modes of these materials have a major amplitude contribution to Al atoms. At the  $\Gamma$  point, there are threefold degenerate optical modes, as there are four atoms in the unit cell. For  $\text{Fe}_2\text{CrAl}$ ,  $\text{Co}_2\text{CrAl}$ , and  $\text{Co}_2\text{ScAl}$ , gaps exist between optical and optical phonon branches. In the phonon DOS, the band gap between the optical and optical branches of  $\text{Fe}_2\text{CrAl}$ ,  $\text{Co}_2\text{CrAl}$ , and  $\text{Co}_2\text{ScAl}$  is around  $0.959$ ,  $2.128$ , and  $0.47$  THz, respectively. For  $\text{Fe}_2\text{ScAl}$ , there is no gap between the optical and optical phonon branches in Fig. 4, as there is a considerable overlap between the optical phonon branches. The optical mode frequencies at the  $\tau$  point are  $10.210$ ,  $8.872$ , and  $6.683$  THz for  $\text{Fe}_2\text{CrAl}$ ;  $10.192$ ,  $7.255$ , and  $7.141$  THz for  $\text{Co}_2\text{CrAl}$ ;  $6.407$ ,  $7.900$ , and  $9.526$  for  $\text{Co}_2\text{ScAl}$ ; and  $4.419$ ,  $8.752$ , and  $9.485$  THz for  $\text{Fe}_2\text{ScAl}$ , respectively. Due to lack of experimental results, the calculated phonon curves could not be compared. The phonon calculations of these materials will certainly be useful for the interpretations of future theoretical and experimental studies. We observed phonon anomalous along with  $\tau$ -K and  $\tau$ -L directions at  $\text{TA}_2$  (acoustic transverse) branches for  $\text{Co}_2\text{ScAl}$  and  $\text{Fe}_2\text{CrAl}$ . The origin of phonon anomalous in  $\text{Ni}_2\text{MnGa}$  has been investigated by first-principle calculations [51]. It has been found that the Raman-active optical modes lower their energy due to the negative force constants, by which the acoustic mode  $\text{TA}_2$  becomes unstable due to the repulsive interaction with the anomalous optical mode. This acoustic-optical interaction might be of more general interest than just for the Heusler compounds [51]. We can conclude that the anomalous for  $\text{Co}_2\text{ScAl}$  and  $\text{Fe}_2\text{CrAl}$  is connected with the acoustic–optical interaction.

The heat capacity,  $C$ , of a material is a fundamental state property of matter, as it measures the ability of a substance that can store heat energy. It is described by the ratio of the energy  $dE$  transferred to a substance to raise its temperature by an amount  $dT$ . The specific heat of a material is closely related to its vibrational and configurational entropy, which is mostly affected by nearest-neighbor configurations. First-principle calculations combined with the quasi-harmonic approximation (QHA) have been employed to predict the temperature dependence of the vibrational contributions to the specific heat capacity at a constant volume ( $C_V$ ) of  $\text{X}_2\text{YAl}$  ( $\text{X} = \text{Co}, \text{Fe}$  and  $\text{Y} = \text{Cr}, \text{Sc}$ ) alloys. The results are illustrated in Fig. 5. It can be seen that the rapid increase in the specific heat is a marked



**Fig. 4** Calculated phonon-dispersion curves and phonon DOS for **a** Fe<sub>2</sub>CrAl **b** Co<sub>2</sub>CrAl **c** Fe<sub>2</sub>ScAl **d** Co<sub>2</sub>ScAl Heusler alloys along several high-symmetry lines in the Brillouin zone





**Fig. 5** The temperature dependence of the specific heat capacity at constant volume ( $C_V$ ) of  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) Heusler alloys

feature of these alloys; when the temperature reaches about 1000 K,  $C_V$  becomes constant following the Dulong–Petit law. Unfortunately, no experimental data are available for the specific heat capacity for these materials. No effect is shown on  $C_V$  by exchanging Cr by the Sc atom in  $Co_2YAl$  alloys.

## Conclusion

The structural, electronic, magnetic, and phonon properties of Heusler alloys  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) have been investigated using the pseudopotential plane-wave method. The exchange–correlation potential is treated using the GGA. The main focus of this work is to study the structural, electronic, elastic, and phonon properties of  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) and to elaborate the changes brought about in the studied properties of  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) by replacing the Fe atom by Co atom and the Cr atom by the Sc atom. The half metallicity and the stability of the ferromagnetic state have been investigated. The band structure calculation shows that  $Co_2CrAl$  and  $Fe_2CrAl$  are true half-metallic ferromagnets with a magnetic moment of 3 and 1  $\mu_B$  per formula unit, respectively, which is characterized by an indirect band gap of about 0.2912 and 0.668 eV for  $Fe_2CrAl$  and  $Co_2CrAl$ , respectively, at around the Fermi level in the minority-spin band. However,  $Fe_2ScAl$  is found to be a false half metal with a false gap in the majority of the spin states. The calculated total spin magnetic moments of all the compounds are found to be in good agreement with other theoretical calculations and in good agreement with the Slater–Pauling rule. Spin-polarized electronic structure

calculations show that the moments are mainly of  $3d$  origin. The minority density of states is exactly zero at the Fermi energy, whereas the majority  $d$  density of states can have a peak or a valley close to  $E_F$ . The slope of acoustic phonon-dispersion curves was used to calculate elastic constants  $C_{ij}$ . Elastically, all the compounds have been found to be stable in the  $L2_1$  phase. The DOS of the minority-spin band as well as of the majority-spin band is affected by replacing Fe by Co. The main change is a small shift of the minority DOS to higher energies and the majority DOS to lower energies with regard to the Fermi level for  $X_2CrAl$  alloys. The phonon frequencies and phonon densities of states in the  $L2_1$  phase in several lines of high symmetry of the Brillouin zone were obtained and discussed using the density-functional perturbation theory. Finally, the specific heat capacity at constant volume  $C_V$  of  $X_2YAl$  ( $X = Co, Fe$  and  $Y = Cr, Sc$ ) alloys is calculated and discussed.

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