

# TEACHERS' DEFINITIONS OF SQUARE, RECTANGLE, TRAPEZOID, AND PARALLELOGRAM

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# Defining in Mathematics

- Defining is giving a name to a concept
- Definition establishes necessary and sufficient conditions for the concept.
- Previously not defined concepts should'nt be used in a definition of new concept
- The set of necessary and sufficient conditions must be minimal (Zazkis and Lekinin, 2008).

# Importance of Definitions in Mathematics

- Mathematical definitions have fundamental importance in the axiomatic structure that characterizes mathematics (Edwards and Ward, 2008)
- Definitions contain only necessary and sufficient conditions required to identify an example of the concept.
- Other critical attributes may be reasoned out from the definition.
- Non-critical attributes include the overall size of the figure (large or small) and orientation (horizontal base).
- In mathematics education “*defining activities*” holds an important position (Mariotti and Fischbein, 1997).

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# Van Hiele Model Of Geometric Thinking

- Students' geometrical thinking progresses through a hierarchy of five levels
  - At the first level, students use visual reasoning, considering the shape as a whole.
  - At the second level students begin to notice that different shapes have different attributes but the properties are not seen as being related.
  - At the third level, relationships between properties are perceived and definitions become meaningful.
  - According to van Hiele, the essence of geometry is understood at the third level.
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- Individuals who base their reasoning on non critical attributes may be operating at the second van Hiele level.
- Those who base their reasoning on critical attributes may at the very least be operating at the second van Hiele level. (Tsamir, Tirosh, & Levenson, 2008)

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# Method

- 36 practicing teachers
    - 12 teachers maths teacher teaching grades 6-8 (mathematics teachers)
    - 24 teachers class teacher teaching grades 1-5 (class teachers)
  - Teaching experience between 1 year and 27 years.
  - Write as many examples as possible for a definition of a square, rectangle, trapezoid, and parallelogram.
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# Analysis of Data: Zazkis and Leikin's framework

- **accessibility and correctness**,
  - logical structure of the mathematical statement and minimality
  - a set of conditions for the concept that is necessary and sufficient.
- **richness**,
  - whether the examples vary in type and structure,
  - whether they are situated in a particular context or drawn from a variety of contexts.
- **generality**

# Appropriate Definitions

- Necessary and Sufficient
  - Rigorous minimal
    - A square is a “rhombus with a right angle”
  - Rigorous barely-not-minimal
    - A square is a “a four sided polygon having all sides equal and all angles equal 90°”
  - Non-rigorous: considerably not-minimal, and/or improper terminology
    - A square is a “shape that has four sides of equal length with angles of between adjoining sides equal to 90°, giving a total interior angle measure of 360°”



# Inappropriate Definitions

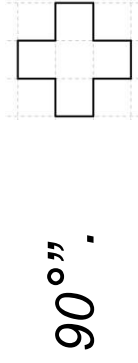
- Necessary but not sufficient

- Listing properties
- More general specific quadrilateral

*A square is a “ a quadrilateral with equal sides”. (rhombus)*

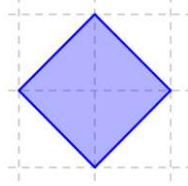
- Not necessarily a quadrilateral: improper original object

*A square is a “ a figure that looks the same every time it is rotated*



# Inappropriate Definitions

- Sufficient but not necessary



- Restriction of location, orientation or size

*A square has four vertices placed at  $(x,y)$   $(x+1,y)$   $(x+1,y+1)$*

- Not necessary and not sufficient

- Inappropriate shape or drawing

*Two parallel horizontal lines perpendicular to two parallel vertical lines*

- Irrelevant

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# Results

- The number of definitions for each concept provided by each participant varied between one and eight.
- While most of the class teachers provided one or two statements for each concept, majority of the maths teachers listed at least three statements for each concept under investigation.

	Appropriate	Inappropriate	Total
Square	52	69	121
Rectangle	36	51	87
Parallelogram	40	41	81
Trapezoid	27	40	67
Total	155	201	356

	Appropriate		Inappropriate	
	Class	Math	Class	Math
Square	17	35	48	21
Rectangle	11	25	33	18
Parallelogram	22	18	20	21
Trapezoid	14	13	28	12
Total	64	91	129	72

# Examples of Generated Definitions

- **not Geometrical or irrelevant**
- Square is a name given to four objects (Class, 18 years)
- Square is a number multiplied by itself (Class, 18 years).
- Trapezoid is a deformed shape. (Class, 18 years).
- Rectangle is the big brother of square (Class, 5 years)
- Trapezoid has no specific geometric shape (Class, 12)
- Square is a closed box with equal sides (Class, 12 years)

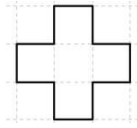
# Examples of Generated Definitions

- **Appropriate**
  - *Square is a quadrilateral with all the angles and all the sides equal (Math, 1 year )*
  - *Rectangle is quadrilateral with 4 right angle and equal opposite sides. (Class, 8 years)*
  - *Trapezoid is a quadrilateral with with one pair of parallel sides (Math, 3 years)*
  - *Parallelogram is a quadrilateral with paralell opposite sides (Class, 20 years)*

# Examples of Generated Definitions

## ■ Inappropriate

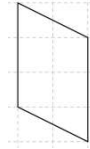
❑ *Square is a geometric shape with all the sides equal and sides intersecting each at right angles (Class, 4 years)*



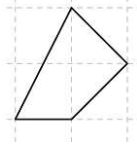
❑ *Rectangle is a geometric shape with two long and two short sides. (Class, 12 years)*



❑ *Rectangle is a nonregular polygon whose opposite sides are parallel, giving a total interior angle measure of 360°. (Class, 27 years)*




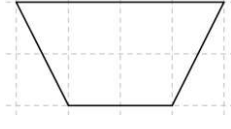
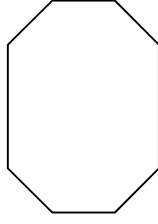
❑ *Trapezoid is a closed shape with four sides and four angles, measures of its angles and sides need not to obey any standard. (Class, 3 years)*





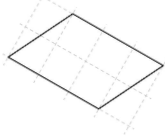
# Examples of Generated Definitions

- **Inappropriate**
  - Square is the most basic and necessary geometric shape of math. (Class, 27 years).
  - A flowerpot turned upside down can be given as an example for a trapezoid. (Class, 10 years). 
  - Trapezoid is a geometric shape in which lower and upper bases are parallel. (Class, 8 years).



# Examples of Generated Definitions

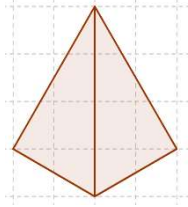
- **Inappropriate**
  - Parallelogram is a special case of a rectangle. (Maths, 1 year).
  - Parallelogram is a slanting rectangle /slapped rectangle. (Class, 16 years)
  - Parallelogram is a closed geometric figure which consists of two parallel lines and two vertical parallel lines intersecting them. (Class, 3 years)
  - Trapezoid is a closed shape with unequal sides/sides of different lengths (Class, 12 years).



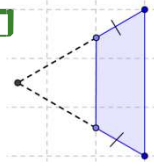
# Examples of Generated Definitions

## ■ Inappropriate

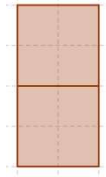
- Parallelogram is a geometric shape which is constructed by combining two congruent triangle. (Maths, 1 years).



- Trapezoid is a shape which is obtained by cutting a small equilateral triangle from one vertices of a larger equilateral triangle. (Class, 1 years). (*isosceles trapezoid*)



- Rectangle is a shape which can be constructed by combining two congruent square (Class, 9 years)



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# Examples of Generated Definitions

- Rich
  - Square is a rhombus with equal angles. (Maths, 2 years).
  - Square is a rectangle with four equal sides. (Maths. 1 year)
  - Rectangle is a parallelogram with four right angles. (Maths, 3 years).
  - A parallelogram is a trapezoid in which opposite sides are parallel. (Maths, 3 years)

# Examples of Generated Definitions

- Rich
  - All squares are rectangle (Maths, 3 years).
  - Square is a rectangle in which diagonals intersect at right angle and bisects each other (Maths, 1 year)
  - Square is a plane figure with equal diagonals (Class, 10)
  - Parallelogram is a quadrilateral with equal opposite sides and angles, and diagonal bisecting each other (Math, 2)

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# Conclusions

- One third of elementary school teachers were unable to generate an appropriate definition for each of the concepts of square, rectangle, trapezoid, and parallelogram.
  - Classroom teachers' definitions were more sympathetic but less mathematically appropriate.
  - Most of the class teachers were not aware that what a mathematical definition is.
  - In order to be able to guide students in defining activities, their teachers should have the ability to engage in this process themselves.
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# Conclusions

- Class teachers definitions' attended mostly non-critical attributes.
  - An individual's reference to non-critical attributes has an element of visual reasoning.
  - Overexposure to prototypes may impede the growth of fuller concept acquisition (Tsamir, Tirosh, & Levenson, 2008).
  - In teacher training programs, by exposing students to nonexamples with the same non-critical attributes, they may begin to differentiate between critical and non-critical attributes.
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- Teachers in grades 1-8, which deal with mostly level 1 and 2 thinking, should provide children with experiences that lay the ground work for thinking about shapes in terms of relationships between properties and definitions
- One of our major aims, as educators, is to bring our students to use only critical attributes as the deciding factor in identifying examples and forming geometrical concepts.



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# Thank you...

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