ISSN 0036-0244, Russian Journal of Physical Chemistry A, 2015, Vol. 89, No. 13, pp. 2426–2431. © Pleiades Publishing, Ltd., 2015.

STRUCTURE OF MATTER
AND QUANTUM CHEMISTRY

Determination of Critical Linear Lattice Size for the Four Dimensional Ising Model on the Creutz Cellular Automaton[1]

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Received January 26, 2015

Abstract—The four-dimensional Ising model is simulated on the Creutz cellular automaton (CCA) near the infinite-lattice critical temperature for the lattice with the linear dimension $4 \leqslant L \leqslant 22$. The temperature dependence of Binder parameter (g_L) are analyzed for the lattice with the linear dimension $4 \leqslant L \leqslant 22$. In this study conducted highly detailed, two different types of behavior were determined as a result of varying linear lattice dimension. The infinite lattice critical temperatures are obtained to be $T_c = 6.6845 \pm 0.0005$ in interval $4 \leqslant L \leqslant 12$ and $T_c = 6.6807 \pm 0.0024$ in interval $14 \leqslant L \leqslant 22$. The finite and infinite lattice critical exponents for the order parameter, the magnetic susceptibility and the specific heat are computed from the results of simulations by using finite-size scaling relations. Critical linear lattice size have been identified as $L = 14$.

Keywords: Ising model, cellular automaton, Binder parameter, finite-size scaling relations, magnetic phase transition.

DOI: 10.1134/S0036024415130221

INTRODUCTION

While the four-dimensional Ising model is not directly applicable to real magnetic systems, it is useful to investigate the influence of dimensionality on phase transitions [1]. The Creutz cellular automaton [2] has proven to be a fast alternative research tool in Ising model investigations near the critical regions of the lattices [3, 6]. As the dimension or the lattice size increases, the simulation of the Ising model by the conventional Monte Carlo method becomes impractical and faster algorithms are need. The Creutz cellular automaton does not require high-quality random numbers, it is an order of magnitude faster than the conventional Monte Carlo method, and compared to Q2R cellular automaton [4], it has the advantage of fluctuating internal energy from which the specific heat can be computed.

The purpose of this study is determined to critical linear lattice dimension.

EXPERIMENTAL

The four-dimensional Ising model with nearest-neighbor pair interactions is simulated on the Creutz cellular automaton near the infinite-lattice critical temperature for the lattice with the linear dimension $4 \leqslant L \leqslant 22$. Three variables are associated with each site of the lattice. The value of each site is determined from its value and those of its nearest-neighbors at the previous time step. The updating rule, which defines a deterministic cellular automaton, is as follows: of the three variables on each site, the first one is the Ising spin B_i. Its value may be 0, or 1, or 2. The Ising spin energy of the lattice for the model is given with $H_I = -J \sum_{\langle i,j \rangle} S_i S_j$, where $S_i = 2B_i - 1$, $\langle i,j \rangle$ denotes the sum over all nearest neighbor pairs of sites. The second variable is for the momentum variable conjugate to the spin (the demon). The kinetic energy associated with the demon, H_K, is an integer, which is equal to the change in the Ising spin energy for any spin flip. The total energy $E = H_I + H_K$, is conserved. The third variable provides a checkerboard style updating, and so it allows the simulation of the Ising model on a cellular automaton.

The simulations are carried out on simple hypercubic lattices L^4 of linear lattice dimensions $4 \leqslant L \leqslant 22$ with periodic boundary conditions. The cellular automaton develops 10^6 sweeps for each run with 1 run for each total energy. The simulations were studied for kT/J and E/J values in the interval $2 \leq kT/J \leq 8$ and $-2 \leq E/J \leq 6$, respectively. Differ from our pre-

[1] The article is published in the original.

Table 1. The critical exponent values of the order parameter and the magnetic susceptibility for $T_c\ (L)$ and $T_c\ (\infty)$ on each value of L

L	$\beta(T_c^\chi(L))$	$\beta(T_c^\chi(\infty))$	$\bar{\beta}(T_c^\chi(L))$	$\bar{\beta}(T_c\ (\infty))$
4	0.4745 ± 0.0007	0.4908 ± 0.0009	0.3594 ± 0.0025	0.3840 ± 0.0035
6	0.4839 ± 0.0001	0.4954 ± 0.0001	0.3723 ± 0.0012	0.3941 ± 0.0020
8	0.4861 ± 0.0038	0.4979 ± 0.0001	0.3823 ± 0.0007	0.3966 ± 0.0012
10	0.4886 ± 0.0001	0.4984 ± 0.0001	0.3854 ± 0.0014	0.3998 ± 0.0024
12	0.4888 ± 0.0008	0.5004 ± 0.0014	0.3882 ± 0.0017	0.3994 ± 0.0027
14	0.4924 ± 0.0001	0.4949 ± 0.0002	0.3680 ± 0.0001	0.3997 ± 0.0027
16	0.4935 ± 0.0001	0.4962 ± 0.0001	0.3704 ± 0.0001	0.4020 ± 0.0004
18	0.4954 ± 0.0001	0.4978 ± 0.0003	0.3731 ± 0.0002	0.4055 ± 0.0006
20	0.4967 ± 0.0001	0.4989 ± 0.0001	0.3743 ± 0.0006	0.4067 ± 0.0027
22	0.4976 ± 0.0001	0.4999 ± 0.0001	0.3769 ± 0.0004	0.4078 ± 0.0003

L	$\gamma(T_c^\chi(L))$	$\gamma(T_c^\chi(\infty))$	$\bar{\gamma}(T_c^\chi(L))$	$\bar{\gamma}(T_c\ (\infty))$
4	1.0610 ± 0.0238	1.1047 ± 0.0254	1.4163 ± 0.0095	1.4743 ± 0.0096
6	1.0446 ± 0.0004	1.0777 ± 0.0001	1.3403 ± 0.0046	1.4106 ± 0.0089
8	1.0368 ± 0.0002	1.0589 ± 0.0002	1.3198 ± 0.0065	1.3967 ± 0.0071
10	1.0215 ± 0.0005	1.0490 ± 0.0003	1.3193 ± 0.0013	1.3290 ± 0.0013
12	1.0129 ± 0.0004	1.0314 ± 0.0001	1.3011 ± 0.0014	1.3080 ± 0.0014
14	1.0531 ± 0.0013	1.0678 ± 0.0003	1.4254 ± 0.0158	1.4749 ± 0.0162
16	1.0468 ± 0.0004	1.0564 ± 0.0002	1.4237 ± 0.0036	1.4113 ± 0.0353
18	1.0400 ± 0.1762	1.0446 ± 0.1772	1.3987 ± 0.0008	1.4071 ± 0.0008
20	1.0369 ± 0.0708	1.0418 ± 0.0712	1.3402 ± 0.0125	1.3217 ± 0.0054
22	1.0322 ± 0.0019	1.0361 ± 0.0009	1.3238 ± 0.0462	1.3008 ± 0.0245

vious work [8], simulations were carried out by scanning more frequently the energy (E/J) interval.

RESULTS AND DISCUSSION

The temperature dependence of the functions for the magnetic susceptibility (χ), the specific heat (C), and order parameter (M) analyzed for the lattice with the linear dimension $4 \leqslant L \leqslant 22$ are illustrated in Fig. 1.

Differ from our previous work [8], the temperature dependence of Binder parameter (g_L) are analyzed for the lattice with the linear dimension $4 \leqslant L \leqslant 22$. The temperature variation of the Binder parameter has been shown in Fig. 2. The intersection point of the g_L curves gives the infinite lattices critical temperature T_c as $L \to \infty$ [5]. As shown in Figs. 2a and 2b, the g_L curves indicate two different behaviors in interval $4 \leqslant L \leqslant 22$. For $L = 4, 6, 8, 10, 12$ the intersection point of curves gives $T_c = 6.6845 \pm 0.0006$ (Fig. 2c) and for $L = 14, 16, 18, 20, 22$ the intersection point of curves gives $T_c = 6.6807 \pm 0.0024$ (Fig. 2d) approximately.

The finite-size lattice critical temperatures obtained from the susceptibility maxima $T_c^\chi(L)$ and

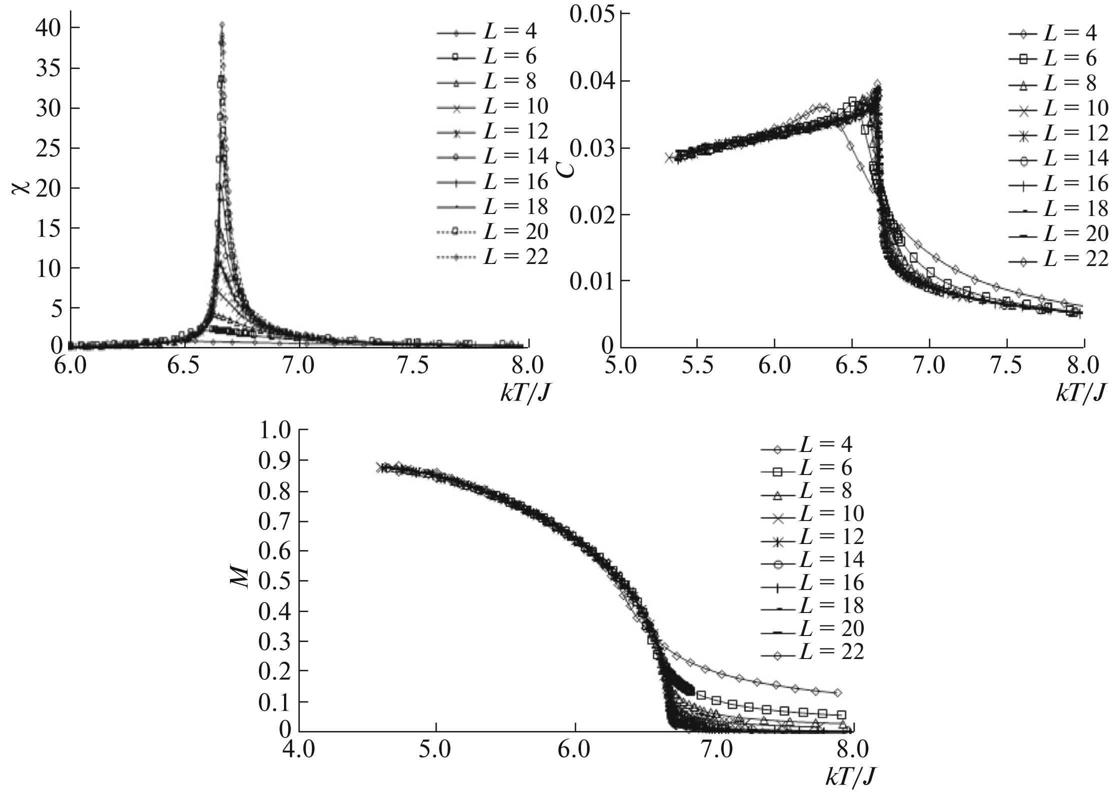

Fig. 1. The temperature dependence of the order parameter (M). the magnetic susceptibility (χ), and (C) the specific heat.

the specific heat maxima $T_c^C(L)$ are determined. The dependences of the critical temperatures $T_c^\chi(L)$ and $T_c^C(L)$ obtained from the magnetic susceptibility or the specific heat maxima are given with $T_c(\infty) - T_c(L) \alpha L^{-1/\nu} \log^{-1/6}(L)$. The finite-size scaling relation for $T_c^\chi(L)$ and $T_c^C(L)$ are used to get the critical temperatures of the infinite lattice in Fig. 3. As can be seen from the Fig. 3, by varying lattice linear dimension in range $4 \leqslant L \leqslant 22$ two different types of $T_c^\chi(L)$ and $T_c^C(L)$ behavior have been detected.

The computed values of the critical temperatures ($T_c(\infty)$) for the infinite lattice are $T_c(\infty) = 6.6807 \pm 0.0134$ ($4 \leqslant L \leqslant 12$), $T_c(\infty) = 6.6844 \pm 0.0339$ ($14 \leqslant L \leqslant 22$), for T_c^χ and $T_c(\infty) = 6.6810 \pm 0.0394$ ($4 \leqslant L \leqslant 12$), $T_c(\infty) = 6.6871 \pm 0.0255$ ($14 \leqslant L \leqslant 22$) for T_c^C.

These critical temperatures is in agreement with the results that obtained from the intersection point of the g_L curves.

The critical exponents [6–8] for the order parameter and the magnetic susceptibility are computed from

Table 2. The infinite lattice critical exponents values of the order parameter and the magnetic susceptibility for $T_c(L)$ and $T_c(\infty)$

L	$\beta(T_c^\chi(L))$	$\beta(T_c^\chi(\infty))$	$\bar{\beta}(T_c^\chi(L))$	$\bar{\beta}(T_c^\chi(\infty))$
$4 \le L \le 12$	0.474 ± 0.0007	0.4908 ± 0.0009	0.3594 ± 0.0025	0.3840 ± 0.0035
$14 \le L \le 22$	0.4839 ± 0.0001	0.4954 ± 0.0001	0.3723 ± 0.0012	0.3941 ± 0.0020
L	$\gamma(T_c^\chi(L))$	$\gamma(T_c^\chi(\infty))$	$\bar{\gamma}(T_c^\chi(L))$	$\bar{\gamma}(T_c^\chi(\infty))$
$4 \le L \le 12$	1.0610 ± 0.0238	1.1047 ± 0.0254	1.4163 ± 0.0095	1.4743 ± 0.0096
$14 \le L \le 22$	1.0446 ± 0.0004	1.0777 ± 0.0001	1.3403 ± 0.0046	1.4106 ± 0.0089

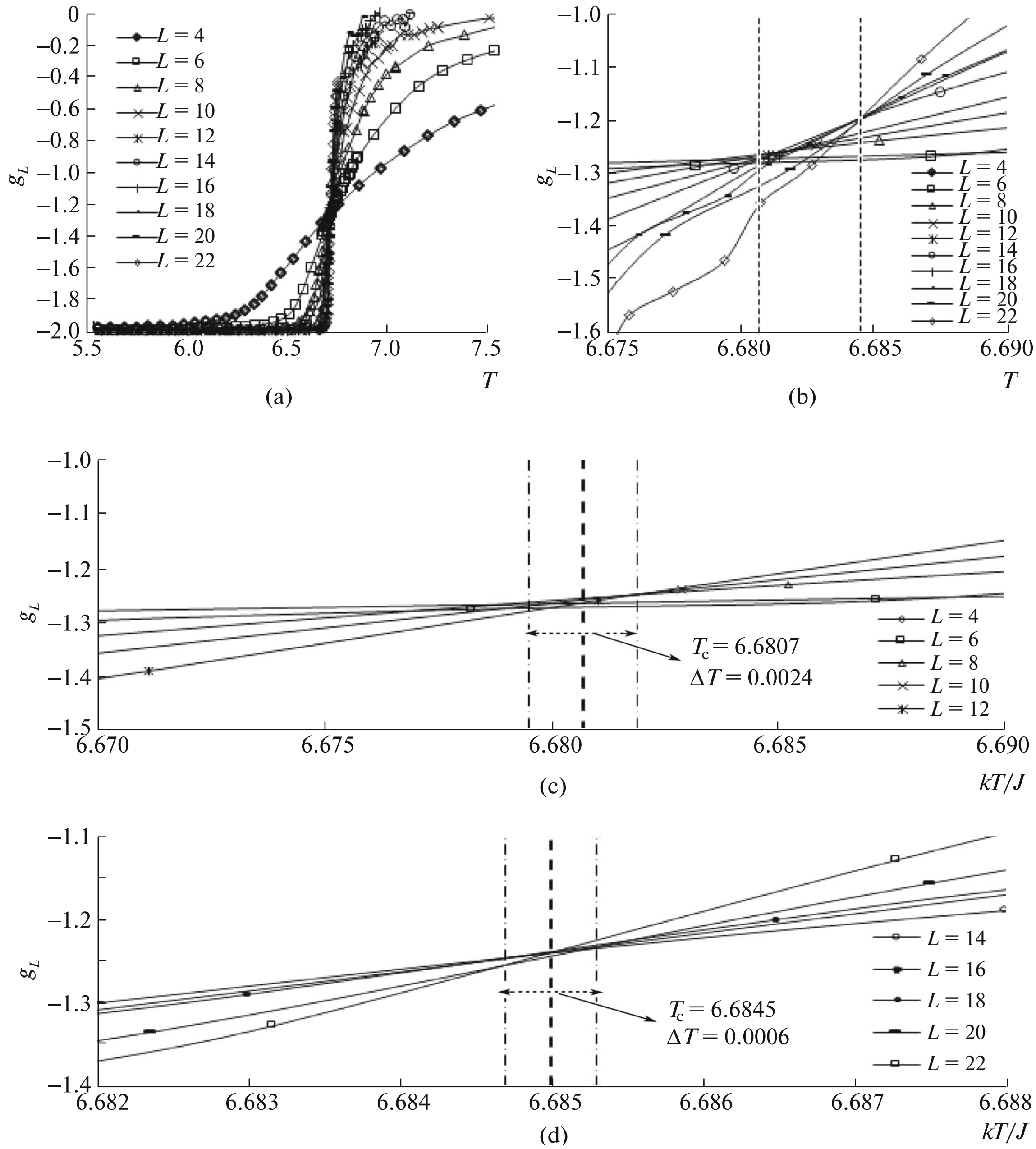

Fig. 2. (a) The Binder cumulant as a function of kT/J in interval $4 \le L \le 22$, (b) the figure on the right side is enlargement around the critical point, (c) the Binder cumulant as a function of kT/J in interval $4 \le L \le 12$, and (d) the Binder cumulant as a function of kT/J in interval $14 \le L \le 22$.

the results of simulations on each lattice for these $T_c\,(L)$ and $T_c\,(\infty)$ (Table 1).

These critical exponents are plotted against $1/L$. The data lie on straight lines, and their extrapolations to $1/L \to 0$ give the infinite lattice critical exponents (Table 2).

The critical exponents β/ν, γ/ν, and α/ν for the order parameter, the magnetic susceptibility and the specific heat are obtained from the results of simulations by using finite-size scaling relations [6] (Table 3).

As seen from Tables 2 and 3 the critical exponents and finite-size scaling functions show two different behaviors to be in interval $4 \leqslant L \leqslant 12$ and in interval $14 \leqslant L \leqslant 22$.

The temperature dependence of finite-size scaling functions [6–8] for magnetic susceptibility (χ), the specific heat (C) (Fig. 4), order parameter (M), and

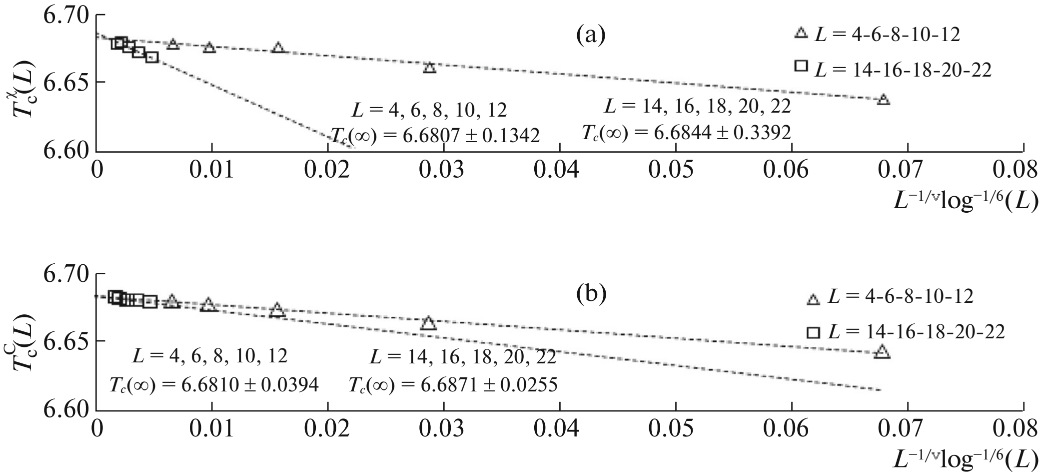

Fig. 3. (a) The plots of $T_c^{\chi}(L)$ against $L^{-1/\nu}\log^{-1/6}(L)$ with $\nu = 1/2$. (b) The plots of $T_c^{C}(L)$ against $L^{-1/\nu}\log^{-1/6}(L)$ with $\nu = 1/2$.

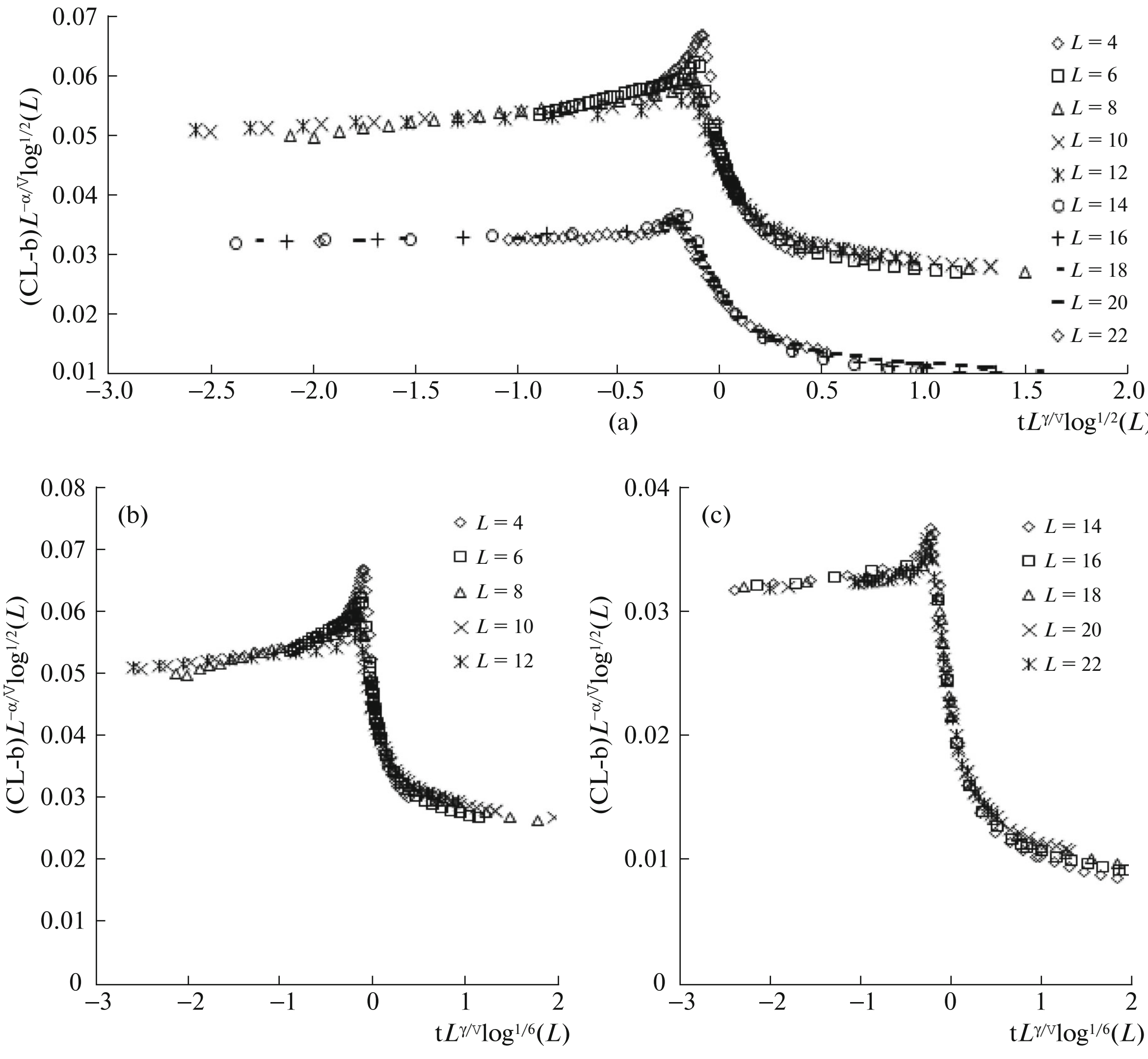

Fig. 4. The temperature dependence of finite-size scaling functions for the specific heat in interval $4 \le L \le 22$ (a), $14 \le L \le 22$ (b), $4 \le L \le 12$ (c).

Table 3. The values of critical exponents according to finite-size scaling functions for the four-dimensional Ising model

L	β/ν	γ/ν	α/ν
$4 \leq L \leq 12$	1.0025 ± 0.0001	1.9935 ± 0.0001	0.0577 ± 0.0011
$14 \leq L \leq 22$	1.0459 ± 0.0033	1.8751 ± 0.0003	0.0326 ± 0.0176

Binder parameter (g_L) show two different behaviors to be in interval $4 \leqslant L \leqslant 12$ and in interval $14 \leqslant L \leqslant 22$.

CONCLUSION

The Ising model in $d = 4$ dimension is simulated on the Creutz cellular automaton for finite-size lattices with the linear dimensions $4 \leqslant L \leqslant 22$. In this study conducted with great precision, two different types of behavior were determined as a result of varying linear lattice dimension. Critical size dimension have been identified as $L = 14$. The average value of these two different behaviors is in agreement with the results of other studies [7, 8].

ACKNOWLEDGMENTS

This study was supported by the Ahi Evran University under Scientific Support, project no. PYO-Fen.4010.14.009.

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