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## ON THE $F$ -CONTRACTION PROPERTIES OF MULTIVALUED INTEGRAL TYPE TRANSFORMATIONS

DERYA SEKMAN AND VATAN KARAKAYA

**ABSTRACT.** The main purpose of this work is to extend the properties of multivalued transformations to the integral type transformations and to obtain the existence of fixed points under  $F$ -contraction. In addition, the results of this study were evaluated with some interesting example.

### 1. INTRODUCTION AND PRELIMINARIES

In the development process of fixed point theory, every new problem has contributed to the expansion of this theory and it has been possible to incorporate new fields and new concepts into it. In this sense, the existence of fixed points of multivalued transformations is an important research area. These studies, which started with Nadler [10], leded many new ideas. In this way, many authors including Gordji [6], Berinde [4], Sekman et al. [12] the others studied and generalized set-valued mappings. The fact that the transformation classes have the contraction condition, first given by Banach [3], has an important role in the existence and uniqueness of the fixed point. Quiet recently, Wardowski [14] introduced the concept of  $F$ -contraction and examined the properties of this transformation for single valued transformations. Later, many authors have done a lot of work using this idea. One can find in list [1, 7, 9, 13]. Afterward, by combining the concepts of  $F$ -contraction and multivalued mapping, Altun et al. [2] defined the multivalued  $F$ -contraction. Our goal in this work is to determine the behavior of multivalued integral mappings under  $F$ -contraction and to investigate their some properties, which is a open problem in the current literature. It also supports the results of the study with some interesting examples.

Let  $(X, d)$  be a metric space. Denote by  $P(X)$  the family of all nonempty subsets of  $X$ ,  $CB(X)$  the family of all nonempty, closed and bounded subsets of  $X$  and  $K(X)$  the family of all nonempty compact subsets of  $X$ .

**Definition 1.** Let  $X$  be a nonempty set.  $T$  is said to be a multivalued mapping if  $T$  is a mapping from  $X$  to the power set of  $X$ . We denote a multivalued map by  $T : X \rightarrow P(X)$ .

**Definition 2** (see [8]). Let  $(X, d)$  be a complete metric space. We define the Hausdorff metric on  $CB(X)$  by

$$H(A, B) := \max \left\{ \sup_{x \in A} D(x, B), \sup_{y \in B} D(y, A) \right\}$$

for all  $A, B \in CB(X)$ , where  $D(x, B) := \inf_{b \in B} d(x, b)$  for all  $x \in X$ . Mapping  $H$  is said to be a *Hausdorff metric* induced by  $d$ .

**Lemma 1** (see [5]). Let  $A, B \in CB(X)$ , then for any  $a \in A$ ,

$$D(a, B) \leq H(A, B).$$

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**Definition 3** (see [10]). Let  $(X, d)$  be a metric space. A map  $T : X \rightarrow CB(X)$  is said to be multivalued contraction if there exists  $0 \leq \lambda < 1$  such that

$$H(Tx, Ty) \leq \lambda d(x, y)$$

for all  $x, y \in X$ .

**Definition 4** (see [10]). A point  $x_0 \in X$  is said to be a fixed point of a multivalued mapping  $T : X \rightarrow CB(X)$  such that  $x_0 \in T(x_0)$ .

**Theorem 1** (see [11]). Let  $(X, d)$  be a complete metric space. Suppose  $T : X \rightarrow CB(X)$  is a contraction mapping for some  $0 \leq \alpha < 1$ ,

$$\int_0^{H(Tx, Ty)} \varphi(t) dt \leq \alpha \int_0^{M(x, y)} \varphi(t) dt,$$

where

$$M(x, y) = \max \left\{ d(x, y), D(x, Tx), D(y, Ty), \frac{1}{2} [D(x, Ty) + D(y, Tx)] \right\}$$

for all  $x, y \in X$ . Then there exists a point  $x \in X$  such that  $x \in Tx$  (i.e.,  $x$  is a fixed point of  $T$ ).

Now, we will give a new type of contraction called  $F$ -contraction by introduced Wardowski [14] and a fixed point theorem concerning  $F$ -contraction.

**Definition 5** (see [14]). Let  $F : \mathbb{R}^+ \rightarrow \mathbb{R}$  be a mapping satisfying:

- (F1)  $F$  is strictly increasing, i.e. for all  $\alpha, \beta \in \mathbb{R}^+$  such that  $\alpha < \beta, F(\alpha) < F(\beta)$ ,
- (F2) For each sequence  $\{\alpha_n\}_{n \in \mathbb{N}}$  of positive numbers  $\lim_{n \rightarrow \infty} \alpha_n = 0$  if and only if  $\lim_{n \rightarrow \infty} F(\alpha_n) = -\infty$ ,
- (F3) There exists  $k \in (0, 1)$  such that  $\lim_{\alpha \rightarrow 0^+} \alpha^k F(\alpha) = 0$ .

A mapping  $T : X \rightarrow X$  is said to be an  $F$ -contraction if there exists  $\tau > 0$  such that

$$(1.1) \quad d(Tx, Ty) > 0 \Rightarrow \tau + F(d(Tx, Ty)) \leq F(d(x, y))$$

for all  $x, y \in X$ .

**Remark 1.** From (F1) and (1.1) it is easy to conclude that every  $F$ -contraction  $T$  is a contractive mapping, i.e.

$$d(Tx, Ty) < d(x, y) \quad \text{for all } x, y \in X, \quad Tx \neq Ty.$$

Thus every  $F$ -contraction is a continuous mapping.

**Remark 2.** Let  $F_1, F_2$  be the mappings satisfying (F1-F3). If  $F_1(\alpha) \leq F_2(\alpha)$  for all  $\alpha > 0$  and a mapping  $G : F_1$ - $F_2$  is nondecreasing then every  $F_1$ -contraction  $T$  is  $F_2$ -contraction.

**Theorem 2** (see [14]). Let  $(X, d)$  be a complete metric space and let  $T : X \rightarrow X$  be an  $F$ -contraction. Then  $T$  has a unique fixed point  $x^* \in X$  and for every  $x_0 \in X$  a sequence  $\{T^n x_0\}_{n \in \mathbb{N}}$  is convergent to  $x^*$ .

**Definition 6** (see [1]). Let  $(X, d)$  be a metric space and  $T : X \rightarrow CB(X)$  be a mapping. Then  $T$  is said to be a generalized multivalued  $F$ -contraction if there exists  $\tau > 0$  such that

$$H(Tx, Ty) > 0 \Rightarrow \tau + F(H(Tx, Ty)) \leq F(M(x, y)),$$

where

$$M(x, y) = \max \left\{ d(x, y), D(x, Tx), D(y, Ty), \frac{1}{2} [D(x, Ty) + D(y, Tx)] \right\}$$

for all  $x, y \in X$ .

**Theorem 3** (see [1]). *Let  $(X, d)$  be a complete metric space and  $T : X \rightarrow K(X)$  be a generalized multivalued  $F$ -contraction. If  $T$  or  $F$  is continuous, then  $T$  has a fixed point in  $X$ .*

**Theorem 4** (see [2]). *Let  $(X, d)$  be a complete metric space and  $T : X \rightarrow K(X)$  be a multivalued  $F$ -contraction, then  $T$  has a fixed point in  $X$ .*

**Theorem 5** (see [2]). *Let  $(X, d)$  be a complete metric space and let  $T : X \rightarrow CB(X)$  be a  $F$ -contraction. Suppose that  $F$  also satisfies*

(F4)  $F(\inf A) = \inf F(A)$  for all  $A \subset (0, \infty)$  with  $\inf A > 0$ . Then  $T$  has a fixed point.

## 2. MAIN RESULT

**Definition 7.** Let  $(X, d)$  be complete metric space and  $T : X \rightarrow K(X)$  be a  $F$ -contraction of generalized multivalued integral type mapping if there exists  $\tau > 0$  such that, for all  $x, y \in X$ ,

$$(2.1) \quad H(Tx, Ty) > 0 \Rightarrow \tau + F \left( \int_0^{H(Tx, Ty)} \varphi(t) dt \right) \leq F \left( \int_0^{M(x, y)} \varphi(t) dt \right),$$

where

$$M(x, y) = \max \left\{ d(x, y), D(x, Tx), D(y, Ty), \frac{1}{2} [D(x, Ty) + D(y, Tx)] \right\},$$

where  $\varphi : [0, +\infty) \rightarrow [0, +\infty)$  is a Lebesgue-integrable mapping which is summable on each compact subset of  $[0, +\infty)$ , non-negative, and such that for each  $\varepsilon > 0$ ,  $\int_0^\varepsilon \varphi(t) dt > 0$ .

**Example 1.** Let  $F : \mathbb{R}^+ \rightarrow \mathbb{R}$  be a mapping given by  $F(\alpha) = \ln \alpha$ . It is clear that  $F$  satisfies (F1)–(F3) for any  $k \in (0, 1)$ . Every  $F$ -contraction for generalized multivalued integral type mapping satisfies

$$\tau + F \left( \int_0^{H(Tx, Ty)} \varphi(t) dt \right) \leq F \left( \int_0^{M(x, y)} \varphi(t) dt \right).$$

Now we have

$$\ln e^\tau + \ln \left( \int_0^{H(Tx, Ty)} \varphi(t) dt \right) \leq \ln \left( \int_0^{M(x, y)} \varphi(t) dt \right),$$

$$\int_0^{H(Tx, Ty)} \varphi(t) dt \leq e^{-\tau} \int_0^{M(x, y)} \varphi(t) dt$$

for all  $x, y \in X$ ,  $Tx \neq Ty$ . It is clear that for  $x, y \in X$  such that  $Tx = Ty$  the inequality

$$\int_0^{H(Tx, Ty)} \varphi(t) dt \leq e^{-\tau} \int_0^{M(x, y)} \varphi(t) dt$$

also holds, i.e.  $T$  is a contraction.

**Theorem 6.** *Let  $(X, d)$  be a complete metric space and  $T : X \rightarrow K(X)$  be a  $F$ -contraction of generalized multivalued integral type mapping. If  $T$  or  $F$  is continuous, then  $T$  has a fixed point in  $X$ .*

*Proof.* Let  $x_0 \in X$  be an arbitrary point and define a sequence  $\{x_n\}$  by  $x_{n+1} \in Tx_n$  for  $n = 1, 2, \dots$ . As  $Tx$  is nonempty for all  $x \in X$ , we can choose  $x_1 \in Tx_0$ . If  $x_1 \in Tx_1$ , then  $x_1$  is a fixed point of  $T$ . Let  $x_1 \notin Tx_1$ , then  $D(x_1, Tx_1) > 0$  since  $Tx_1$  is compact. By using (F1), from Lemma 1 and (2.1), we can write that

$$\begin{aligned}
 & F\left(\int_0^{D(x_1, Tx_1)} \varphi(t) dt\right) \\
 & \leq F\left(\int_0^{H(Tx_0, Tx_1)} \varphi(t) dt\right) \leq F\left(\int_0^{M(x_0, x_1)} \varphi(t) dt\right) - \tau \\
 (2.2) \quad & = F\left(\int_0^{\max\{d(x_0, x_1), D(x_0, Tx_0), D(x_1, Tx_1), \frac{1}{2}[D(x_0, Tx_1) + D(x_1, Tx_0)]\}} \varphi(t) dt\right) - \tau \\
 & \leq F\left(\int_0^{\max\{d(x_0, x_1), \frac{1}{2}D(x_0, Tx_1)\}} \varphi(t) dt\right) - \tau \\
 & \leq F\left(\int_0^{\max\{d(x_0, x_1), D(x_1, Tx_1)\}} \varphi(t) dt\right) - \tau \\
 & \leq F\left(\int_0^{d(x_0, x_1)} \varphi(t) dt\right) - \tau.
 \end{aligned}$$

Also, since  $Tx_1$  is compact, we obtain that  $x_2 \in Tx_1$  such that  $d(x_1, x_2) = D(x_1, Tx_1)$ . From (2.2) we have

$$\begin{aligned}
 & F\left(\int_0^{d(x_1, x_2)} \varphi(t) dt\right) \leq F\left(\int_0^{H(Tx_0, Tx_1)} \varphi(t) dt\right) \leq F\left(\int_0^{d(x_0, x_1)} \varphi(t) dt\right) - \tau \\
 (2.3) \quad & F\left(\int_0^{d(x_2, x_3)} \varphi(t) dt\right) \leq F\left(\int_0^{d(x_1, x_2)} \varphi(t) dt\right) - \tau \leq F\left(\int_0^{d(x_0, x_1)} \varphi(t) dt\right) - 2\tau \\
 & \quad \vdots \\
 & F\left(\int_0^{d(x_n, x_{n+1})} \varphi(t) dt\right) \leq F\left(\int_0^{d(x_{n-1}, x_n)} \varphi(t) dt\right) - \tau \leq F\left(\int_0^{d(x_0, x_1)} \varphi(t) dt\right) - n\tau.
 \end{aligned}$$

So we obtain a sequence  $\{x_n\}$  in  $X$  such that  $x_{n+1} \in Tx_n$  and for all  $n \in \mathbb{N}$ .

If there exists  $n_0 \in \mathbb{N}$  for which  $x_{n_0} \in Tx_{n_0}$ , then  $x_{n_0}$  is a fixed point of  $T$  and so the proof is completed. Thus, suppose that for every  $n \in \mathbb{N}$ ,  $x_n \notin Tx_n$ . Denote

$$\gamma_n = \int_0^{d(x_n, x_{n+1})} \varphi(t) dt$$

for  $n = 0, 1, 2, \dots$ .

Then  $\gamma_n > 0$  for all  $n$  and by using (2.3)

$$(2.4) \quad F(\gamma_n) \leq F(\gamma_{n-1}) - \tau \leq F(\gamma_{n-2}) - 2\tau \leq \dots \leq F(\gamma_0) - n\tau.$$

From (2.4), we get  $\lim_{n \rightarrow \infty} F(\gamma_n) = -\infty$ . Thus, from (F2), we have

$$\lim_{n \rightarrow \infty} \gamma_n = 0.$$

From (F3), there exists  $k \in (0, 1)$  such that

$$\lim_{n \rightarrow \infty} \gamma_n^k F(\gamma_n) = 0.$$

By (2.4), the following inequality holds for all  $n \in \mathbb{N}$

$$(2.5) \quad \gamma_n^k F(\gamma_n) - \gamma_n^k F(\gamma_0) \leq -\gamma_n^k n\tau \leq 0.$$

Letting  $n \rightarrow \infty$  in (2.5), we obtain that

$$(2.6) \quad \lim_{n \rightarrow \infty} n\gamma_n^k = 0.$$

From (2.6), there exists  $n_1 \in \mathbb{N}$  such that  $n\gamma_n^k \leq 1$  for all  $n \geq n_1$ . So we obtain that

$$(2.7) \quad \gamma_n \leq \frac{1}{n^{1/k}}.$$

Now, let  $m, n \in \mathbb{N}$  such that  $m > n > n_1$  to show that  $\{x_n\}$  is a Cauchy sequence. By using the triangle inequality for the metric and from (2.7), we have

$$\begin{aligned} \int_0^{d(x_n, x_m)} \varphi(t) dt &\leq \int_0^{d(x_n, x_{n+1})} \varphi(t) dt \\ &\quad + \int_0^{d(x_{n+1}, x_{n+2})} \varphi(t) dt + \dots + \int_0^{d(x_{m-1}, x_m)} \varphi(t) dt \\ &= \gamma_n + \gamma_{n+1} + \dots + \gamma_{m-1} \leq \sum_{i=n}^{\infty} \frac{1}{i^{1/k}}. \end{aligned}$$

By the convergence of the series  $\sum_{i=n}^{\infty} \frac{1}{i^{1/k}}$ , we get  $\int_0^{d(x_n, x_m)} \varphi(t) dt \rightarrow 0$  as  $n \rightarrow \infty$ . As a result,  $\{x_n\}$  is a Cauchy sequence in  $(X, d)$ . Since  $(X, d)$  is a complete metric space, the sequence  $\{x_n\}$  converges to some point  $z \in X$ , that is,  $\lim_{n \rightarrow \infty} x_n = z$ .

If  $T$  is compact, then we have  $Tx_n \rightarrow Tz$  and from Lemma 1

$$D(x_n, Tz) \leq H(Tx_{n-1}, Tz),$$

so  $D(z, Tz) = 0$  and  $z \in Tz$ .

Now, suppose  $F$  is continuous. In this case, we claim that  $z \in Tz$ . Assume the contrary, that is,  $z \notin Tz$ . In this case, there exists an  $n_0 \in \mathbb{N}$  and a subsequence  $\{x_{n_k}\}$  of  $\{x_n\}$  such that  $D(x_{n_k+1}, Tz) > 0$  for all  $n_k \geq n_0$ . (Otherwise, there exists  $n_1 \in \mathbb{N}$  such that  $x_n \in Tz$  for all  $n \geq n_1$ , which implies that  $z \in Tz$ . This is a contradiction).

Since  $D(x_{n_k+1}, Tz) > 0$  for all  $n_k \geq n_0$ , then we have

$$\begin{aligned} &\tau + F\left(\int_0^{D(x_{n_k+1}, Tz)} \varphi(t) dt\right) \\ &\leq \tau + F\left(\int_0^{H(Tx_{n_k}, Tz)} \varphi(t) dt\right) \leq F\left(\int_0^{M(x_{n_k}, z)} \varphi(t) dt\right) \\ &= F\left(\int_0^{\max\{d(x_{n_k}, z), D(x_{n_k}, Tx_{n_k}), D(z, Tz), \frac{1}{2}[D(x_{n_k}, Tz) + D(z, Tx_{n_k})]\}} \varphi(t) dt\right). \end{aligned}$$

Taking the limit as  $k \rightarrow \infty$  and using the continuity of  $F$ , we have

$$\tau + F\left(\int_0^{D(z, Tz)} \varphi(t) dt\right) \leq F\left(\int_0^{D(z, Tz)} \varphi(t) dt\right),$$

which is a contradiction. Thus, we get  $z \in Tz$ . That is,  $T$  has a fixed point. □

**Example 2.** Let  $X = [0, 1]$  and  $d(x, y) = |x - y|$ . Define  $T : X \rightarrow K(X)$  multivalued mapping by

$$Tx = \left[ \frac{x}{4}, \frac{x+1}{2} \right].$$

For  $x_0 = 0$  and  $x_1 = 1$  arbitrary points, we have  $T_0 = [0, \frac{1}{2}]$  and  $T_1 = [\frac{1}{4}, 1]$ .

$$\begin{aligned} H(T_0, T_1) &= \max \left\{ \sup_{x \in T_0} D(x, T_1), \sup_{y \in T_1} D(y, T_0) \right\} \\ &= \max \left\{ \sup_{x \in T_0} \inf_{y \in T_1} d(x, y), \sup_{y \in T_1} \inf_{x \in T_0} d(x, y) \right\} = \frac{1}{4}. \end{aligned}$$

Now, we show that mapping  $T$  under this condition is not generalized multivalued integral type mapping. Afterward, we will show that it is a contraction together with  $F$ . By using Theorem 1 and  $\varphi(t) = 1$  for all  $t \in \mathbb{R}$ , we have

$$\int_0^{H(T_0, T_1)} dt \leq \alpha \int_0^{d(x_0, x_1)} dt \quad \text{and} \quad \alpha \geq \frac{1}{4}.$$

It can be shown that multivalued integral type mapping above holds for  $\alpha \in [\frac{1}{4}, 1)$ . In Theorem 1, it holds  $0 \leq \alpha < 1$ . This is a contraction. Under same condition; we will apply  $F$ -contraction. Let us take  $F(\alpha) = \ln \alpha$  and  $\tau \in (0, 1.39)$  also  $\varphi(t) = 1$  for all  $t \in \mathbb{R}$ , we have

$$F \left( \int_0^{H(T_0, T_1)} dt \right) \leq F \left( \int_0^{d(x_0, x_1)} dt \right) - \tau.$$

This inequality holds three condition of  $F$ -contraction:

- i) It is easy to see ( $F1$ ).
- ii) Let  $x_n = (\frac{1}{n}) \in [0, 1]$ ,  $n = 1, 2, \dots$  and  $x_0 = 0$ . By taking  $x_n = (\frac{1}{n})$ , we get

$$\begin{aligned} Tx_n &= \left[ \frac{1}{4n}, \frac{n+1}{2n} \right], \\ H(Tx_n, Tx_{n+1}) &= \frac{1}{4n(n+1)}. \end{aligned}$$

If we denote that  $\gamma_n = \int_0^{\frac{1}{4n(n+1)}} \varphi(t) dt$ ,  $\varphi(t) = 1$  for all  $t \in \mathbb{R}$ , then  $\gamma_n = (\frac{1}{4n(n+1)})$ . By using some calculations, we conclude that following inequality:

$$(2.8) \quad F \left( \frac{1}{4n(n+1)} \right) \leq F \left( \frac{1}{4(n-1)n} \right) - \tau \leq \dots \leq F(1) - n\tau.$$

By taking  $F(\alpha) = \ln \alpha$ , we have

$$0 \leq F \left( \frac{1}{4n(n+1)} \right) \leq \ln 1 - n\tau.$$

From (2.8), letting  $n \rightarrow \infty$ , we obtain that

$$\lim_{n \rightarrow \infty} \ln \left( \frac{1}{4n(n+1)} \right) = -\infty.$$

Also for  $\varphi(t) = 1$ ,  $\gamma_n = (\frac{1}{4n(n+1)})$  and  $\lim_{n \rightarrow \infty} \frac{1}{4n(n+1)} = 0$ .

- iii) We will show that  $\lim_{n \rightarrow \infty} \gamma_n^k F(\gamma_n) = 0$ . Letting  $n \rightarrow \infty$ , we have

$$\lim_{n \rightarrow \infty} \gamma_n^k F(\gamma_n) = \lim_{n \rightarrow \infty} \frac{1}{[4n(n+1)]^k} \ln \left( \frac{1}{4(n+1)} \right) = 0.$$

Since  $x_n = \left(\frac{1}{n}\right) \in [0, 1]$  is a Cauchy sequence and  $(X, d)$  is complete metric space, taking the limit as  $n \rightarrow \infty$  we have  $\frac{1}{n} \rightarrow 0$  in  $[0, 1]$ . However, since  $Tx_n = \left[\frac{1}{4n}, \frac{n+1}{2n}\right]$  and  $Tx_n \in K(X)$ , then  $\lim_{n \rightarrow \infty} Tx_n = Tx$ , that is, we have

$$\lim_{n \rightarrow \infty} \left[ \frac{1}{4n}, \frac{n+1}{2n} \right] = \left[ 0, \frac{1}{2} \right].$$

As a result we get  $0 \in \left[0, \frac{1}{2}\right]$ , that is,  $T$  has a fixed point under  $F$ -contraction.

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DEPARTMENT OF MATHEMATICS, FACULTY OF ARTS AND SCIENCES, AHI EVRAN UNIVERSITY, 40100 KIRŞEHİR, TURKEY

*E-mail address:* derya.sekman@ahievran.edu.tr

DEPARTMENT OF MATHEMATICAL ENGINEERING, FACULTY OF CHEMISTRY-METALLURGICAL, YILDIZ TECHNICAL UNIVERSITY, 34210 ISTANBUL, TURKEY

*E-mail address:* vkkaya@yildiz.edu.tr

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