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FURTHER GENERALIZATIONS OF GAMMA, BETA AND RELATED FUNCTIONS

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ABSTRACT. In this paper, we present further generalizations of gamma and beta functions by adding extra parameters to their integral representations. Then we introduce new generalizations of hypergeometric and confluent hypergeometric functions by using new beta function. We also obtain many interesting properties such as integral transforms, differentiation formulas, Mellin transforms, transformation formulas, differential and difference relations and a summation formula.

1. INTRODUCTION

Recently, many mathematicians obtained different generalizations of gamma and beta functions by adding some extra parameters to their integral representations. They also examined many properties of these functions. For further reading please see [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

Here, we introduce new generalizations of gamma and beta functions by using an exponential function with four parameter p, q, κ, μ as follows:

$$\Gamma_{p,q}^{(\kappa,\mu)}(x) := \int_0^\infty t^{x-1} e^{\left(-\frac{t^\kappa}{p}-\frac{q}{t^\mu}\right)} dt \tag{1.1}$$

$$(\Re(p) > 0, \Re(q) > 0, \Re(\kappa) > 0, \Re(\mu) > 0),$$

$$B_{p,q}^{(\kappa,\mu)}(x,y) := \int_0^1 t^{x-1}(1-t)^{y-1} e^{\left(-\frac{p}{t^\kappa}-\frac{q}{(1-t)^\mu}\right)} dt \tag{1.2}$$

$$(\Re(p) > 0, \Re(q) > 0, \Re(\kappa) > 0, \Re(\mu) > 0).$$

The special cases of (1.1) are $\Gamma_{1,0}^{(1,1)}(x) = \Gamma(x)$, $\Gamma_{1,p}^{(1,1)}(x) = \Gamma_p(x)$ and (1.2) are $B_{0,0}^{(\kappa,\mu)}(x,y) = B(x,y)$, $B_{p,p}^{(1,1)}(x,y) = B_p(x,y)$, $B_{p,p}^{(m,m)}(x,y) = B_{p:m}(x,y)$, $B_{p,q}^{(1,1)}(x,y) = B_{p,q}(x,y)$ where $\Gamma(x), B(x,y)$ are classical gamma and beta functions (see, [13]), $\Gamma_p(x), B_p(x,y)$ are extended gamma and beta functions defined

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in [1], $B_{p:m}(x,y), B_{p,q}(x,y)$ are the generalized beta functions given in [5], [3] respectively.

Throughout this paper, we assume that $\min\{\Re(p),\Re(q),\Re(\kappa),\Re(\mu)\}>0$.

Using $B_{p,q}^{(\kappa,\mu)}(x,y)$, we obtain further generalizations of Gauss and confluent hypergeometric functions

$$F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z):=\sum_{n=0}^{\infty}(\alpha)_n\frac{B_{p,q}^{(\kappa,\mu)}(\beta+n,\gamma-\beta)}{B(\beta,\gamma-\beta)}\frac{z^n}{n!},\ |z|<1 \tag{1.3}$$

$$\Phi_{p,q}^{(\kappa,\mu)}(\beta;\gamma;z):=\sum_{n=0}^{\infty}\frac{B_{p,q}^{(\kappa,\mu)}(\beta+n,\gamma-\beta)}{B(\beta,\gamma-\beta)}\frac{z^n}{n!} \tag{1.4}$$

where $\Re(\gamma)>\Re(\beta)>0$.

2. Integral Representations

Theorem 2.1. *The integral representations of* (1.3) *and* (1.4) *are*

$$F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z)=\frac{1}{B(\beta,\gamma-\beta)}\times\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}e^{\left(-\frac{p}{t^\kappa}-\frac{q}{(1-t)^\mu}\right)}(1-zt)^{-\alpha}dt, \tag{2.1}$$

$$F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z)=\frac{1}{B(\beta,\gamma-\beta)}\times\int_0^\infty u^{\beta-1}(1+u)^{\alpha-\gamma}e^{\left(-\frac{p(u+1)^\kappa}{u^\kappa}-q(u+1)^\mu\right)}(1+u(1-z))^{-\alpha}du, \tag{2.2}$$

$$F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z)=\frac{2}{B(\beta,\gamma-\beta)}\times\int_0^{\frac{\pi}{2}}\frac{\sin^{2\beta-1}\theta\cos^{2\gamma-2\beta-1}\theta}{\left(1-z\sin^2\theta\right)^{-\alpha}}e^{\left(-p\csc^{2\kappa}\theta-q\sec^{2\mu}\theta\right)}d\theta, \tag{2.3}$$

$$F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z)=\frac{1}{B(\beta,\gamma-\beta)}\int_0^\infty\Big[\sinh^{2\beta-2}\theta\cosh^{2\alpha-2\gamma-2}\theta\times e^{\left(-p\coth^{2\kappa}\theta-q\cosh^{2\mu}\theta\right)}\left(\cosh^2\theta-z\sinh^2\theta\right)^{-\alpha}d\theta\Big], \tag{2.4}$$

$$\Phi_{p,q}^{(\kappa,\mu)}(\beta;\gamma;z)=\frac{1}{B(\beta,\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}e^{\left(zt-\frac{p}{t^\kappa}-\frac{q}{(1-t)^\mu}\right)}dt, \tag{2.5}$$

$$\Phi_{q,p}^{(\mu,\kappa)}(\beta;\gamma;z)=\frac{e^z}{B(\beta,\gamma-\beta)}\int_0^1 t^{\gamma-\beta-1}(1-t)^{\beta-1}e^{-\left(zt+\frac{p}{(1-t)^\kappa}+\frac{q}{t^\mu}\right)}dt. \tag{2.6}$$

Proof. The integral formulas (2.1) and (2.5) are easily obtained by replacing the definition of generalized beta function (1.2) in (1.3) and (1.4). The integrals in (2.2), (2.3) and (2.4) can be found by making transformations $t=\frac{u}{u+1}$, $t=\sin^2\theta$, $t=\tanh^2\theta$ in (2.1), respectively. The integral formula (2.6) is also obtained by changing t with $1-t$ in (2.5). □

3. Differentiation Formulas

Theorem 3.1. *The differentiation formulas of* (1.3) *and* (1.4) *are*

$$\begin{aligned}\frac{d^n}{dz^n}F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z) =&\frac{(\alpha)_n(\beta)_n}{(\gamma)_n}F_{p,q}^{(\kappa,\mu)}(\alpha+n,\beta+n;\gamma+n;z), \\ \frac{d^n}{dz^n}\Phi_{p,q}^{(\kappa,\mu)}(\beta;\gamma;z) =&\frac{(\beta)_n}{(\gamma)_n}\Phi_{p,q}^{(\kappa,\mu)}(\beta+n;\gamma+n;z).\end{aligned} \tag{3.1}$$

Proof. Differentiation formulas can be found by taking the derivatives of (1.3) and (1.4), using the identities

$$\begin{aligned}B(x,y-x) &= \frac{y}{x}B(x+1,y-x), \\ (\nu)_{n+1} &= \nu(\nu+1)_n\end{aligned}$$

which given in [1] and induction. □

4. Mellin Transforms

Theorem 4.1. *The following double Mellin transformation of generalized beta function is*

$$\mathfrak{M}\left[B_{p,q}^{(\kappa,\mu)}(x,y);r,s\right] = \Gamma(s)\Gamma(r)B(x+\kappa s,y+\mu r)$$

$$(\Re(s)>0,\ \Re(r)>0,\ \Re(\beta+r)>0,\ \Re(\gamma+s)>0).$$

Proof. Double Mellin transform of generalized beta function is

$$\mathfrak{M}\left[B_{p,q}^{(\kappa,\mu)}(x,y);r,s\right] = \int_0^\infty\int_0^\infty q^{r-1}p^{s-1}B_{p,q}^{(\kappa,\mu)}(x,y)\,dpdq.$$

Considering (1.2) in above equality and making necessary calculations we have

$$\begin{aligned}\mathfrak{M}\left[B_{p,q}^{(\kappa,\mu)}(x,y);r,s\right] =& \int_0^\infty\int_0^\infty q^{r-1}p^{s-1} \\ &\times\left[\int_0^1 t^{x-1}(1-t)^{y-1}e^{\left(-\frac{p}{t^\kappa}-\frac{q}{(1-t)^\mu}\right)}dt\right]dpdq \\ =&\int_0^1 t^{x-1}(1-t)^{y-1} \\ &\times\left[\int_0^\infty q^{r-1}e^{\left(-\frac{q}{(1-t)^\mu}\right)}dq\int_0^\infty p^{s-1}e^{\left(-\frac{p}{t^\kappa}\right)}dp\right]dt \\ =&\Gamma(s)\Gamma(r)\int_0^1 t^{x+\kappa s-1}(1-t)^{y+\mu r-1}dt \\ =&\Gamma(s)\Gamma(r)B(x+\kappa s,y+\mu r).\end{aligned}$$

□

Theorem 4.2. *For $x_1 > 0, x_2 > 0$, the following Mellin-Barnes contour integrals hold true:*

$$F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z) = \frac{1}{(2\pi i)^2 B(\beta,\gamma-\beta)} \int_{x_1-i\infty}^{x_1+i\infty} \int_{x_2-i\infty}^{x_2+i\infty} \Big[\Gamma(s)\Gamma(r) \times B(\beta+\kappa s, \gamma+\kappa s+\mu r) \times F(\alpha, \beta+\kappa s; \gamma+\kappa s+\mu r; z)\, q^{-\kappa s} p^{-\mu r} dr ds \Big], \tag{4.1}$$

$$\Phi_{p,q}^{(\kappa,\mu)}(\beta;\gamma;z) = \frac{1}{(2\pi i)^2 B(\beta,\gamma-\beta)} \int_{x_1-i\infty}^{x_1+i\infty} \int_{x_2-i\infty}^{x_2+i\infty} \Big[\Gamma(s)\Gamma(r) \times B(\beta+\kappa s, \gamma+\kappa s+\mu r) \times \Phi(\beta+\kappa s; \gamma+\kappa s+\mu r; z)\, q^{-\kappa s} p^{-\mu r} dr ds \Big]. \tag{4.2}$$

Proof. The double Mellin transform of generalized beta function is

$$\begin{aligned}\mathfrak{M}\Big[F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z); r, s\Big] &= \int_0^\infty \int_0^\infty p^{s-1} q^{r-1} F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z)\, dp dq \\ &= \frac{1}{B(\beta,\gamma-\beta)} \int_0^1 \Big[t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-zt)^{-\alpha} \\ &\quad \times \int_0^\infty q^{r-1} e^{\left(-\frac{q}{(1-t)^\mu}\right)} dq \int_0^\infty p^{s-1} e^{\left(-\frac{p}{t^\kappa}\right)} dp \Big] dt.\end{aligned}$$

Using (2.1) in above equality and making necessary calculations we obtain

$$\begin{aligned}&\mathfrak{M}\Big[F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z); r, s\Big] \\ &= \frac{\Gamma(r)\Gamma(s)}{B(\beta,\gamma-\beta)} \int_0^1 t^{\beta+\kappa s-1}(1-t)^{\gamma-\beta+\mu r-1}(1-zt)^{-\alpha} dt \\ &= \frac{\Gamma(r)\Gamma(s) B(\beta+\kappa s, \gamma+\kappa s+\mu r)}{B(\beta,\gamma-\beta)} F(\alpha, \beta+\kappa s; \gamma+\kappa s+\mu r; z).\end{aligned} \tag{4.3}$$

Using inverse Mellin transforms of (4.3) we have (4.1). We can also prove (4.2) with similar calculations. □

Theorem 4.3. *Difference formula of generalized Gauss hypergeometric function is*

$$\begin{aligned}&(\beta-1)B(\beta-1,\gamma-\beta+1)\, F_{p,q}^{(\kappa,\mu)}(\alpha,\beta-1;\gamma;z) \\ &= (\gamma-\beta-1)\, B(\beta,\gamma-\beta-1)\, F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma-1;z) \\ &\quad - \alpha z B(\beta,\gamma-\beta)\, F_{p,q}^{(\kappa,\mu)}(\alpha+1,\beta;\gamma;z) \\ &\quad - \kappa p B(\beta-\kappa-1,\gamma-\beta)\, F_{p,q}^{(\kappa,\mu)}(\alpha,\beta-\kappa-1;\gamma-\kappa-1;z) \\ &\quad + \mu q B(\beta,\gamma-\beta-\mu-1)\, F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma-\mu-1;z).\end{aligned}$$

Proof. We have

$$\begin{aligned}
&\frac{d}{dt}\left[H\left(1-t\right)\left(1-t\right)^{\gamma-\beta-1}\left(1-zt\right)^{-\alpha}e^{\left(-\frac{p}{t^{\kappa}}-\frac{q}{(1-t)^{\mu}}\right)}\right]\\
&\quad=-\left(1-zt\right)^{-\alpha}\left(1-t\right)^{\gamma-\beta-1}H\left(1-t\right)\left[\delta\left(1-t\right)+\frac{\left(\gamma-\beta-1\right)}{\left(1-t\right)}\right.\\
&\qquad\left.-\alpha z\left(1-zt\right)^{-1}-\left(\frac{\kappa p}{t^{\kappa+1}}-\frac{\mu q}{\left(1-t\right)^{\mu+1}}\right)\right]e^{\left(-\frac{p}{t^{\kappa}}-\frac{q}{(1-t)^{\mu}}\right)}.
\end{aligned}$$

Taking Mellin transform of both sides and using the following equalities

$$\mathfrak{M}\left[f'(t);r\right]=-(r-1)\mathfrak{M}\left[f(t);r-1\right]$$

and

$$\begin{aligned}
&\mathfrak{M}\left[\frac{d}{dt}\left(H\left(1-t\right)\left(1-t\right)^{\gamma-\beta-1}\left(1-zt\right)^{-\alpha}e^{\left(-\frac{p}{t^{\kappa}}-\frac{q}{(1-t)^{\mu}}\right)}\right);\beta\right]\\
&\qquad=B\left(\beta,\gamma-\beta\right)F_{p,q}^{(\kappa,\mu)}\left(\alpha,\beta;\gamma;z\right)
\end{aligned}$$

in [13], we get the desired result. □

The next theorem can be proved with similar calculations of the above theorem. That's why we omit its proof.

Theorem 4.4. *Difference formula of generalized confluent hypergeometric function is*

$$\begin{aligned}
(\beta-1)B\left(\beta-1,\gamma-\beta+1\right)\Phi_{p,q}^{(\kappa,\mu)}\left(\beta-1;\gamma;z\right)&=-zB\left(\beta,\gamma-\beta\right)\Phi_{p,q}^{(\kappa,\mu)}\left(\beta;\gamma;z\right)\\
&+\left(\gamma-\beta-1\right)B\left(\beta,\gamma-\beta-1\right)\Phi_{p,q}^{(\kappa,\mu)}\left(\beta;\gamma-1;z\right)\\
&-\kappa pB\left(\beta-\kappa-1,\gamma-\beta\right)\Phi_{p,q}^{(\kappa,\mu)}\left(\beta-\kappa-1;\gamma-\kappa-1;z\right)\\
&+\mu qB\left(\beta,\gamma-\beta-\mu-1\right)\Phi_{p,q}^{(\kappa,\mu)}\left(\beta;\gamma-\mu-1;z\right).
\end{aligned}$$

5. Transformation Formulas

Theorem 5.1. *The transformation formulas of* (1.3) *and* (1.4) *are*

$$F_{p,q}^{(\kappa,\mu)}\left(\alpha,\beta;\gamma;z\right)=\left(1-z\right)^{-\alpha}F_{q,p}^{(\mu,\kappa)}\left(\alpha,\gamma-\beta;\gamma;-\frac{z}{1-z}\right)\tag{5.1}$$
$$\left(\left|\arg(1-z)\right|<\pi,\ \Re\left(\gamma\right)>\Re\left(\beta\right)>0\right),$$

$$F_{p,q}^{(\kappa,\mu)}\left(\alpha,\beta;\gamma;1-\tfrac{1}{z}\right)=z^{\alpha}F_{q,p}^{(\mu,\kappa)}\left(\alpha,\gamma-\beta;\gamma;1-z\right)\tag{5.2}$$
$$\left(\left|\arg(1-z)\right|<\pi,\ \Re\left(\gamma\right)>\Re\left(\beta\right)>0\right),$$

$$F_{p,q}^{(\kappa,\mu)}\left(\alpha,\beta;\gamma;\frac{z}{z+1}\right)=\left(1+z\right)^{\alpha}F_{q,p}^{(\mu,\kappa)}\left(\alpha,\gamma-\beta;\gamma;-z\right)\tag{5.3}$$
$$\left(\left|\arg(1-z)\right|<\pi,\ \Re\left(\gamma\right)>\Re\left(\beta\right)>0\right),$$

$$\Phi_{p,q}^{(\kappa,\mu)}\left(\beta;\gamma;z\right)=e^{z}\Phi_{q,p}^{(\mu,\kappa)}\left(\gamma-\beta;\gamma;-z\right)\tag{5.4}$$
$$\left(\Re\left(\gamma\right)>\Re\left(\beta\right)>0\right).$$

Proof. By using the equality

$$[1-z(1-u)]^{-\alpha}=(1-z)^{-\alpha}\left(1+\frac{z}{1-z}u\right)^{-\alpha}$$

in (2.1) and taking $1-u$ in stead of t, we find

$$F_{q,p}^{(\mu,\kappa)}(\alpha,\beta;\gamma;z)=\frac{(1-z)^{-\alpha}}{B(\beta,\gamma-\beta)}\int_0^1\left[u^{\gamma-\beta-1}(1-u)^{\beta-1}\right.$$
$$\left.\times e^{\left(-\frac{q}{u^{\mu}}-\frac{p}{(1-u)^{\kappa}}\right)}\left(1+\frac{z}{1-z}u\right)^{-\alpha}du\right].$$

This proves (5.1). For (5.2) and (5.3) one can replace z by $1-\dfrac{1}{z}$ and $\dfrac{z}{z+1}$ in (5.1), respectively. Finally, equation (5.4) can be easily obtained from (2.5) and (2.6). □

6. Differential and Difference Relations

Theorem 6.1. *The differential and difference relations of* (1.3) *and* (1.4) *are*

$$\Delta_\alpha F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z)=\frac{bz}{c}F_{p,q}^{(\kappa,\mu)}(\alpha+1,\beta+1;\gamma+1;z)\,, \tag{6.1}$$

$$\frac{d}{dz}F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z)=\frac{\alpha}{z}\Delta_\alpha F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z)\,, \tag{6.2}$$

$$\beta\Delta_\beta\Phi_{p,q}^{(\kappa,\mu)}(\beta;\gamma+1;z)+\gamma\Delta_\gamma\Phi_{p,q}^{(\kappa,\mu)}(\beta;\gamma;z)=0, \tag{6.3}$$

$$\frac{d}{dz}\Phi_{p,q}^{(\kappa,\mu)}(\beta;\gamma;z)=\frac{\beta}{\gamma}\Phi_{p,q}^{(\kappa,\mu)}(\beta;\gamma+1;z)-\Delta_\gamma\Phi_{p,q}^{(\kappa,\mu)}(\beta;\gamma;z)\,, \tag{6.4}$$

where Δ_α *is the well-known difference operator*

$$\Delta_\alpha f(\alpha,...)=f(\alpha+1,...)-f(\alpha,...)\,.$$

Proof. In (2.1), if we write $\alpha+1$, $\beta+1$, and $\gamma+1$ in stead of α, β, and γ and using

$$\Delta_\alpha F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z)=F_{p,q}^{(\kappa,\mu)}(\alpha+1,\beta;\gamma;z)-F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;z)$$

we obtain (6.1). To prove (6.2) we can use (3.1) in (6.1). By making similar calculations we can prove (6.3) and (6.4). □

7. Summation Formula

We also give a summation formula for the special case of generalized Gauss hypergeometric function when $z=1$.

Theorem 7.1. *The summation formula of* (1.3) *is*

$$F_{p,q}^{(\kappa,\mu)}(\alpha,\beta;\gamma;1)=\frac{B_{p,q}^{(\kappa,\mu)}(\beta,\gamma-\alpha-\beta)}{B(\beta,\gamma-\beta)}$$
$$(\Re(\gamma-\beta-\alpha)>0).$$

Proof. The result can obtained easily from (2.1) by taking $z=1$ and using (1.2). □

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