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The extended Srivastava's triple hypergeometric functions and their integral representations

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Abstract

We introduce the extended Srivastava's triple hypergeometric functions by using an extension of beta function. Furthermore, some integral representations are given for these new functions.

Keywords: Beta function, Srivastava's triple hypergeometric functions, Appell's hypergeometric function, Exton's function.
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1. Introduction

Recently, various extensions of beta and related functions have appeared in the literature [1–3, 9–13, 17, 19]. Particularly, the following extension of beta function was introduced by Chaudhry et al. in [2] as

$$B_p(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}\exp\left(-\frac{p}{t(1-t)}\right)dt, \tag{1.1}$$

$$(\Re(p)>0;\ \Re(x)>0,\ \Re(y)>0 \text{ when } p=0).$$

Later, by using this extension of beta function, Chaudhry et al. [3] extended the hypergeometric function as follows:

$$F_p(a,b;c;z)=\sum_{n=0}^{\infty}(a)_n\frac{B_p(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!},$$

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$$(p \geq 0;\ |z| < 1;\ \Re(c) > \Re(b) > 0).$$

In [12], Özarslan et al. defined the extended Appell's hypergeometric function as

$$F_{1,p}(a,b,c;d;x,y) = \sum_{m,n=0}^{\infty} (b)_n (c)_m \frac{B_p(a+m+n, d-a)}{B(a,d-a)} \frac{x^n}{n!} \frac{y^m}{m!}, \tag{1.2}$$

$$(p \geq 0;\ \max\{|x|,|y|\} < 1),$$

and obtained the following integral representation

$$F_{1,p}(a,b,c;d;x,y) = \frac{\Gamma(d)}{\Gamma(a)\Gamma(d-a)} \int_0^1 t^{a-1}(1-t)^{d-a-1}(1-xt)^{-b}(1-yt)^{-c} \exp\left(-\frac{p}{t(1-t)}\right) dt, \tag{1.3}$$

$$(p > 0;\ p = 0 \ \text{ and } |\arg(1-x)| < \pi,\ |\arg(1-y)| < \pi;\ \Re(d) > \Re(a) > 0).$$

Note that these extended functions are reduced to their original forms for $p = 0$.

2. Extended Srivastava's triple hypergeometric functions

Srivastava defined triple hypergeometric functions H_A, H_B and H_C in [15, 16] and then many authors have studied some integral representations of these functions [4–8, 16].

In this paper, we introduce the extensions of Srivastava's triple hypergeometric functions as follows:

$$H_{A,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2;x,y,z) := \sum_{m,n,k=0}^{\infty} \frac{(\alpha)_{m+k}(\beta_1)_{m+n}}{(\gamma_1)_m} \frac{B_p(\beta_2+n+k, \gamma_2-\beta_2)}{B(\beta_2,\gamma_2-\beta_2)} \frac{x^m}{m!} \frac{y^n}{n!} \frac{z^k}{k!}, \tag{2.1}$$

$$(p \geq 0;\ \mathfrak{r} < 1,\ \mathfrak{s} < 1,\ \mathfrak{t} < (1-\mathfrak{r})(1-\mathfrak{s})),$$

$$H_{B,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x,y,z) := \sum_{m,n,k=0}^{\infty} \frac{(\alpha+\beta_1)_{2m+n+k}(\beta_2)_{n+k}}{(\gamma_1)_m(\gamma_2)_n(\gamma_3)_k} \frac{B_p(\alpha+m+k, \beta_1+m+n)}{B(\alpha,\beta_1)} \frac{x^m}{m!} \frac{y^n}{n!} \frac{z^k}{k!}, \tag{2.2}$$

$$(p \geq 0;\ \mathfrak{r}+\mathfrak{s}+\mathfrak{t}+2\sqrt{\mathfrak{r}\mathfrak{s}\mathfrak{t}} < 1),$$

and

$$H_{C,p}(\alpha,\beta_1,\beta_2;\gamma;x,y,z) := \sum_{m,n,k=0}^{\infty} \frac{(\beta_1)_{m+n}(\beta_2)_{n+k}}{(\gamma)_n} \frac{B_p(\alpha+m+k, \gamma+n-\alpha)}{B(\alpha,\gamma+n-\alpha)} \frac{x^m}{m!} \frac{y^n}{n!} \frac{z^k}{k!}, \tag{2.3}$$

$$(p \geq 0;\ \mathfrak{r} < 1,\ \mathfrak{s} < 1,\ \mathfrak{t} < 1,\ \mathfrak{r}+\mathfrak{s}+\mathfrak{t}-2\sqrt{(1-\mathfrak{r})(1-\mathfrak{s})(1-\mathfrak{t})} < 2),$$

where $\mathfrak{r} := |x|, \mathfrak{s} := |y|, \mathfrak{t} := |z|$. Obviously for $p = 0$, these functions are reduced to the well-known Srivastava's triple hypergeometric functions H_A, H_B and H_C, respectively. The extended Srivastava's triple hypergeometric functions defined by (2.1) and (2.3) can also be given with the following series representations:

$$H_{A,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2;x,y,z) = \sum_{m=0}^{\infty} \frac{(\alpha)_m(\beta_1)_m}{(\gamma_1)_m} F_{1,p}(\beta_2, \beta_1+m, \alpha+m; \gamma_2; y, z) \frac{x^m}{m!}, \tag{2.4}$$

and

$$H_{C,p}(\alpha,\beta_1,\beta_2;\gamma;x,y,z) = \sum_{n=0}^{\infty} \frac{(\beta_1)_n(\beta_2)_n}{(\gamma)_n} F_{1,p}(\alpha, \beta_1+n, \beta_2+n; \gamma+n; x, z) \frac{y^n}{n!}, \tag{2.5}$$

where $F_{1,p}$ is the extended Appell's hypergeometric function given by (1.2). Throughout this paper, we assume that p is any nonnegative real number.

3. Integral representations for $H_{A,p}$

Theorem 3.1. *The integral representations* (3.1), (3.4)-(3.7) *of* $H_{A,p}$ *hold for* $\Re(\gamma_2) > \Re(\beta_2) > 0$ *and the others hold for* $\Re(\gamma_j) > \Re(\beta_j) > 0$, $j = 1, 2$*:*

$$\begin{aligned} H_{A,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2;x,y,z) &= \frac{\Gamma(\gamma_2)}{\Gamma(\beta_2)\Gamma(\gamma_2-\beta_2)} \\ &\times \int_0^1 t^{\beta_2-1}(1-t)^{\gamma_2-\beta_2-1}(1-yt)^{-\beta_1}(1-zt)^{-\alpha} \\ &\times \exp\left(-\frac{p}{t(1-t)}\right) {}_2F_1\left(\alpha,\beta_1;\gamma_1;\frac{x}{(1-yt)(1-zt)}\right) dt, \end{aligned} \tag{3.1}$$

$$\begin{aligned} H_{A,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2;x,y,z) &= \frac{\Gamma(\gamma_1)\Gamma(\gamma_2)}{\Gamma(\beta_1)\Gamma(\beta_2)\Gamma(\gamma_1-\beta_1)\Gamma(\gamma_2-\beta_2)} \\ &\times \int_0^1\int_0^1 \xi^{\beta_1-1}t^{\beta_2-1}(1-\xi)^{\gamma_1-\beta_1-1}(1-t)^{\gamma_2-\beta_2-1} \\ &\times (1-yt)^{\alpha-\beta_1}[(1-yt)(1-zt)-x\xi]^{-\alpha}\exp\left(-\frac{p}{t(1-t)}\right) d\xi dt, \end{aligned} \tag{3.2}$$

$$\begin{aligned} H_{A,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2;x,y,z) &= \frac{\Gamma(\gamma_1)\Gamma(\gamma_2)}{\Gamma(\beta_1)\Gamma(\beta_2)\Gamma(\gamma_1-\beta_1)\Gamma(\gamma_2-\beta_2)} \\ &\times \int_0^1\int_0^1 \xi^{\beta_1-1}t^{\beta_2-1}(1-\xi)^{\gamma_1-\beta_1-1}(1-t)^{\gamma_2-\beta_2-1}(1-yt)^{-\beta_1} \\ &\times (1-x\xi-zt)^{-\alpha}\left(1-\frac{xy\xi t}{(1-yt)(1-x\xi-zt)}\right)^{-\alpha}\exp\left(-\frac{p}{t(1-t)}\right) d\xi dt, \end{aligned} \tag{3.3}$$

$$\begin{aligned} H_{A,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2;x,y,z) &= \frac{\Gamma(\gamma_2)}{\Gamma(\beta_2)\Gamma(\gamma_2-\beta_2)} \\ &\times \int_0^\infty \xi^{\beta_2-1}(1+\xi)^{\alpha+\beta_1-\gamma_2}(1+\xi-y\xi)^{-\beta_1}(1+\xi-z\xi)^{-\alpha} \\ &\times \exp\left(-\frac{p(1+\xi)^2}{\xi}\right) {}_2F_1\left(\alpha,\beta_1;\gamma_1;\frac{x(1+\xi)^2}{(1+\xi-y\xi)(1+\xi-z\xi)}\right) d\xi, \end{aligned} \tag{3.4}$$

$$\begin{aligned} H_{A,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2;x,y,z) &= \frac{2\Gamma(\gamma_2)}{\Gamma(\beta_2)\Gamma(\gamma_2-\beta_2)} \\ &\times \int_0^{\pi/2} (\sin^2\xi)^{\beta_2-\frac{1}{2}}(\cos^2\xi)^{\gamma_2-\beta_2-\frac{1}{2}}(1-y\sin^2\xi)^{-\beta_1}(1-z\sin^2\xi)^{-\alpha} \\ &\times \exp\left(-\frac{p}{\sin^2\xi\cos^2\xi}\right) {}_2F_1\left(\alpha,\beta_1;\gamma_1;\frac{x}{(1-y\sin^2\xi)(1-z\sin^2\xi)}\right) d\xi, \end{aligned} \tag{3.5}$$

$$\begin{aligned} H_{A,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2;x,y,z) &= \frac{\Gamma(\gamma_2)}{\Gamma(\beta_2)\Gamma(\gamma_2-\beta_2)}\frac{(b-c)^{\beta_2}(a-c)^{\gamma_2-\beta_2}}{(b-a)^{\gamma_2-\alpha-\beta_1-1}} \\ &\times \int_a^b \frac{(\xi-a)^{\beta_2-1}(b-\xi)^{\gamma_2-\beta_2-1}}{(\xi-c)^{\gamma_2-\alpha-\beta_1}}[\sigma(\xi,y)]^{-\beta_1}[\sigma(\xi,z)]^{-\alpha} \\ &\times \exp\left(-\frac{p(b-a)^2(\xi-c)^2}{(a-c)(b-c)(\xi-a)(b-\xi)}\right) {}_2F_1\left(\alpha,\beta_1;\gamma_1;\rho(\xi,y,z)x\right) d\xi, \end{aligned} \tag{3.6}$$

where $\sigma(\xi,x)=(b-a)(\xi-c)-(b-c)(\xi-a)x$, $\rho(\xi,y,z)=\frac{(b-a)^2(\xi-c)^2}{\sigma(\xi,y)\sigma(\xi,z)}$, $c<a<b$, *and*

$$\begin{aligned}H_{A,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2;x,y,z)=&\frac{\Gamma(\gamma_2)(1+\lambda)^{\beta_2}}{\Gamma(\beta_2)\Gamma(\gamma_2-\beta_2)}\\&\times\int_0^1\frac{\xi^{\beta_2-1}(1-\xi)^{\gamma_2-\beta_2-1}}{(1+\lambda\xi)^{\gamma_2-\alpha-\beta_1}}[\tau(\xi,y)]^{-\beta_1}[\tau(\xi,z)]^{-\alpha}\\&\times\exp\left(-\frac{p(1+\lambda\xi)^2}{(1+\lambda)\xi(1-\xi)}\right){}_2F_1\left(\alpha,\beta_1;\gamma_1;\frac{(1+\lambda\xi)^2x}{\tau(\xi,y)\tau(\xi,z)}\right)d\xi,\end{aligned}\tag{3.7}$$

where $\tau(\xi,x)=1+\lambda\xi-(1+\lambda)\xi x$, $\lambda>-1$.

Proof. To get (3.1), it is enough to use (1.3) in (2.4). For the second integral representation (3.2), it is enough to use the following integral representation [14]

$${}_2F_1(a,b;c;x)=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-xt)^{-a}dt,\quad \Re(c)>\Re(b)>0,$$

in (3.1). The integral representation (3.3) can be immediately gotten by putting

$$[(1-yt)(1-zt)-x\xi]^{-\alpha}=(1-yt)^{-\alpha}(1-x\xi-zt)^{-\alpha}\left(1-\frac{xy\xi t}{(1-yt)(1-x\xi-zt)}\right)^{-\alpha},$$

in (3.2). The integral representations (3.4)-(3.7) can be easily proved by directly using the transformations $t=\frac{\xi}{1+\xi}$, $t=\sin^2\xi$, $t=\frac{(b-c)(\xi-a)}{(b-a)(\xi-c)}$ and $t=\frac{(1+\lambda)\xi}{1+\lambda\xi}$ in (3.1), respectively. □

4. Integral representations for $H_{B,p}$

Theorem 4.1. *The function* $H_{B,p}$ *has the following integral representations for* $\min\{\Re(\alpha),\Re(\beta_1)\}>0$*:*

$$\begin{aligned}H_{B,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x,y,z)=&\frac{\Gamma(\alpha+\beta_1)}{\Gamma(\alpha)\Gamma(\beta_1)}\\&\times\int_0^1 t^{\alpha-1}(1-t)^{\beta_1-1}\exp\left(-\frac{p}{t(1-t)}\right)\\&\times X_4(\alpha+\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;xt(1-t),y(1-t),zt)dt,\end{aligned}\tag{4.1}$$

$$\begin{aligned}H_{B,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x,y,z)=&\frac{2\Gamma(\alpha+\beta_1)}{\Gamma(\alpha)\Gamma(\beta_1)}\\&\times\int_0^{\pi/2}(\sin^2\xi)^{\alpha-\frac12}(\cos^2\xi)^{\beta_1-\frac12}\exp\left(-\frac{p}{\sin^2\xi\cos^2\xi}\right)\\&\times X_4(\alpha+\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x\sin^2\xi\cos^2\xi,y\cos^2\xi,z\sin^2\xi)d\xi,\end{aligned}\tag{4.2}$$

$$\begin{aligned}H_{B,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x,y,z)=&\frac{\Gamma(\alpha+\beta_1)}{\Gamma(\alpha)\Gamma(\beta_1)}\frac{(b-c)^{\alpha}(a-c)^{\beta_1}}{(b-a)^{\alpha+\beta_1-1}}\\&\times\int_a^b(\xi-a)^{\alpha-1}(b-\xi)^{\beta_1-1}(\xi-c)^{-\alpha-\beta_1}\exp\left(-\frac{p}{\sigma(1-\sigma)}\right)\\&\times X_4(\alpha+\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x\sigma(1-\sigma),y(1-\sigma),z\sigma)d\xi,\end{aligned}\tag{4.3}$$

where $\sigma=\frac{(b-c)(\xi-a)}{(b-a)(\xi-c)}$, $c<a<b$,

$$\begin{aligned}H_{B,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x,y,z) &= \frac{2\Gamma(\alpha+\beta_1)(1+\lambda)^\alpha}{\Gamma(\alpha)\Gamma(\beta_1)}\\ &\times\int_0^{\pi/2}\frac{(\sin^2\xi)^{\alpha-\frac{1}{2}}(\cos^2\xi)^{\beta_1-\frac{1}{2}}}{(1+\lambda\sin^2\xi)^{\alpha+\beta_1}}\exp\left(-\frac{p}{\sigma(1-\sigma)}\right)\\ &\times X_4(\alpha+\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x\sigma(1-\sigma),y(1-\sigma),z\sigma)d\xi,\end{aligned}\tag{4.4}$$

where $\sigma=\frac{(1+\lambda)\sin^2\xi}{1+\lambda\sin^2\xi},\ \lambda>-1,$

$$\begin{aligned}H_{B,p}(\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x,y,z) &= \frac{2\Gamma(\alpha+\beta_1)\lambda^\alpha}{\Gamma(\alpha)\Gamma(\beta_1)}\\ &\times\int_0^{\pi/2}\frac{(\sin^2\xi)^{\alpha-\frac{1}{2}}(\cos^2\xi)^{\beta_1-\frac{1}{2}}}{(\cos^2\xi+\lambda\sin^2\xi)^{\alpha+\beta_1}}\exp\left(-\frac{p}{\sigma(1-\sigma)}\right)\\ &\times X_4(\alpha+\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x\sigma(1-\sigma),y(1-\sigma),z\sigma)d\xi,\end{aligned}\tag{4.5}$$

where $\sigma=\frac{\lambda\sin^2\xi}{\cos^2\xi+\lambda\sin^2\xi},\lambda>0$. *Here, Exton's function* X_4 *is defined by* [18]

$$X_4(a_1,a_2;c_1,c_2,c_3;x,y,z)=\sum_{m,n,k=0}^{\infty}\frac{(a_1)_{2m+n+k}(a_2)_{n+k}}{(c_1)_m(c_2)_n(c_3)_k}\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^k}{k!},$$

where $2\sqrt{\mathfrak{r}}+(\sqrt{\mathfrak{s}}+\sqrt{\mathfrak{t}})^2<1$.

Proof. To obtain the first representation (4.1), it is enough to use (1.1) in (2.2). The other representations can be easily obtained by using the transformations $t=\sin^2\xi, t=\frac{(b-c)(\xi-a)}{(b-a)(\xi-c)}, t=\frac{(1+\lambda)\sin^2\xi}{1+\lambda\sin^2\xi}$ and $t=\frac{\lambda\sin^2\xi}{\cos^2\xi+\lambda\sin^2\xi}$ respectively. □

5. Integral representations for $H_{C,p}$

Theorem 5.1. *The function* $H_{C,p}$ *has the following integral representations under the assumption* $\Re(\gamma)>\Re(\alpha)>0$ *for* (5.1), (5.3)-(5.6) *and the assumption* $\min\{\Re(\alpha),\Re(\beta_1),\Re(\gamma-\alpha-\beta_1)\}>0$ *for* (5.2)*:*

$$\begin{aligned}H_{C,p}(\alpha,\beta_1,\beta_2;\gamma;x,y,z) &= \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)}\\ &\times\int_0^1 t^{\alpha-1}(1-t)^{\gamma-\alpha-1}(1-xt)^{-\beta_1}(1-zt)^{-\beta_2}\exp\left(-\frac{p}{t(1-t)}\right)\\ &\times{}_2F_1\left(\beta_1,\beta_2;\gamma-\alpha;\frac{y(1-t)}{(1-xt)(1-zt)}\right)dt,\end{aligned}\tag{5.1}$$

$$\begin{aligned}H_{C,p}(\alpha,\beta_1,\beta_2;\gamma;x,y,z) &= \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\beta_1)\Gamma(\gamma-\alpha-\beta_1)}\\ &\times\int_0^1\int_0^1 t^{\alpha-1}\xi^{\beta_1-1}(1-t)^{\gamma-\alpha-1}(1-\xi)^{\gamma-\alpha-\beta_1-1}(1-xt)^{\beta_2-\beta_1}\\ &\times(1-xt-y\xi-zt+yt\xi+zxt^2)^{-\beta_2}\exp\left(-\frac{p}{t(1-t)}\right)dtd\xi,\end{aligned}\tag{5.2}$$

$$\begin{aligned}H_{C,p}(\alpha,\beta_1,\beta_2;\gamma;x,y,z) &= \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)}\\ &\times\int_0^\infty \xi^{\alpha-1}(1+\xi)^{\beta_1+\beta_2-\gamma}(1+\xi-x\xi)^{-\beta_1}(1+\xi-z\xi)^{-\beta_2}\\ &\times\exp\left(-\frac{p(1+\xi)^2}{\xi}\right){}_2F_1\left(\beta_1,\beta_2;\gamma-\alpha;\frac{y(1+\xi)}{(1+\xi-x\xi)(1+\xi-z\xi)}\right)d\xi,\end{aligned}\tag{5.3}$$

$$H_{C,p}(\alpha,\beta_1,\beta_2;\gamma;x,y,z) = \frac{2\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)} \times \int_0^{\pi/2} (\sin^2\xi)^{\alpha-\frac{1}{2}}(\cos^2\xi)^{\gamma-\alpha-\frac{1}{2}}(1-x\sin^2\xi)^{-\beta_1}(1-z\sin^2\xi)^{-\beta_2} \times \exp\left(-\frac{p}{\sin^2\xi\cos^2\xi}\right) {}_2F_1\left(\beta_1,\beta_2;\gamma-\alpha;\frac{y\cos^2\xi}{(1-x\sin^2\xi)(1-z\sin^2\xi)}\right)d\xi, \tag{5.4}$$

$$H_{C,p}(\alpha,\beta_1,\beta_2;\gamma;x,y,z) = \frac{\Gamma(\gamma)(1+\lambda)^{\alpha}}{\Gamma(\alpha)\Gamma(\gamma-\alpha)} \times \int_0^1 \frac{\xi^{\alpha-1}(1-\xi)^{\gamma-\alpha-1}}{(1+\lambda\xi)^{\gamma-\beta_1-\beta_2}}[\tau(\xi,x)]^{-\beta_1}[\tau(\xi,z)]^{-\beta_2} \times \exp\left(-\frac{p(1+\lambda\xi)^2}{(1+\lambda)\xi(1-\xi)}\right) {}_2F_1\left(\beta_1,\beta_2;\gamma-\alpha;\frac{y(1+\lambda\xi)(1-\xi)}{\tau(\xi,x)\tau(\xi,z)}\right)d\xi, \tag{5.5}$$

where $\tau(\xi,x) = 1+\lambda\xi-(1+\lambda)\xi x,\ \lambda > -1,$

$$H_{C,p}(\alpha,\beta_1,\beta_2;\gamma;x,y,z) = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\gamma-\alpha)}\frac{(b-c)^{\alpha}(a-c)^{\gamma-\alpha}}{(b-a)^{\gamma-\beta_1-\beta_2-1}} \times \int_a^b \frac{(\xi-a)^{\alpha-1}(b-\xi)^{\gamma-\alpha-1}}{(\xi-c)^{\gamma-\beta_1-\beta_2}}[\sigma(\xi,x)]^{-\beta_1}[\sigma(\xi,z)]^{-\beta_2} \times \exp\left(-\frac{p(b-a)^2(\xi-c)^2}{(a-c)(b-c)(\xi-a)(b-\xi)}\right) {}_2F_1\left(\beta_1,\beta_2;\gamma-\alpha;\rho(\xi,x,z)y\right)d\xi, \tag{5.6}$$

where $\sigma(\xi,x) = (b-a)(\xi-c)-(b-c)(\xi-a)x,\ \rho(\xi,x,z) = \frac{(a-c)(b-a)(b-\xi)(\xi-c)}{\sigma(\xi,x)\sigma(\xi,z)},\ c < a < b.$

Proof. All the integral representations presented here can be easily obtained as in the proof of Theorem 3.1. □

6. Conclusions

In this work, the extended Srivastava's triple hypergeometric functions denoted by $H_{A,p}$, $H_{B,p}$ and $H_{C,p}$ are defined by using an extension of beta function. Besides, the single series representations of functions $H_{A,p}$ and $H_{C,p}$ are given in terms of extended Appell's hypergeometric function $F_{1,p}$. Finally, some integral representations for each of the extended Srivastava's triple hypergeometric functions are presented. The closed-form expressions of the integrals presented here, are presumably not available in the existing literature.

For $p = 0$, the special cases of all representations given in this paper can be found in [4, 8, 16, 18]. Furthermore, a variety of different integral representations of these new functions can also be provided by using the same transformations in [5–7].

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