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Localization property on absolute matrix summability factors of Fourier series

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Abstract

In this paper, a known theorem dealing with $\left|\bar{N}, p_n\right|_k$ summability methods of Fourier series is generalized to $|A, \theta_n|_k$ summability method by taking normal matrices.

Keywords: Weighted arithmetic mean, summability factors, absolute matrix summability, trigonometric Fourier series, infinite series, Hölder inequality, Minkowski inequality, almost increasing sequence, quasi-power increasing sequence.

1 Introduction

Let f be a periodic function with period 2π and integrable (L) over $(-\pi, \pi)$. Without any loss of generality the constant term in the constant term in the Fourier series of f can be taken to be zero, so that

$$(1) \quad f(t) \sim \sum_{n=1}^{\infty}(a_n cosnt + b_n sinnt) = \sum_{n=1}^{\infty} C_n(t).$$

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We write

(2) $\varphi(t) = \frac{1}{2}\{f(x+t) + f(x-t)\}$.

It is well known that the convergence of the Fourier series at $t = x$ is a local property of f (i.e., depends only on the behaviour of f in an arbitrarily small neighbourhood of x), and so the summability of the Fourier series $t = x$ by any regular linear summability method is also a local property of f.

2 The Known Results

It has been pointed out by Bosanquet [1] that for the case $\lambda_n = logn$, the definition of *absolutely summable* $(R, logn, 1)$ or *summable* $|R, logn, 1|$ is equivalent to the definition of the summability $|R, \lambda_n, 1|$ used by Mohanty [9], λ_n being a monotonic increasing sequence tending to infinity with n.
Matsumoto [7] improved this result by replacing the series $\sum (logn)^{-1} C_n(t)$ by $\sum (loglogn)^{-p} C_n(t), \quad p > 1$. Bhatt [2] showed that the factor $(loglogn)^{-p}$ in the above series can be replaced by the more general factor $\gamma_n logn$ where (γ_n) is a convex sequence such that $\sum n^{-1}\gamma_n$ is convergent. Borwein [6] generalized Bhatt's result by proving that (λ_n) is a sequence for which $\sum_{n=1}^{\infty} \frac{p_n}{P_n}|\lambda_n| < \infty \quad and \quad \sum_{n=1}^{\infty} |\Delta\lambda_n| < \infty$, then the summability $|R, P_n, 1|$ of the factored Fourier series $\sum_{n=1}^{\infty} \lambda_n C_n(t)$ at any point is a local property of f. On the other hand, Mishra [8] proved that if (γ_n) is as above, and if

(3) $P_n = O(np_n) \quad and \quad P_n \Delta p_n = O(p_n p_{n+1})$,

the summability $|\bar{N}, p_n|$ of the series $\sum_{n=1}^{\infty} \gamma_n \frac{P_n}{np_n} C_n(t)$, at any point is a local property of f. Bor [4] showed that $|\bar{N}, p_n|$ in Mishra's result can be replaced by a more general summability method $|\bar{N}, p_n|_k$ (see [3]), and Bor ([5]) introduced the main theorem on the local property of the summability $|\bar{N}, p_n|_k$ of the factored Fourier series, which generalizes most of the above results under more appropriate conditions then those given in them.

3 The Main Result

Many studies have been done for matrix generalization of Fourier series (see [11]-[26]). The aim of this paper is to extend Bor's main theorem in ([5]) for $|A, \theta_n|_k$ summability method (see [10], [18]) by taking normal matrices and by using Fourier series.

Theorem 3.1 Let $A = (a_{nv})$ be a positive normal matrix such that

(4) $\overline{a}_{n0} = 1, \ n = 0, 1, ...,$

(5) $a_{n-1,v} \geq a_{nv}, \ for \ n \geq v+1,$

(6) $\sum_{v=1}^{n-1} a_{vv} \hat{a}_{n,v+1} = O(a_{nn}).$

Let $(\theta_n a_{nn})$ be a non increasing sequence. If (λ_n) and (X_n) are sequences satisfying the following conditions:

(7) $\sum_{n=1}^{\infty} (\theta_n a_{nn})^{k-1} n^{-1} \left\{ |\lambda_n|^k + |\lambda_{n+1}|^k \right\} X_n^{k-1} < \infty,$

(8) $\sum_{n=1}^{\infty} (\theta_n a_{nn})^{k-1} (X_n^k + 1)|\Delta\lambda_n| < \infty,$

(9) $\Delta X_n = O(1/n),$

where $X_n = (na_{nn})^{-1}$, and (θ_n) is any sequence of positive constants, then the summability $|A, \theta_n|_k$, $k \geq 1$ of the series $\sum \lambda_n X_n C_n(t)$, at a point can be ensured by a local property.

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