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Absolute Matrix Summability Factors of Fourier Series with Quasi-f-Power Increasing Sequences

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Abstract

In this paper, we have generalized a main theorem dealing with quasi-f-power increasing sequence to $|A, \theta_n|_k$ summability method by using Fourier series.

Keywords: Weighted arithmetic mean, summability factors, absolute matrix summability, trigonometric Fourier series, infinite series, Hölder inequality, Minkowski inequality, almost increasing sequence, quasi-power increasing sequence.

1 Introduction

A positive sequence (b_n) is said to be an almost increasing sequence if there exists a positive increasing sequence (c_n) and two positive constants M and N such that $Mc_n \leq b_n \leq Nc_n$ (see [1]). A positive sequence $X = (X_n)$ is said to be quasi-f-power increasing sequence if there exists a constant $K = K(X, f) \geq 1$ such that $Kf_nX_n \geq f_mX_m$ for all $n \geq m \geq 1$, where

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$f = \{f_n(\sigma,\beta)\} = \left\{n^\sigma (logn)^\beta), \quad \beta \geq 0, \quad 0 < \sigma < 1\right\}$ (see [18]).
If we take $\beta = 0$, then we have a quasi-σ-power increasing sequence. Let (p_n) be a sequence of positive real numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty \quad as \quad n \to \infty, \quad (P_{-i} = p_{-i} = 0, \quad i \geq 1).$$

The sequence-to-sequence transformation $w_n = \frac{1}{P_n}\sum_{v=0}^{n} p_v s_v$ defines the sequence (w_n) of the weighted arithmetic mean or simply the $(\bar{N}, p_n)$ mean of the sequence (s_n) generated by the sequence of coefficients (p_n) (see [9]).
The series $\sum a_n$ is said to be summable $\left|\bar{N}, p_n\right|_k$, $k \geq 1$, if (see [2])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} \mid w_n - w_{n-1} \mid^k < \infty.$$

Let $A = (a_{nv})$ be a normal matrix, i.e., a lower triangular matrix of nonzero diagonal entries. Then A defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $As = (A_n(s))$, where

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v, \quad n = 0, 1, ...$$

The series $\sum a_n$ is said to be summable $|A, \theta_n|_k$, $k \geq 1$, if (see [10],[17])

$$\sum_{n=1}^{\infty} \theta_n^{k-1} \left|\bar{\Delta} A_n(s)\right|^k < \infty,$$

where (θ_n) is any sequence of positive constants and

$$\bar{\Delta} A_n(s) = A_n(s) - A_{n-1}(s).$$

2 An application of absolute matrix summability of trigonometric Fourier series

Let f be a periodic function with period 2π and integrable (L) over $(-\pi, \pi)$. The trigonometric Fourier series of f is defined as

$$f(t) \sim \sum_{n=1}^{\infty} (a_n cosnt + b_n sinnt) = \sum_{n=1}^{\infty} C_n(t)$$

where
$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)dt, \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t) cos(nt)dt, \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t) sin(nt)dt.$
We write $\phi(t) = \frac{1}{2}\left\{f(x+t) + f(x-t)\right\}, \quad \phi_\alpha(t) = \frac{\alpha}{t^\alpha}\int_0^t (t-u)^{\alpha-1}\phi(u)\, du, \quad (\alpha >$

0). It is well known that if $\phi(t) \in \mathcal{BV}(0,\pi)$, then $t_n(x) = O(1)$, where $t_n(x)$ is the $(C,1)$ mean of the sequence $(nC_n(x))$ (see [8]).
The Fourier series play an important role in many areas of applied mathematics and mechanics. Recently some papers have been done concerning absolute matrix summability of infinite series and Fourier series (see [3]-[6], [11]-[26]). Using this fact Bor proved the following theorem concerning a quasi-f-power increasing sequence.

Theorem 2.1 [7] *Let (X_n) be a quasi-f-power increasing sequence. If $\phi_1(t) \in \mathcal{BV}(0,\pi)$, and the sequences (X_n), (λ_n), and (p_n) satisfy the following conditions*

(1) $$\lambda_m X_m = O(1) \quad as \quad m \to \infty,$$

(2) $$\sum_{n=1}^{m} n X_n |\Delta^2 \lambda_n| = O(1) \quad as \quad m \to \infty,$$

(3) $$\sum_{n=1}^{m} \frac{P_n}{n} = O(P_m)$$

(4) $$\sum_{n=1}^{m} \frac{p_n}{P_n} |t_n(x)|^k = O(X_m) \quad as \quad m \to \infty,$$

(5) $$\sum_{n=1}^{m} \frac{|t_n(x)|^k}{n} = O(X_m) \quad as \quad m \to \infty,$$

(6) $$\sum_{n=1}^{m} \frac{p_n}{P_n} \frac{|t_n|^k}{X_n^{k-1}} = O(X_m) \quad as \quad m \to \infty,$$

(7) $$\sum_{n=1}^{m} \frac{|t_n(x)|^k}{n X_n^{k-1}} = O(X_m) \quad as \quad m \to \infty,$$

then the series $\sum C_n(x)\lambda_n$ is summable $\left|\bar{N}, p_n\right|_k$, $k \geq 1$.

The aim of this paper is to generalize Bor's the above main theorem involving quasi-f-power increasing sequence (see ([7])) for $|A, \theta_n|_k$ summability method. Also, we use the following lemma for the proof of our theorem.

Lemma 2.2 [3] *Under the conditions of Theorem 2.1 we have that*

(8) $$n X_n |\Delta \lambda_n| = O(1) \quad as \quad n \to \infty,$$

(9) $$\sum_{n=1}^{\infty} X_n |\Delta \lambda_n| < \infty.$$

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