Variations on lacunary statistical quasi Cauchy sequences

Cite as: AIP Conference Proceedings **2086**, 030045 (2019); https://doi.org/10.1063/1.5095130
Published Online: 02 April 2019

Sebnem Yildiz

Variations on lacunary statistical quasi Cauchy sequences

Sebnem Yildiz

Sebnem Yildiz, Ahi Evran University, Department of Mathematics, Kirsehir-Turkey

sebnemyildiz@ahievran.edu.tr

Abstract. In this paper, we introduce a concept of lacunary statistically p-quasi-Cauchyness of a real sequence in the sense that a sequence (α_k) is lacunary statistically p-quasi-Cauchy if $\lim_{r\to\infty} \frac{1}{h_r}|\{k \in I_r : |\alpha_{k+p} - \alpha_k| \geq \varepsilon\}| = 0$ for each $\varepsilon > 0$. A function f is called lacunary statistically p-ward continuous on a subset A of the set of real numbers $\mathbb{R}$ if it preserves lacunary statistically p-quasi-Cauchy sequences, i.e. the sequence $f(\mathbf{x}) = (f(\alpha_n))$ is lacunary statistically p-quasi-Cauchy whenever $\alpha = (\alpha_n)$ is a lacunary statistically p-quasi-Cauchy sequence of points in A. It turns out that a real valued function f is uniformly continuous on a bounded subset A of $\mathbb{R}$ if there exists a positive integer p such that f preserves lacunary statistically p-quasi-Cauchy sequences of points in A.

Keywords: Lacunary statistical convergence, quasi-Cauchy sequences, continuity
PACS: 02.30.Lt, 02.30.Sa

INTRODUCTION

In [19] (see also [20]) Fridy and Orhan introduced the concept of lacunary statistically convergence in the sense that a sequence (α_k) of points in $\mathbb{R}$ is called lacunary statistically convergent, or S_θ-convergent, to an element L of $\mathbb{R}$ if $\lim_{r\to\infty} \frac{1}{h_r}|\{k \in I_r : |\alpha_k - L| \geq \varepsilon\}| = 0$ for every positive real number ε where $I_r = (k_{r-1}, k_r]$ and $k_0 = 0, h_r : k_r - k_{r-1} \to \infty$ as $r \to \infty$ and $\theta = (k_r)$ is an increasing sequence of positive integers (see also [28], [33], [26], and [27]). In this case we write $S_\theta - lim\alpha_k = L$. The set of lacunary statistically convergent sequences of points in $\mathbb{R}$ is denoted by S_θ. In the sequel, we will always assume that $\lim inf_r\ q_r > 1$. A sequence (α_k) of points in $\mathbb{R}$ is called lacunary statistically quasi-Cauchy if $S_\theta - lim\Delta\alpha_k = 0$, where $\Delta\alpha_k = \alpha_{k+1} - \alpha_k$ for each positive integer k. The set of lacunary statistically quasi-Cauchy sequences will be denoted by ΔS_θ. Using the idea of continuity of a real function in terms of sequences in the sense that a function preserves a certain kind of sequences, many kinds of continuities were introduced and investigated, not all but some of them we recall in the following: slowly oscillating continuity ([3]), quasi-slowly oscillating continuity ([18]), ward continuity ([4]), δ-ward continuity ([6]), statistical ward continuity ([7]), [9], [31]) and N_θ-ward continuity ([8]) which enabled some authors to obtain conditions on the domain of a function for some characterizations of uniform continuity (see [32, Theorem 6],[2, Theorem 1 and Theorem 2],[18, Theorem 2.3], [2, Theorem 1], and [15, Theorem 5].

The purpose of this paper is to introduce lacunary statistically p-quasi-Cauchy sequences, and prove some theorems.

Variations on lacunary statistical p ward compactness

Definition 1 *A sequence (α_k) of points in $\mathbb{R}$ is called lacunary statistically p-quasi-Cauchy if* $st{-}\lim_{k\to\infty} \Delta_p\alpha_k = 0$, *ie.* $\lim_{r\to\infty} \frac{1}{h_r}|\{k \in I_r : |\Delta_p\alpha_k| \geq \varepsilon\}| = 0$ *for each $\varepsilon > 0$, where $\Delta_p\alpha_k = \alpha_{k+p} - \alpha_k$ for every $k \in \mathbb{N}$.*

Definition 2 *A subset A of $\mathbb{R}$ is called lacunary statistically p-ward compact if any sequence of points in A has a lacunary statistically p-quasi-Cauchy subsequence.*

Since any lacunary statistically quasi-Cauchy sequence is lacunary statistically p-quasi-Cauchy we see that any lacunary statistically ward compact subset of $\mathbb{R}$ is lacunary statistically p-ward compact for any $p \in \mathbb{N}$. A finite subset of

International Conference of Mathematical Sciences (ICMS 2018)
AIP Conf. Proc. 2086, 030045-1–030045-4; https://doi.org/10.1063/1.5095130
Published by AIP Publishing. 978-0-7354-1816-5/$30.00

$\mathbb{R}$ is lacunary statistically p-ward compact, the union of finite number of lacunary statistically p-ward compact subsets of $\mathbb{R}$ is lacunary statistically p-ward compact, and the intersection of any family of lacunary statistically p-ward compact subsets of $\mathbb{R}$ is lacunary statistically p-ward compact. Furthermore any subset of a lacunary statistically p-ward compact set of $\mathbb{R}$ is lacunary statistically p-ward compact and any bounded subset of $\mathbb{R}$ is lacunary statistically p-ward compact. These observations above suggest to us the following.

Theorem 1 *A subset A of $\mathbb{R}$ is bounded if and only if there exists some $p \in \mathbb{N}$ such that A is lacunary statistically p-ward compact.*

Corollary 1 A subset of $\mathbb{R}$ is statistically p ward compact if and only if it is statistically q ward compact for any $p, q \in \mathbb{N}$.

Corollary 2 A subset of $\mathbb{R}$ is statistically p ward compact if and only if it is both statistically upward half compact and statistically downward half compact.

Corollary 3 A subset of $\mathbb{R}$ is statistically p ward compact for some $p \in \mathbb{N}$ if and only if it is both lacunary statistically upward half compact and lacunary statistically downward half compact.

Variations on lacunary statistical p ward continuity

In this section, we investigate connections between uniformly continuous functions and lacunary statistically p-ward continuous functions. A function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is continuous if and only if it preserves statistically convergent sequences. Using this idea, we introduce statistical p-ward continuity.

Definition 3 *A function f is called lacunary statistically p-ward continuous on a subset A of $\mathbb{R}$ if it preserves lacunary statistically p-quasi-Cauchy sequences, i.e. the sequence $f(\boldsymbol{x}) = (f(\alpha_n))$ is lacunary statistically p-quasi-Cauchy whenever $\alpha = (\alpha_n)$ is lacunary statistically p-quasi-Cauchy of points in A.*

Theorem 2 *If f is lacunary statistically p-ward continuous on a subset A of $\mathbb{R}$ for some $p \in \mathbb{N}$, then it is lacunary statistically ward continuous on A.*

Corollary 4 *If f is lacunary statistically p-ward continuous on a subset A of $\mathbb{R}$, then it is continuous on A in the ordinary case.*

Theorem 3 *Lacunary statistical p-ward continuous image of any lacunary statistically p-ward compact subset of $\mathbb{R}$ is lacunary statistically p-ward compact.*

Corollary 5 *Lacunary statistical p-ward continuous image of any G-sequentially connected subset of $\mathbb{R}$ is G-sequentially connected for a regular subsequential method G (see [5], [22], and [13]).*

Theorem 4 *If f is uniformly continuous on a subset A of $\mathbb{R}$, then $(f(\alpha_n))$ is lacunary statistically p-quasi-Cauchy whenever (α_n) is a p-quasi-Cauchy sequence of points in A.*

Corollary 6 *If f is slowly oscillating continuous on a bounded subset A of $\mathbb{R}$, then $(f(\alpha_n))$ is lacunary statistically p-quasi-Cauchy whenever (α_n) is a p quasi-Cauchy sequence of points in A.*

It is well-known that any continuous function on a compact subset A of $\mathbb{R}$ is uniformly continuous on A. We have an analogous theorem for a lacunary statistically p-ward continuous function defined on a lacunary statistically p-ward compact subset of $\mathbb{R}$.

Theorem 5 *If a function is lacunary statistically p-ward continuous on a lacunary statistically p-ward compact subset of $\mathbb{R}$, then it is uniformly continuous on A.*

Corollary 7 *If a function defined on a bounded subset of $\mathbb{R}$ is lacunary statistically p-ward continuous, then it is uniformly continuous.*

We note that when the domain of a function is restricted to a bounded subset of $\mathbb{R}$, lacunary statistically p-ward continuity implies not only ward continuity, but also slowly oscillating continuity.

It is well-known that the uniform limit of continuous functions on a subset A of $\mathbb{R}$ is continuous on A. We have an analogous theorem for a lacunary statistically p-ward continuous function that the uniform limit of lacunary statistically p-ward continuous functions on a subset A of $\mathbb{R}$ is lacunary statistically p-ward continuous on A.

Theorem 6 *If a sequence of lacunary statistically p-ward continuous function is uniformly convergent to a function f on a subset of $\mathbb{R}$, then f is lacunary statistically p-ward continuous*

Theorem 7 *The set of lacunary statistically p-ward continuous functions on a subset A of R is a closed subset of the space of continuous functions.*

Theorem 8 *The set of lacunary statistically p-ward continuous functions on a subset A of R is a complete subset of the space of continuous functions on A.*

Conclusion

In this paper, mainly a new types of continuity, namely lacunary statistically p-ward continuity of a real function are introduced and investigated. In this investigation we have obtained results related to lacunary statistically p-ward continuity, some other kinds of continuities via lacunary statistically p-quasi Cauchy sequences, convergent sequences, statistical convergent sequences, lacunary statistical convergent sequences of points in **R**. We not that the set of lacunary statistically p-ward continuous functions is a proper subset of the set of ordinary continuous functions. We suggest to investigate lacunary statistically p-quasi Cauchy sequences of fuzzy points in fuzzy spaces, and in soft spaces (see [10], [21] for the definitions and related concepts in fuzzy setting, and see [1], and [17] for the soft setting). We also suggest to investigate lacunary statistically p-quasi Cauchy double sequences (see for example [11], [24], [16] and [25] for the definitions and related concepts in the double sequences case). For another further study, we suggest to investigate lacunary statistically p-quasi Cauchy sequences of points in a cone metric space ([23], [12], [29], [30], and [14]).
Acknowledgment: The author acknowledges that the full paper of the results of this extended abstract is to appear in Maltepe Journal of Mathematics ([34]).

REFERENCES

[1] C.G. Aras, A. Sonmez, H. Cakalli, An approach to soft functions, J. Math. Anal. **8**, 2, 129-138, (2017).
[2] D. Burton, and J. Coleman, Quasi-Cauchy Sequences, Amer. Math. Monthly **117**, 4, 328-333, (2010).
[3] H.Çakallı, Slowly oscillating continuity, Abstr. Appl. Anal. Hindawi Publ. Corp., New York, ISSN 1085-3375, Volume 2008, Article ID 485706, (2008). MR **2009b**:26004
[4] H. Çakalli, Forward continuity, J. Comput. Anal. Appl. **13** (2011), 225-230.
[5] H. Çakallı, On *G*-continuity, Comput. Math. Appl. **61** 2 (2011), 313-318.
[6] H. Çakallı, *δ-statistically quasi-Cauchy sequences*, Math. Comput. Modelling, **53**, no. 1-2, 397-401, (2011).
[7] H. Çakalli, Statistical ward continuity. Appl. Math. Lett. **24** (2011) 1724-1728.
[8] H. Cakalli, N-theta-ward continuity, Abstr. Appl. Anal. **2012** (2012), Article ID 680456 8 pages.
[9] H. Cakalli, A new approach to statistically quasi Cauchy sequences, Maltepe Journal of Mathematics, **1**, 1, 1-8, (2019).
[10] H. Çakalli and Pratulananda Das, *Fuzzy compactness via summability*, Appl. Math. Lett. **22**, 11, 1665-1669, (2009).
[11] H. Çakalli, R.F. Patterson, Functions preserving slowly oscillating double sequences, An. Stiint. Univ. Al. I. Cuza Iasi. Mat. (N.S.) **62**, 2, vol. 2. 531-536, (2016).
[12] H. Çakallı, A. Sonmez, and Ç. Genç, On an equivalence of topological vector space valued cone metric spaces and metric spaces, Appl. Math. Lett. **25**, 3, 429-433, (2012).
[13] H. Cakalli and O. Mucuk, On connectedness via a sequential method, Revista de la Union Matematica Argentina, **54**(2) (2013), 101-109.
[14] H. Cakalli, A. Sonmez, *Slowly oscillating continuity in abstract metric spaces* Filomat, **27**, 5, 925-930, (2013).

[15] H. Çakalli, A. Sonmez, and C.G. Aras, λ-statistical ward continuity, An. Stiint. Univ. Al. I. Cuza Iasi. Mat. (N.S.) DOI: 10.1515/aicu-2015-0016 March 2015.

[16] D. Djurcic, Ljubisa D.R. Kocinac, M.R. Zizovic, Double sequences and selections, Abstr. Appl. Anal. Art. ID 497594, 6 pp, (2012).

[17] A.E. Coskun, C.G Aras, H. Cakalli, and A. Sonmez, Soft matrices on soft multisets in an optimal decision process, AIP Conference Proceedings, **1759**, 1, 020099 (2016); doi: 10.1063/1.4959713

[18] I. Canak, and M. Dik, New Types of Continuities, Abstr. Appl. Anal., Hindawi Publ. Corp., New York, ISSN 1085-3375, Volume 2010, Article ID 258980, (2010). doi:10.1155/2010/258980

[19] Fridy, J.A. and Orhan, C., Lacunary statistical convergence, Pacific J. Math., **160** 1, 43-51 (1993)

[20] Fridy, J.A. and Orhan, C., Lacunary statistical summability, J. Math. Anal. Appl, **173** 2, 497-504 (1993)

[21] Ljubisa D.R. Kočinac, Selection properties in fuzzy metric spaces, Filomat, **26**(2) (2012), 99-106.

[22] O. Mucuk, T. Şahan On G-Sequential Continuity, Filomat, **28**(6) (2014), 1181-1189.

[23] S.K. Pal, E. Savas, and H. Cakalli, I-convergence on cone metric spaces, Sarajevo J. Math. **9**, 85-93, (2013).

[24] R.F. Patterson and H. Cakalli, Quasi Cauchy double sequences, Tbilisi Math. J., **8**, 2, 211-219, (2015).

[25] Richard F. Patterson, and E. Savas, Asymptotic equivalence of double sequences, Hacet. J. Math. Stat. **41**, 4, 487-497, (2012).

[26] H.Sengul, M.Et, On (λ, I)-statistical convergence of order α of sequences of function. Proc. Nat. Acad. Sci.India Sect. A, **88**, no.2, 181-186, (2018).

[27] H.Sengul, M.Et, On I-lacunary statistical convergence of order α of sequences of sets. Filomat **31**, no.8, 2403-2412, (2018).

[28] M.Et, H.Sengul, On pointwise lacunary statistical convergence of order α of sequences of function. Proc. Nat. Acad. Sci.India Sect. A., **85**, no.2, 253-258, (2015).

[29] A. Sonmez, On paracompactness in cone metric spaces, Appl. Math. Lett. **23**, 494-497, (2010).

[30] A. Sonmez, and H. Cakalli, Cone normed spaces and weighted means, Math. Comput. Modelling **52** (2010) 1660-16660.

[31] I. Taylan, Abel statistical delta quasi Cauchy sequences of real numbers, Maltepe Journal of Mathematics, **1**, 1, 18-23, (2019).

[32] R.W. Vallin, Creating slowly oscillating sequences and slowly oscillating continuous functions, With an appendix by Vallin and H. Cakalli, Acta Math. Univ. Comenianae, **25**, 1, 71-78, (2011).

[33] Ş. Yıldız, İstatistiksel boşluklu delta 2 quasi Cauchy dizileri, Sakarya University Journal of Science, **21**, 6, (2017). DOI: 10.16984/saufenbilder.336128 , http://www.saujs.sakarya.edu.tr/issue/26999/336128

[34] Ş. Yıldız, Lacunary statistical p-quasi Cauchy sequences, Maltepe Journal of Mathematics, **1**, 1, 9-17, (2019).