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Corrigendum

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Corrigendum to: Parabolic oblique derivative problem with discontinuous coefficients in generalized weighted Morrey spaces

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We rectify an error in the proof of the global weighted Morrey-type regularity of the solution of the regular oblique derivative problem for linear uniformly parabolic operators with VMO coefficients.

Let $\Omega \subset \mathbb{R}^n$, $n \geq 1$ be a bounded $C^{1,1}$-domain, $Q = \Omega \times (0,T)$ be a cylinder in $\mathbb{R}_+^{n+1}$, and $S = \partial\Omega \times (0,T)$ stands for the lateral boundary of Q. We consider the problem

$$\begin{cases} \mathcal{P}u := u_t - a^{ij}(x)D_{ij}u = f & \text{a.e. in} \quad Q, \\ \mathfrak{I}u := u(x',0) = 0, & \text{on} \quad \Omega, \\ \mathfrak{B}u := \partial u/\partial l = l^i(x)D_i u = 0 & \text{on} \quad S, \end{cases} \tag{3}$$

under the following conditions (i) and (ii) in [5].

The unique strong solvability of this problem was proved in [9]. In [10] Softova studied the regularity of the solution in the Morrey spaces and in [11] extended these studies on generalized Morrey spaces $M^{p,\varphi}$ with a Morrey function φ satisfying Guliyev type conditions considered in [6].

Remark A. *The density of the C_0^∞ functions in the weighted Lebesgue space L_w^p is proved in [8, Chapter 3, Theorem 3.11].*

Definition 2.9. *A measurable function $\mathfrak{K}(x;\xi) : \mathbb{R}^{n+1} \times \mathbb{R}^{n+1} \setminus \{0\} \to \mathbb{R}$ is called variable parabolic Calderón-Zygmund kernel ($PCZK$) if:*

i) $\mathfrak{K}(x;\cdot)$ *is a PCZK for a.a.* $x \in \mathbb{R}^{n+1}$*:*
  a) $\mathfrak{K}(x;\cdot) \in C^\infty(\mathbb{R}^{n+1} \setminus \{0\})$,
  b) $\mathfrak{K}(x;(\mu y', \mu^2\tau)) = \mu^{-(n+2)}\mathfrak{K}(x;y), \ \forall \mu > 0, \ y = (y',\tau)$,
  c) $\int_{\mathbb{S}^n} \mathfrak{K}(x;y)d\sigma_y = 0, \ \int_{\mathbb{S}^n} |\mathfrak{K}(x;y)|d\sigma_y < +\infty$.

ii) $\left\| D_y^\beta \mathfrak{K} \right\|_{\infty;\mathbb{R}^{n+1}\times\mathbb{S}^n} \leq M(\beta) < \infty$
for each multi-index β.

Note that, the Theorems 2.10, Theorem 2.13 and Corollaries 2.11, 2.12, 2.14 in [5] the isotropic case are obtained in [4] and treat continuity in $M^{p,\varphi}(\mathbb{R}^{n+1},\omega)$ of certain parabolic singular and parabolic nonsingular integrals.

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3 Proof of the main result

Suppose that $u \in \overset{\circ}{W}{}^{p}_{2,1}(Q,\omega)$ is a solution of (3). Note that the solution of (3) exists according to Remark A (see also, [1], [12]).

We are going to show that $f \in M^{p,\varphi}(Q,\omega)$ implies $u \in \overset{\circ}{W}{}^{p,\varphi}_{2,1}(Q,\omega)$. For this goal we obtain an a priori estimate of u. Following the method used by Chiarenza, Frasca and Longo in [2] and [3], we prove the results considering two steps: interior estimate and boundary estimate.

In the proof of the interior estimate after inequality (23) in [5] we need the following interpolation inequality.

Lemma 3.1 (Interpolation inequality)**.** *There exists a constant C independent of r such that*

$$\Theta_1 \le \varepsilon\Theta_2 + \frac{C}{\varepsilon}\Theta_0 \qquad \textit{for any } \varepsilon \in (0,2).$$

Proof. For functions $u \in W^{p,\omega}_{2,1}(\mathcal{C}_r)$, $p \in (1,\infty)$ and $\omega \in A_p$ we dispose with the following interpolation inequality which may be proved analogously in [7].

$$\|Du\|_{p,\omega;\mathcal{C}_r} \le C\left(\|u\|_{p,\omega;\mathcal{C}_r} + \|u\|^{1/2}_{p,\omega;\mathcal{C}_r}\left(\|D_t u\|_{p,\omega;\mathcal{C}_r} + \|D^2u\|_{p,\omega;\mathcal{C}_r}\right)^{1/2}\right).$$

Then for any $\epsilon > 0$ we have

$$\|Du\|_{p,\omega;\mathcal{C}_r} \le C\left(\left(1+\frac{1}{2\epsilon}\right)\|u\|_{p,\omega;\mathcal{C}_r} + \frac{\epsilon}{2}\left(\|D_t u\|_{p,\omega;\mathcal{C}_r} + \|D^2u\|_{p,\omega;\mathcal{C}_r}\right)\right).$$

Then for any $\epsilon > 0$ we have

$$\|Du\|_{p,\omega;\mathcal{C}_r} \le C\left(\left(1+\frac{1}{2\epsilon}\right)\|u\|_{p,\omega;\mathcal{C}_r} + \frac{\epsilon}{2}\left(\|D_t u\|_{p,\omega;\mathcal{C}_r} + \|D^2u\|_{p,\omega;\mathcal{C}_r}\right)\right).$$

Choosing ϵ small enough, taking $\delta = \frac{C\epsilon}{2} < 1$, dividing all terms of $\varphi(x,r)w(\mathcal{C}_r)^{1/p}$ and taking the supremum over $\mathcal{C}_r$ we get the desired interpolation inequality in $M_{p,\varphi}(\omega)$

$$\|Du\|_{p,\varphi,\omega;\mathcal{C}_r} \le \delta\left(\|D_t u\|_{p,\varphi,\omega;\mathcal{C}_r} + \|D^2u\|_{p,\varphi,\omega;\mathcal{C}_r}\right) + \frac{C}{\delta}\|u\|_{p,\varphi,\omega;\mathcal{C}_r}. \tag{$*$}$$

We can always find some $\theta_0 \in (0,1)$ such that

$$\begin{aligned}\Theta_1 &\le 2[\theta_0(1-\theta_0)r]\|Du\|_{p,\varphi,\omega;\mathcal{C}_{\theta_0 r}}\\ &\le 2[\theta_0(1-\theta_0)r]\left(\delta\left(\|D_t u\|_{p,\varphi,\omega;\mathcal{C}_r} + \|D^2u\|_{p,\varphi,\omega;\mathcal{C}_r}\right) + \frac{C}{\delta}\|u\|_{p,\varphi,\omega;\mathcal{C}_{\theta_0 r}}\right).\end{aligned}$$

The assertion follows choosing $\delta = \frac{\varepsilon}{2}[\theta_0(1-\theta_0)r] < \theta_0 r$ for any $\varepsilon \in (0,2)$. □

The interpolation inequality (see Lemma 3.1) gives that there exists a positive constant C independent of r such that

$$\Theta_1 \le \varepsilon\Theta_2 + \frac{C}{\varepsilon}\Theta_0 \quad \text{for any } \varepsilon \in (0,2).$$

Thus from (23) in [5] becomes

$$[\theta(1-\theta)r]^2\left(\|D_t u\|_{p,\varphi,\omega;\mathcal{C}_r} + \|D^2u\|_{p,\varphi,\omega;\mathcal{C}_r}\right) \le \Theta_2 \le C(r^2\|f\|_{p,\varphi,\omega;Q} + \Theta_0)$$

for each $\theta \in (0,1)$. Taking $\theta = 1/2$ we get the Caccioppoli-type estimate

$$\|D_t u\|_{p,\varphi,\omega;\mathcal{C}_{r/2}(x_0)} + \|D^2u\|_{p,\varphi,\omega;\mathcal{C}_{r/2}(x_0)} \le C\left(\|f\|_{p,\varphi,\omega;Q} + \frac{1}{r^2}\|u\|_{p,\varphi,\omega;\mathcal{C}_r(x_0)}\right).$$

Consider cylinders $Q' = \Omega' \times (0,T)$ and $Q'' = \Omega'' \times (0,T)$ with $\Omega' \Subset \Omega'' \Subset \Omega$, by standard covering procedure and partition of the unity we get

$$\|u\|_{W^{p,\varphi}_{2,1}(Q',\omega)} \le C\left(\|f\|_{p,\varphi,\omega;Q} + \|u\|_{p,\varphi,\omega;Q''}\right), \tag{24}$$

where C depends on $n,\ p,\ [\omega]_{A_p},\ \Lambda,\ T,\ \|D\Gamma\|_{\infty;Q},\ \eta_{\mathbf{a}}(r),\ \|\mathbf{a}\|_{\infty,Q}$ and $dist(\Omega',\partial\Omega'')$.

In the proof of the boundary estimates after equality (27) in [5] applying Theorem 2.10 and Corollary 2.11 in [5], the interpolation inequality $(*)$, taking into account the VMO properties of the coefficients a^{ij}'s, it is possible to choose R_0 small

$$\|u\|_{W^{p,\varphi}_{2,1}(\mathcal{C}^+_R,\omega)} \le C\|f\|_{p,\varphi,\omega;\mathcal{C}^+_R} \tag{30}$$

for all $R < R_0$ (see [9, 11] for details). Making a covering $\{\mathcal{C}^+_\alpha\}$, $\alpha \in \mathcal{A}$ such that $Q \setminus Q' \subset \bigcup_{\alpha\in\mathcal{A}} \mathcal{C}^+_\alpha$, considering a partition of unity subordinated to that covering and applying (30) for each $\mathcal{C}^+_\alpha$ we get

$$\|u\|_{W^{p,\varphi}_{2,1}(Q\setminus Q',\omega)} \le C\|f\|_{p,\varphi,\omega;Q} \tag{31}$$

with a constant depending on $n,\ p,\ [\omega]_{A_p},\ \Lambda,\ T,\ diam\,\Omega,\ \|D\Gamma\|_{\infty;Q},\ \eta_{\mathbf{a}},\ \|\mathbf{a}\|_{\infty;Q},\ \|l\|_{Lip(\bar{\mathcal{S}})}$, and $\|Dl\|_{\infty,\mathcal{S}}$.

The main estimate (11) in the Theorem 2.8 in [5] follows from (24) and (31).

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