A Novel Method for Numerical Analysis of Microwave Surface Resistance of Type-II Superconductors

Sukru Yildiz, Fedai Inanir, and Ahmet Cicek

***Abstract*—Microwave surface resistance of type-II superconductors is investigated via the finite-element method by calculating the electromagnetic field distribution within a superconductor. The software allows changing the geometrical parameters in a facile way. Using a finite-element procedure and the approximation by Bonura *et al.* [Eur. Phys. J. B 52 (4), 459 (2006); Eur. Phys. J. B 53 (3), 315 (2006)], it is shown that the microwave surface resistance of type-II superconductors can be numerically calculated in the framework of the critical-state model.**

***Index Terms*—Component modeling, electromagnetic (EM) simulation, finite-element method (FEM), microwave superconductivity.**

I. Introduction

RESEARCHERS have paid a great deal of attention to the electromagnetic response of superconducting materials for both explanation of superconductivity and their broad range of applications. Microwave surface resistance and critical current density play fundamental roles in the performance of superconductors in the possible microwave applications as active or passive components.

First, the phenomenological model for the electrodynamics of the superconductors in the mixed state was introduced by Gittleman and Rosenblum [1], in which the model has been improved by Coffey and Clem (CC) [2]–[4] and Brandt (B) [5]. The CC theory, apart from the flux motion, considers magnetic field dependence of densities of normal and superconducting fluids. When critical-state models are considered to be based on flux distribution caused by strong pinning effects, it can be concluded that calculation of microwave surface resistance via CC or B theories can be adapted to the critical-state model. From this consideration, Bonura *et al.* [6], [7] proposed a method that takes into account the flux distribution inside a superconducting sample in the critical state in the framework of the CC theory. Yildiz *et al.* [8], [9] successfully applied the aforementioned procedure to the microwave surface resistance considering the effect of different flux dynamics. The CC theory [2]–[4] and subsequently the method of Bonura *et al.* [6], [7] can only be applied to bulk samples since the flux distribution is unknown in the thin sample limit where the distribution significantly differs from that in bulk materials. This type of geometrical dependence could possibly be eliminated by employing the finite-element method (FEM). Recently, the FEM has been intensively applied to develop numerical solutions of levitation, magnetization, and ac loss problems in superconductors [10]–[19].

In this paper, a numerical model is proposed to simulate the performance of a superconducting sample exposed to a microwave field. The model allows both prediction of field profiles inside the superconductor in the critical state and calculation of microwave surface resistance. Results of surface resistance calculations as a function of applied magnetic field are presented.

II. Model for Microwave Surface Resistance

In the superconducting state, complex surface impedance is defined by the complex penetration depth, i.e., $\tilde{\lambda}$, as follows:

$$Z_S = i\mu_0\omega\tilde{\lambda}(\omega, B, T). \tag{1}$$

Here, μ_0 and ω are permeability of vacuum ($\mu_0 = 4\pi.10^{-7}$ Hm^{-1}) and the working angular frequency, respec tively. The surface resistance, i.e., R_s, can be extracted when the imaginary value of complex penetration depth ($\tilde{\lambda}$) is known, i.e.,

$$R_S = -\mu_0\omega\mathrm{Im}\left[\tilde{\lambda}(\omega, B, T)\right]. \tag{2}$$

Different approximations have been proposed for the calculation of the general behavior of $\tilde{\lambda}$ [2]–[4], [7]. One of the most popular theories is elaborated by Coffey and Clem based on the two-fluid model. According to the theory, the complex penetration depth is defined by

$$\tilde{\lambda}(\omega, B, T) = \sqrt{\frac{\lambda^2(B,T) + \left(\frac{i}{2}\right)\tilde{\delta}_v^2(\omega, B, T)}{1 - 2i\lambda^2(B,T)/\tilde{\delta}_{nf}^2(\omega, B, T)}} \tag{3}$$

with

$$\lambda(B, T) = \frac{\lambda_0}{\sqrt{\left[1 - \left(\frac{T}{T_c}\right)^4\right]\left[1 - \left(\frac{B}{B_{c2}(T)}\right)\right]}} \tag{4}$$

$$\delta_{nf}(\omega, B, T) = \frac{\delta_0}{\sqrt{1 - \left[1 - \left(\frac{T}{T_c}\right)^4\right]\left[1 - \left(\frac{B}{B_{c2}(T)}\right)\right]}} \tag{5}$$

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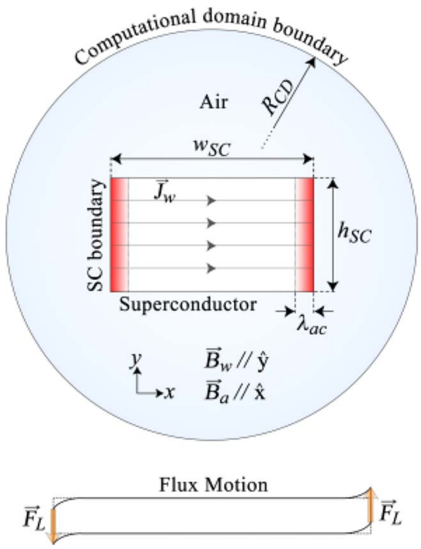

Fig. 1. (Top) Schematic diagram of the computational domain for the calculation of R_s and (bottom) a sketch of flux motion. R_{CD}, w_{SC}, and h_{SC} are the radius of the computational domain and the width and height of a type-II superconductor, respectively. Shaded stripes denoted by λ_{ac} represent regions in which vortices are subject to Lorentz forces and, thus, contribute to microwave surface resistance. The x-direction is pointing into the superconductor surface. The computational domain is not drawn to scale for clarity.

where λ_0 and δ_0 are the London penetration depth at $T = 0$ K and the normal-fluid skin depth at $T = T_c$, respectively. One of the most important parameters in (3)–(5) is the effective complex skin depth $\tilde{\delta}_v$ arising from the vortex motions. Field-induced manipulation of surface resistance is deeply affected by $\tilde{\delta}_v$ depending on the ratio ω_0/ω as follows: $\tilde{\delta}_v$ can be written in terms of characteristic lengths, δ_f due to the contributions of viscous force, i.e.,

$$\tilde{\delta}_v^2 = \delta_f^2 \left(1 + i\frac{\omega_0}{\omega}\right)^{-1} \tag{6}$$

where $\delta_f^2 = 2B\phi_0/\mu_0\omega\eta$ with η and ϕ_0 being the viscous-drag coefficient and the quantum of flux, respectively, and ω_0 is the depinning frequency.

The mixed-state energy losses in type-II superconductors are originated from both the presence of fluxons and their motion (see Fig. 1). Below the critical temperature, vortex pinning takes place, and dissipations essentially come from vortex motions. Therefore, the main contribution to R_s is due to sample regions in which fluxons experience the Lorentz force, i.e., where $\vec{B}_a \times \vec{J}_\omega \neq 0$ ($\vec{B}_a$ and $\vec{J}_\omega$ beign applied magnetic field and microwave current density, respectively).

In contrast to the CC theory [2]–[4], under the assumption that magnetic field distribution is not uniform over the sample, Bonura *et al.* [6] proposed the following simple averaging scheme over the whole sample to take the critical-state effects into account:

$$R_s = \frac{1}{A}\int_\Omega R_s\left(|B(x)|\right) dA \tag{7}$$

where Ω is the sample surface, A is its area, and x denotes the surface element.

The FEM approach developed in this work resembles the methodology used in ac loss calculations in [11] and [12], reported in detail. In calculation of current densities and field distributions inside the superconductor, the governing equation of the FEM calculations is $\vec{\nabla} \times ((1/\mu_0)\vec{\nabla} \times \vec{A}) = \vec{J}$ (where $\vec{A}$ is magnetic vector potential). Despite the fact that contribution to surface resistance is negligibly small when a component of the applied field ($\vec{B}_a$) is parallel to the applied microwave magnetic field ($\vec{B}_\omega$) [6], we used critical current density to take into account the anisotropy of field dependence as follows:

$$J_c(B) = \frac{J_{c0}}{\left(1 + \frac{\sqrt{k^2 B_x^2 + B_y^2}}{B_0}\right)^\beta} \tag{8}$$

where J_{c0} is the current density under zero magnetic field; B_0 and β are the scaling parameters for the field dependence and its exponent, respectively; whereas k is the anisotropy quotient ($k = 1$ for an isotropic material). Under an increasing applied field, evolution of current and field distribution inside the superconductor were calculated by an equation of J_{s} written in terms of a step function, i.e.,

$$J_{s,\mathrm{incr}}(x, y) = J_c \tanh\left(\frac{-A(x, y)}{A_n}\right). \tag{9}$$

In contrast, under a decreasing applied magnetic field, the difference between the calculated vector potential, $A_p(x, y)$, from the previous ac instant and actual $A(x, y)$ calculated for $B_{\max}$ at the current instant is employed in the calculation of J_s as follows:

$$J_{s,\mathrm{decr}}(x, y) = J_c \tanh\left(\frac{A_p(x, y) - A(x, y)}{A_n}\right) \tag{10}$$

where J_{c} is the critical current density [20], and A_{n} is a properly adjusted scaling factor [15].

Calculation of surface resistance using (2)–(5) requires knowledge of particular physical parameters, such as the λ_0/δ_0 ratio, depinning frequency, and the lower (B_{c1}) and upper (B_{c2}) critical fields. Moreover, in considering the critical-state effects by (7), it is also essential to know the B profile inside the sample, which is determined by $J_c(B)$. Geometry and the field profile in the sample can be computed by the FEM utilizing the commercial COMSOL Multiphysics software (through the ac/dc module).[1] The width of our 2-D rectangular superconductor is $w_{SC} = 1.3$ mm, and its ratio to the diameter ($2R_{CD}$) of the computational domain is chosen as 1/25. To predict the magnetic field distribution inside the superconductor by COMSOL, the computational domain is divided into approximately 100 000 triangular elements. Fifty elements per layer thickness are employed to obtain a finer mesh structure. We utilize the software to estimate magnetic field distributions due to the applied magnetic field on the superconductor as a function of position. The estimations can be serialized and automated with the help of the language of MATLAB technical computing package.[2]

[1] [Online]. Available: http://www.comsol.com
[2] [Online]. Available: http://www.mathworks.com

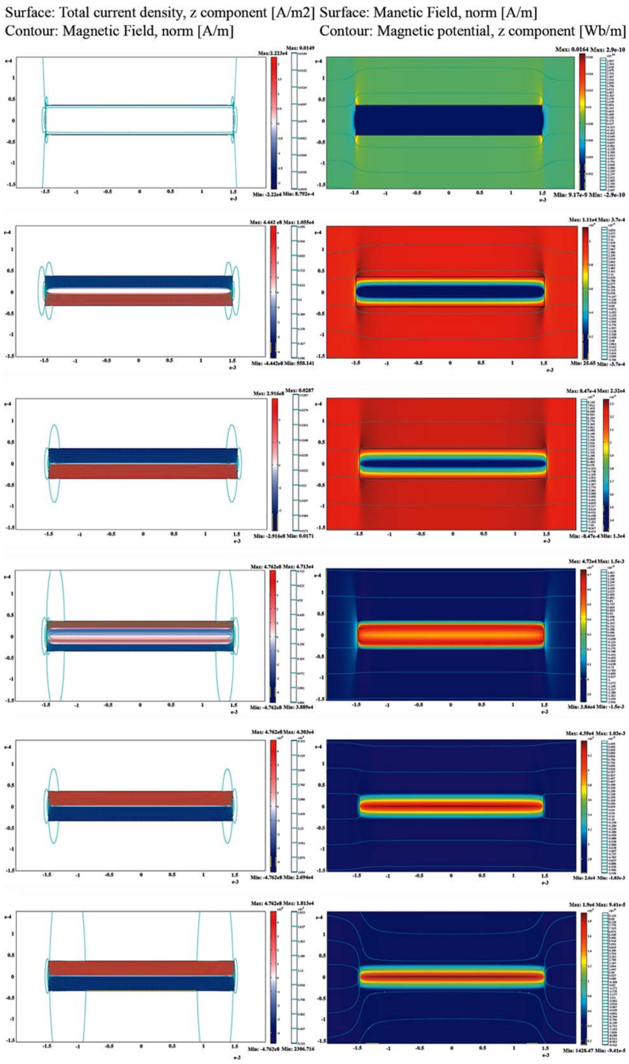

Fig. 2. Demonstration of the magnetic field and current distribution within the superconducting sample. In this calculation, the external magnetic field is applied along the x-direction.

III. Results and Discussion

Hysteretic behavior of microwave surface resistance of type-II superconductors due to critical state is determined through the model discussed in the previous section. The novelty in this work is utilizing FEM microwave surface resistance computations via COMSOL software for conversion of field profiles into MATLAB codes for processing. Hence, it facilitates a more accurate calculation of microwave surface resistance of type-II superconductors.

COMSOL Multiphysics is used to obtain the current distribution in the superconductor and to calculate its microwave surface resistance in the critical state. Fig. 2 depicts the magnetic flux density and current distributions in a rectangular type-II superconductor exposed to an increasing applied field. R_s can easily be calculated by means of the codes developed in this work and the proposition by Bonura *et al.* [6] for the model by Coffey and Clem [2]–[4]. Our calculations are carried out for temperatures sufficiently lower than the critical temperature of the superconductor ($T = T_c/2$). Hence, field dependence of repinning frequency is ignored, whereas the ω/ω_0 ratio is set to 1. The ratio is intended to be illustrative but not exhaustive and is adopted to be constant in the whole field range presented by Bonura *et al.* [21].

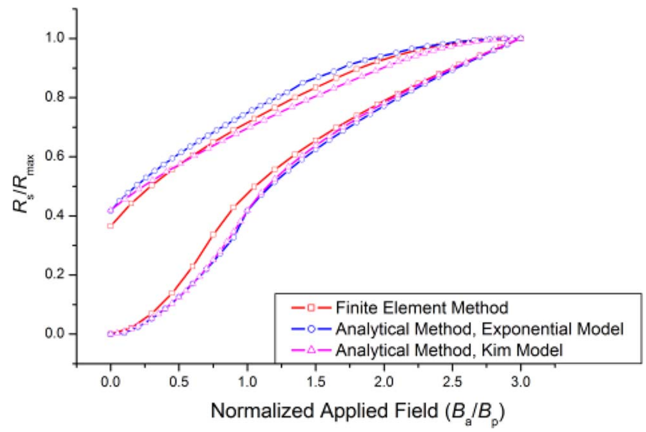

Fig. 3. Comparison of the performance of the proposed FEM-based numerical model with the classical analytical models.

To validate the accuracy of our calculations, the normalized curves obtained in this work are compared in the Mathematica environment against those obtained by solving analytical expressions utilizing the Kim [22] $J_c(B) = J_{c0} \times (1 + (|B|/B_0))^{-\mathrm{n}}$ and exponential [23] $J_c(B) = J_{c0} \times \exp(-(|B|/B_0))$ models for the field dependence of critical current density (see Fig. 3). Here, B_0 and n are positive adjustable parameters. Although (8) resembles the Kim model more, it eliminates anisotropic field dependence of J_c. $R_{\max}$ and $B_p (= \mu_0 H^*)$ in Fig. 3 represent maximum calculated surface resistance and full penetration field, resepctively. It is known that B_p corresponds to a specific point at which the transition from convexity to concavity occurs in surface resistance hysteresis curves. This is demonstrated in the work of Yildiz *et al.* [8] (Fig. 5 therein) where a behavioral investigation was carried out. It is clearly seen that the proposed model displays a good agreement with the existing classical models [8], [9].

Numerical results obtained by the proposed model for different thicknesses (h_{SC}) of the superconducting sample are presented in Fig. 4. R_s decreases with decreasing h_{SC}, and the rate of change is directly proportional to h_{SC}, as shown in the inset of Fig. 4. This stems from the fact that the model is based on flux lines that penetrate into the material. While the lowest surface resistance is 0.2 mΩ for YBCO [24], i.e., the most popular type-II superconductor, the model in this work yields still lower values with decreasing h_{SC}. This suggests that the proposed model should be further investigated. However, the model results for bulk samples are quite satisfactory.

The proposed model can also provide information on how microwave surface resistance varies for different parameters in a relatively simple manner. Variations of surface resistance for different B_{c2} values are provided in Fig. 5. As expected, the higher the B_{c2} value, the smaller the slope of the curve. However, this variation reaches a plateau with still increasing B_{c2} where the influence of the upper critical field on R_s vanishes. This is depicted in the inset of Fig. 5.

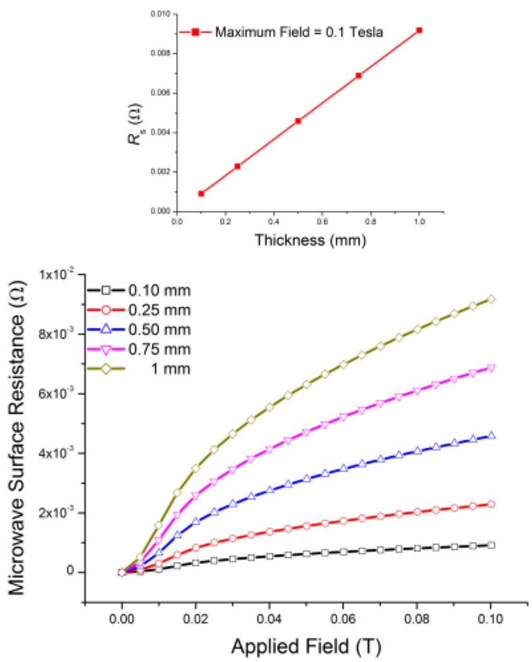

Fig. 4. Surface resistance curves for different superconductor thicknesses (h_{SC}). Simulation parameters are $I_c = 100$ A, $A_n = 10^{-9}$, $w_{SC} = 3$ mm, $R_{CD} = 10w_{SC}$, $\omega/\omega_0 = 1$, $T/T_c = 0.5$, $\lambda_0/\delta_0 = 0.02$, $B_0 = 0.032$, $k = 0.3$, $\beta = 0.5$, $B_{c2} = 70B_p$. $h_{SC} = 0.1, 0.25, 0.5, 0.75, 1$ mm.

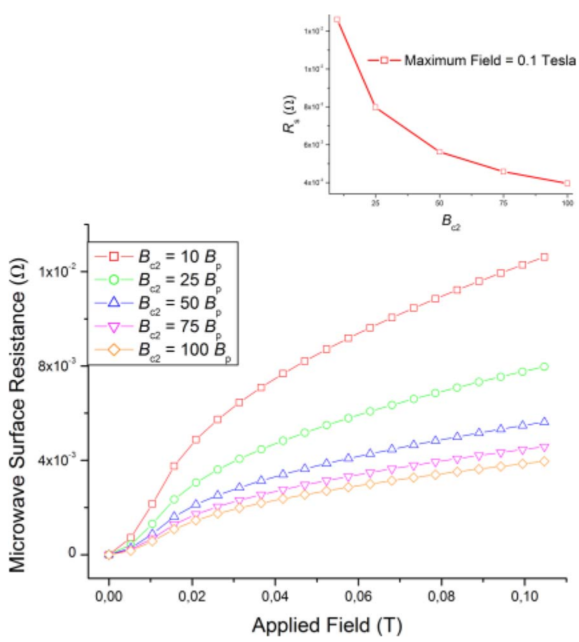

Fig. 5. Surface resistance curves for different B_{c2} values. Simulation parameters are $I_c = 100$ A, $A_n = 10^{-9}$, $w_{SC} = 3$ mm, $h_{SC} = 0.05$ mm, $R_{CD} = 10w_{SC}$, $\omega/\omega_0 = 1$, $T/T_c = 0.5$, $\lambda_0/\delta_0 = 0.02$, $B_0 = 0.032$, $k = 0.3$, $\beta = 0.5$, $B_{c2} = 10, 25, 50, 75, 100B_p$.

IV. Conclusion

Microwave surface resistance of a type-II superconducting material has been numerically obtained by means of a model based on the FEM in two dimensions through the popular COMSOL Multiphysics software. Field-dependent calculations are carried out for different sample thickness and upper critical field values. For sufficiently small film thicknesses approaching the thin-film limit, microwave surface resistance values calculated through the proposed method are considerably smaller than the values in the literature. This may be stemming from the fact that the Coffey–Clem model becomes unsuitable in the thin-film limit.

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