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ABSTRACT

Technological Pedagogical Content Knowledge (TPACK) is defined as the teacher knowledge needed for effective technology integration. This study aimed to investigate preservice elementary mathematics teachers' TPACK development. TPACK survey was administered to 33 preservice teachers (PSTs), and six of them were selected via maximum variation sampling to represent as different cases as possible based on their mathematical knowledge and technological self-assessment. These six PSTs implemented four technology-based lessons (two microteaching sessions in the mathematics teaching method course and two lessons in the student teaching). Participants' TPACK levels were examined based on the development model proposed by Niess, M. L., Sadri, P., & Lee, K. (2007, April). Dynamic spreadsheets as learning technology tools: Developing teachers' technology pedagogical content knowledge (TPCK). Paper presented at the meeting of the *American Educational Research Association Annual Conference*, Chicago, IL. Results showed that participants had not used technology effectively and efficiently in their microteaching sessions and the first lessons in their student teaching. After evaluating their first lessons in schools, PSTs improved their teaching considerably. We suggest giving more opportunities for PSTs to teach with technology in classrooms and to assess their teaching practices reflectively.

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KEYWORDS
Technological pedagogical content knowledge; preservice mathematics teachers; microteaching lesson study; TPACK; GeoGebra

1. Introduction

Technology has a significant impact on almost all areas of life, and individuals need to integrate technology into every part of life. Technology improves students' academic achievement, motivations, self-conceptions, interest, and creativity and promotes higher-order skills such as constructing and testing hypotheses and generalizing (Gökçe & Güner, 2022; Karatas, 2011; Mailizar et al., 2021). National Council of Teachers of Mathematics (NCTM) emphasized the necessity of technology in mathematics classrooms and considered technology an essential resource to help students learn mathematics meaningfully and reason and communicate mathematically (NCTM, 2014). However, recent findings have shown that technology is not integrated effectively into classrooms (Liu et al., 2015;

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Purcell et al., 2013), and teachers have a lack of knowledge in designing and implementing technology-based lessons (Açıkgül, 2021; Agyei & Voogt, 2011; Wang et al., 2018). Teachers' technology use sparks researchers' interest to examine why technology could not be integrated effectively and efficiently. Teachers use technology differently based on their beliefs about the nature of knowledge and learning and beliefs about teaching and learning (Kim et al., 2013). Researchers began to seek answers to these questions: What do teachers need to believe and know to use technology effectively in their classrooms? These questions guided the community of educational technology researchers.

Research shows that knowledge of technology is not enough for successful technology integration (Ertmer & Ottenbreit-Leftwich, 2010; Koehler et al., 2007; Lee & Hollebrands, 2008). The most common idea is that teachers should know about technology-supported pedagogy. Researchers used different names such as ICT-related PCK (Angeli & Valanides, 2005), technology-supported pedagogy knowledge (Hughes, 2005), technological pedagogical content knowledge (Keating & Evans, 2001; Koehler & Mishra, 2005), pedagogical content knowledge of educational technology (Margerum-Leys & Marx, 2002), and technology-enhanced PCK (Niess, 2005) for the new conceptualization of knowledge. The most commonly used framework is technological pedagogical content knowledge developed by Koehler and Mishra (2005), abbreviated as TPACK. Koehler and Mishra (2005) introduced TPACK as a framework to identify the needed teacher knowledge for effective technology integration.

It is possible to infer that curricular knowledge, the third type of knowledge in Shulman's PCK, includes technology integration in teaching (Akyuz, 2018). PCK framework addressed that teachers should use technologies when they need them (Cox & Graham, 2009; Shulman, 1986). It is worth noting that there were limited technological sources when Shulman developed his PCK (Hofer & Grandgenett, 2012). However, the rapid development and evolvement of digital technologies have made these technologies an essential component of teaching and learning (Koehler & Mishra, 2009). Therefore, TPACK was introduced to refer to teacher knowledge needed to teach with technology effectively. TPACK is developed by adding knowledge of technology to Shulman's (1986; 1987) notion of pedagogical content knowledge, knowledge needed to teach content. Teachers should know pedagogy, technology, mathematics and realize the interactions among these knowledge bases and the affordances technology provides to mathematics and teaching practices and vice versa (Earle, 2002; Koehler et al., 2007).

TPACK is an essential framework because it helps PSTs think strategically about when, where, and how to use mathematics-specific technologies (Gillow-Wiles & Niess, 2014). Preservice teachers develop knowledge and form positive beliefs about teaching with technology in their teacher preparation programs. Therefore, it is crucial to determine how to develop TPACK to provide preservice teachers with the necessary training, opportunities, and support in their teacher education programs (Mouza et al., 2014; Polly et al., 2010; Zelkowski et al., 2013). Furthermore, it is still needed longitudinal studies that investigate which approaches lead to TPACK development under which contextual factors (Hofer & Grandgenett, 2012). Considering the mentioned need for further research, we conducted a longitudinal study investigating the effect of TPACK-based workshops and Microteaching Lesson Study (MLS) on preservice elementary mathematics teachers' TPACK development.

2. TPACK development

2.1. TPACK in mathematics education

Technology integration does not deal only with the amount of technology used in class but also is related to how and why technology is used (Earle, 2002). Teachers' decisions about technology use may affect whether technology fosters or hinders student learning (Lee & Hollebrands, 2008). PSTs' prejudices, lack of knowledge and experience, and views about teaching mathematics with technology are the main factors that hinder them from using technology as a learning tool (Niess, 2005). Therefore, it is crucial to understand PSTs' knowledge, attitudes, and beliefs about educational technologies to predict how they will use technology in their future classrooms (Abbitt, 2011; Kartal et al., 2022). TPACK helps teacher educators determine preservice teachers' knowledge and beliefs about technology, refine their teaching practices and prepare PSTs who are tech-savvy and self-confident about teaching with technology. This study examines how preservice teachers' teaching practices with technology evolve during an expanded time, and findings would offer an insight into the possible technology use in mathematics classrooms and preservice mathematics teachers' TPACK development.

The TPACK framework has three main domains. These are pedagogical knowledge, content knowledge, and technological knowledge. These knowledge bases' intersections form other knowledge domains; pedagogical content knowledge, technological content knowledge, technological pedagogical knowledge, and technological pedagogical content knowledge (Figure 1).

Content knowledge (CK) is the knowledge of mathematics that varies by subject matter and grade level (Koehler et al., 2007).

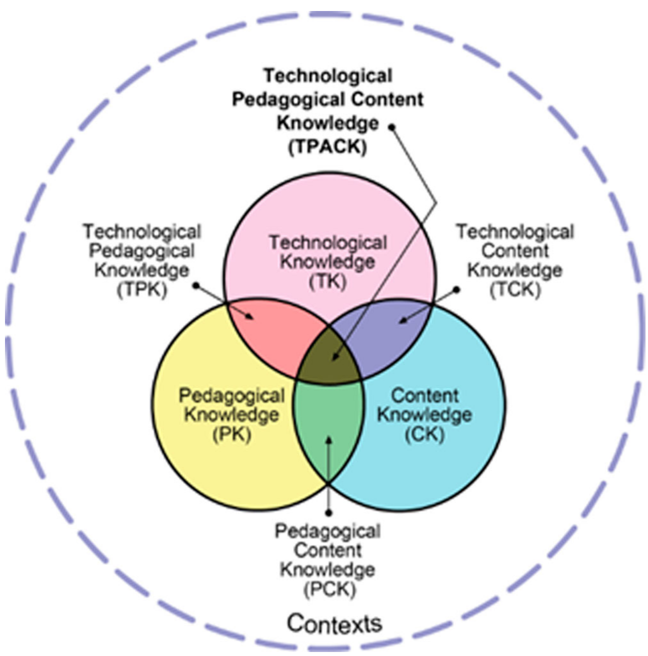

Figure 1. TPACK framework (Reproduced by permission of the publisher, © 2012 by tpack.org).

Technology knowledge (TK) deals with knowledge of using technology-based tools and refers to the adaptability of rapidly changing and new technologies (Ozgun-Koca et al., 2010).

Pedagogical Knowledge (PK) consists of processes, practices, and methods related to teaching objectives, values, and techniques and evaluating student learning strategies (Koehler et al., 2007; Ozgun-Koca et al., 2010).

Pedagogical Content Knowledge (PCK) refers to the same notion as Shulman (1986; 1987). PCK is a way to understand how teachers interpret mathematics content, find multiple representations, and adapt educational materials for students' pre-existing learnings (Mishra & Koehler, 2008).

Technological Content Knowledge (TCK) includes how technology and mathematics reinforce and constrain each other. TCK helps teachers recognize which technology is the most useful in learning mathematics and how mathematics and technology influence each other (Mishra & Koehler, 2008).

Technological Pedagogical Knowledge (TPK) is the knowledge of how teaching and learning processes change when particular technologies are used (Mishra & Koehler, 2008).

Technological Pedagogical Content Knowledge (TPACK) is the knowledge that occurs when three main knowledge domains intersect. TPACK provides an understanding of how these knowledge bases interact instead of considering them as separate domains (Mishra & Koehler, 2008). The more preservice teachers recognize the interactions among pedagogy, subject matter, and technology, the more they can integrate technology effectively (Angeli et al., 2016).

TPACK has a dynamic nature because of rapidly changing technologies, pedagogies, and subject matter. Teachers' TPACK can change or develop based on the subject matter, the technology used, and the pedagogical approach (Agyei & Voogt, 2015; Angeli et al., 2016; Saralar et al., 2018). Therefore, TPACK may differ due to different students and different contexts (Agyei & Voogt, 2011). TPACK should be considered content-specific to understand better the framework (Angeli et al., 2016; Niess, 2005; Schmidt et al., 2009). The elaboration of TPACK for specific subject domains remains an essential research question since considering subject-specific TPACK might shed light on the constraints and affordances of particular technologies for a specific subject matter (Akyuz, 2018; Zambak & Tyminski, 2020). However, the lack of subject-specificity in TPACK might lead to low discriminant validity in TPACK measures (Chai et al., 2016). Given the importance of mathematics-specific TPACK on mathematics teacher education programs, the focus of the study is constrained to preservice teachers' TPACK when teaching the topic of polygons.

TPACK of mathematics teachers includes awareness about how mathematics-specific technologies improve students' mathematics learning and which topics and pedagogical practices align with specific technologies (Grandgenett, 2008). To understand better the interactive relationships between mathematics, mathematics-specific technologies, and pedagogies, Niess (2005; 2013) adapted the four components of Grossman's (1990) PCK. These components belong to the central component in which knowledge of technology, pedagogy, and mathematics merge (TPACK) to describe technology-enhanced PCK;

(1) An overarching conception about the purposes of incorporating technology in teaching mathematics comprises mathematics teachers' beliefs about the role of technology.

PSTs' unexperienced views of how technology can be used to support students' learning belong to this component (Meagher et al., 2011).

(2) Knowledge of students' understandings, thinking, and learning in mathematics with technology refers to teachers' views on how students learn mathematics with technology (Niess, 2013).

(3) Knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics consists of teachers' knowledge about appropriate technological tools to teach mathematics.

(4) Knowledge of instructional strategies and representations for teaching and learning mathematics with technologies involves PSTs' thoughts about whether technology should be used to support known concepts or develop new concepts (Meagher et al., 2011).

Teachers' TPACK plays a crucial role in their decisions about using which technologies and how to use these technologies (Mainali & Key, 2012). Research related to TPACK in mathematics education (Agyei & Voogt, 2012; 2015; Lee & Hollebrands, 2008; Mainali & Key, 2012; Meagher et al., 2011; Ozgun-Koca et al., 2010; Saralar et al., 2018) can provide insight into how, when, and why in/preservice mathematics teachers prefer to use technologies, and which approaches can develop their TPACK.

Teachers can employ digital content (such as websites, video clips), presentation technologies (such as PowerPoint, Prezi), or mathematical software (such as Dynamic Geometry Software (DGS), Computer Algebra Systems, Spreadsheets) in their teaching practices (Mouza et al., 2014). This usage ranges from using technologies as a demonstration and teaching tool to an inquiry and learning tool. One of the most commonly used mathematical technologies is DGS, which is also within the scope of the present study. DGS provides a dynamism that allows students to observe the changes simultaneously and helps students think more deeply about the constructions they make since their feature of preserving the properties of the constructions (Akyuz, 2018). Therefore, students can make and evaluate judgments, communicate mathematically, and formulate mathematical explanations in dynamic geometry environments differently from traditional environments (Gökçe & Güner, 2022). One of the most widely used DGS in mathematics instruction is GeoGebra, which can improve students' problem-solving skills, creativity, and mathematical knowledge (Hohenwarter et al., 2009). GeoGebra serves as a demonstration and visualization tool, construction tool, mathematics discovering tool, and a teaching material tool (Gökçe & Güner, 2022). GeoGebra allows students to reconstruct the objects and rearrange the components beyond simply drawing mathematical concepts (Açıkgül, 2021; Tomic, 2013). GeoGebra is an effective software for PSTs to learn and teach with it.

It is found that most of the PSTs addressed the importance of paper-and-pencil experiences, and their views affected their teaching practices (Mudzimiri, 2010; Ozgun-Koca et al., 2010). Preservice teachers lack experience in teaching mathematics with technology and have difficulty predicting students' solutions and misconceptions with technology, and therefore they may have naïve ideas about teaching with technology (Agyei & Voogt, 2012; Lee & Hollebrands, 2008; Mouza et al., 2014). It is possible to change preservice teachers' conceptions of teaching with technology by providing them with the opportunities to develop and implement technology-integrated lessons and design technology activities. Cavin (2007) reported that participants' views changed from considering technology a reinforcement tool to considering technology a supporting tool that develops students' conceptual understanding. Teachers who value technology are more willing to use technology actively and appropriately in their classrooms (Kartal et al., 2022; Mainali & Key, 2012). Teacher educators and researchers also should try to promote the perceived value of the technology in teachers' belief systems.

Niess (2008) argues that mathematics teachers' practices reflect their TPACK levels. The differences in their knowledge of technology, pedagogy, and mathematics may lead to different decisions and actions, such as accepting or rejecting technology (Mouza et al., 2014). Niess et al. (2007) suggested a TPACK development model with five levels (recognizing, accepting, adapting, exploring, and advancing) in identifying teachers' technology-based decisions and actions (Figure 2).

(1) Recognizing: Teachers can use technology and are aware of the affordances of the technology in mathematics education. However, they do not integrate technology yet.
(2) Accepting: Teachers develop a favorable or unfavorable attitude towards teaching and learning mathematics with technology.
(3) Adapting: Teachers engage with activities that help them decide whether to accept or reject teaching and learning mathematics with technology.
(4) Exploring: Teachers actively integrate technology into teaching and learning mathematics.
(5) Advancing: Teachers evaluate the results of their self-decision about integrating technology.

Mathematics Teachers Development Model is proposed primarily for in-service teachers (Niess et al., 2009). However, some studies employed the development model for preservice mathematics teachers' development of TPACK (Balgalmis et al., 2014; Cavin, 2007; Mudzimiri, 2010; Saralar et al., 2018). Mouza (2016) suggested that little is known about this trajectory among PSTs. This study examined preservice mathematics teachers' TPACK using the Mathematics Teacher Development Model, and the results may be evidence of how this model works for PSTs. The following section describes workshops and microteaching lesson study, which are the approaches to develop TPACK in the present study.

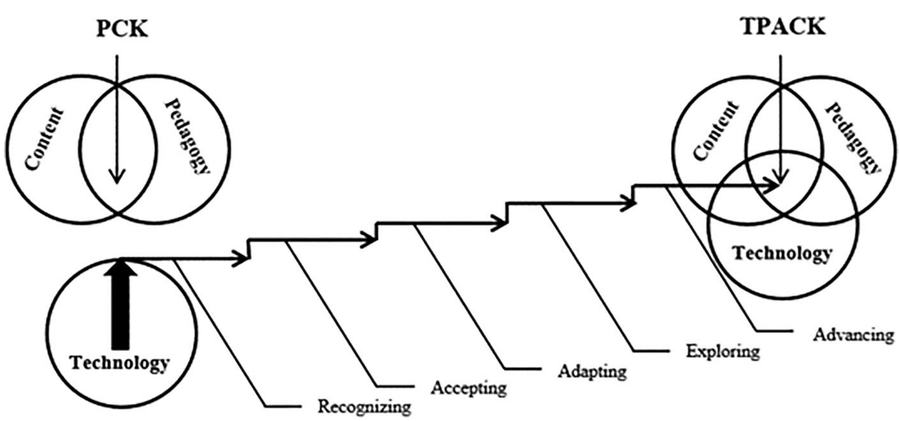

Figure 2. TPACK developmental model extracted from Niess et al. (2009).

2.2. Workshops and microteaching lesson study

Teachers need to use technologies as a learning and inquiry tool to prepare twenty-first-century citizens who utilize technology effectively and efficiently to seek, validate, analyze, synthesize, and interpret information. However, research proposes that teachers need more than technical skills for effective technology integration (Figg & Jaipal, 2012; Mishra & Koehler, 2006; Polly et al., 2010). Teachers should be engaged in thinking about how technology should be integrated into teaching (Mouza et al., 2014) and be provided with enough knowledge and experience to teach mathematics with technology (Zelkowski et al., 2013).

Developing TPACK has attracted researchers' interest for a long time (Cavin, 2007; Hofer & Grandgenett, 2012; Mishra & Koehler, 2006). Determining which approaches contribute to TPACK development (Hofer & Grandgenett, 2012; Shin et al., 2009) is essential to analyze instructional strategies (Koh & Divaharan, 2011; Niess, 2008) and to examine the effectiveness of teacher preparation program (Mouza et al., 2014; Zelkowski et al., 2013). Research showed that teachers and faculty underused technology even when they were aware of the benefits of using technology in teaching and learning (Mudzimiri, 2010; Polly et al., 2010). The underlying factors can be lack of professional development (Mudzimiri, 2012), lack of active learning experiences (Mouza et al., 2014; Niess, 2008), and lack of technological skills (Koh & Divaharan, 2011).

Attempts to develop TPACK should focus on knowledge, beliefs, and skills related to technology, pedagogy, and mathematics (Polly & Orrill, 2012). Therefore, there is a need to shift from techno-centric interventions that focus only on technological skills to content-centric interventions that focus on teaching mathematics with technology (Figg & Jaipal, 2012; Kartal, 2019). Preservice teachers' enthusiasm to teach with technology can improve if they observe effective technology integration by faculty and cooperating teachers (Bandura, 1977). Modeling technology use may be an effective way to demonstrate how to teach with technology (Agyei & Voogt, 2015; Koh & Divaharan, 2011). This modeling should be content-specific to demonstrate the efficacy of technology in different content areas (Mouza et al., 2014; Mudzimiri, 2010; Polly et al., 2010).

Teacher preparation programs may encourage PSTs to use technology, but the experiences of seeing modeled technology integration either in their preparation programs or field experiences will persuade them to use technology (Agyei & Voogt, 2012; Karatas, 2011; Meagher et al., 2011; Voogt et al., 2016). The more preservice teachers have the opportunity to design, enact, and reflect on technology-infused lessons, the more they feel confident and tend to use technology in a student-centered way (Koh & Divaharan, 2011; Mouza et al., 2014).

A helpful model of the cycle of design, enact, and reflect is microteaching lesson study (MLS), in which PSTs work in groups collaboratively to achieve an instructional goal (Fernandez, 2005). Cavin (2007) stated that MLS has three main steps. In the first step, groups come together to plan a research lesson, and then one member of the group enacts the lesson in a real classroom. The group also videotapes the lesson. The research lesson is a mini-lesson for which group members came together to design and teach, and it is the main component of the MLS. PSTs watch the videotape of the research lesson and discuss the effectiveness of the lesson in the second step. PSTs re-plan the lesson and implement the adjusted research lesson in the last step (Figure 3). Planning, implementing, and evaluating

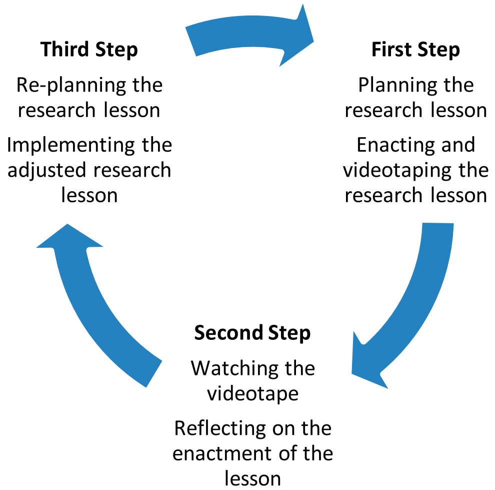

Figure 3. The steps of microteaching lesson study.

a technology-based mathematics lesson may improve PSTs' competencies to teach with technology. Fernandez and Robinson (2006) recommended that it can be helpful for PSTs to watch and analyze the videotapes individually to discuss.

PSTs work collaboratively in a learner-centered environment in MLS (Cavin, 2007). PSTs' cooperation in planning, implementing, and evaluating the lesson makes a learner-and-assessment-centered learning environment (Fernandez & Robinson, 2006). Peer and self-assessments make MLS a unique experience because self-critiquing and peer feedback improve PSTs' learning about teaching (Fernandez, 2005). At the beginning of the MLS process, participants determine a goal to achieve.

MLS is a teaching method in which PSTs learn how to teach with technology (Zhou & Xu, 2017). MLS promotes PSTs' learning and professional development, improves their content knowledge and confidence about teaching (Meng & Sam, 2013; Zhou & Xu, 2017), and enhances their knowledge of pedagogy (Fernandez, 2005). MLS allows PSTs to incorporate theory and practice, collaborate with their peers, and reflect on their teaching practices (Fernandez & Robinson, 2006). MLS is a potential means of PSTs' TPACK development (Cavin, 2007; Meng & Sam, 2013) because guidance, help, and support are given to PSTs (Shafer, 2008). We designed content-centric workshops that focused on teaching different mathematical strands (e.g. numbers, geometry, algebra) with different technologies (calculator, Geometer's SketchPad (GSP), spreadsheets) for six PSTs. We included them in MLS that allowed them to plan, enact, and reflect technology-based teaching practices.

2.3. The significance of the study

Although there has been much recent research investigating preservice mathematics teachers' TPACK regarding lesson planning (Araújo Filho & Gitirana, 2022; Assadi & Hibi, 2020), exploring inquiry-based geometrical tasks using DGS (Segal et al., 2021), using mathematical technologies to create screencast video lessons (Bonafini & Lee, 2021), there is a lack of research regarding TPACK development. The focus of the present study is to examine preservice mathematics teachers' TPACK development based on observations. Therefore, the related literature is limited to preservice mathematics teachers' TPACK development research, indicated in Table 1, demonstrating a short brief of the related literature. Most of the researchers designed courses to develop preservice mathematics teachers' TPACK. A few of them modelled examples of technology use in their courses. Researchers modeled the technology-based lessons in most of these studies (Agyei & Voogt, 2012; Cavin, 2007; Mudzimiri, 2012). However, one study used videos that modelled technology use (Akkaya, 2016). Reflection refers to discussions or self-evaluations that make PSTs reflect on their performances. The courses mainly included the cycle of plan, enact, reflect, and re-enact (Agyei & Voogt, 2015; Cavin, 2007; Kafyulilo et al., 2015; Kafyulilo & Fisser, 2019; Koştur, 2018; Mudzimiri, 2012). PSTs presented their artifacts, such as technology-supported activities, or taught their planned lessons to their peers. However, they had experience in real classrooms within the context of method courses in the studies of Meagher et al. (2011), Ozgun-Koca et al. (2010), and Mudzimiri (2012). Two research (Balgalmis et al., 2014; Saralar et al., 2018) examined the TPACK development of PSTs within the context of student teaching.

One semester may not be enough to prepare PSTs to use technology effectively (Lyublinskaya & Tournaki, 2014). Niess et al. (2007) suggested that PSTs need a long time to display their TPACK in their practices. This study took three semesters. Therefore, this study may have given participants enough time to assimilate this new conceptualization of knowledge (TPACK). This study may be considered topic-and-technology specific. The focus is on the TPACK development of preservice teachers when teaching polygons with GeoGebra. Most studies (Cox & Graham, 2009; Niess, 2005; Schmidt et al., 2009) suggested that TPACK is content-specific. Meagher and his colleagues argued that preservice teachers' beliefs about teaching with technology might vary by topic. Specifying the topic and technology may help better understand how preservice teachers develop TPACK. As seen in Table 1, most research is not restricted to a specific topic, even though they prefer to investigate TPACK development for specific technologies.

Lastly, this study examined TPACK development within the context of the transformative approach. The transformative approach assumes that TPACK is a unique type of knowledge, and it is not easy to restrict to its sub-domains (Angeli et al., 2016; Angeli & Valanides, 2005; Niess, 2013). Conversely, the integrative approach investigates the TPACK sub-domains distinctively (Cox & Graham, 2009). As seen in Table 1, most studies used the integrative approach. Angeli et al. (2016) suggested that research is needed to examine the extent to which TPACK is integrative or transformative. This longitudinal study employs different data collection tools to examine PSTs' TPACK development using the transformative approach.

For this study, TPACK involves formulation and representation of polygons with GeoGebra and how GeoGebra may be employed with appropriate pedagogical strategies constructively in teaching polygons (Mishra & Koehler, 2008). Niess (2005) TPACK components and Mathematics Teachers TPACK development model (Niess et al., 2007) were the theoretical frameworks of this study.

Changes and developments in PSTs' beliefs and actions related to TPACK can be investigated with a study extended over time. This study's primary purpose is to examine the effects of TPACK-based workshops and MLS on PSTs' TPACK development. The research

Table 1. Literature related to TPACK development of preservice mathematics teachers.

	Course						Approach				
Researchers and year	Micro-teaching	Modeling technology-supported lesson	PSTs' lesson plans or technology-based activities	Reflec-tion	Field experience	Work-shops	Integ-rative	Trans-formative	The technology used by participants	Topic	Duration
Cavin, 2007	X	X	X	X				X			One semester
Haciomeroglu et al., 2010			X	X							
Ozgun-Koca et al., 2010			X		X[2]			X	TI-Nspire		One semester
Meagher et al., 2011			X		X[2]			X	TI-Nspire		One semester
Agyei & Voogt, 2012	X			X		X	X		Spreadsheets		
Mudzimiri, 2012	X	X		X	X			X			15 weeks
Tokmak et al., 2013	X		X	X			X				14 weeks
Zhan et al., 2013	X			X			X				Six lessons each lasting 150 min
Balgalmis et al., 2014				X	X			X	GeoGebra		
Agyei & Voogt, 2015	X	X		X			X		Spreadsheets		14 weeks
Kafyulilo et al., 2015	X		X	X			X				
Akkaya, 2016	X	X[1]					X		GeoGebra Cabri		14 weeks
Kurt, 2016	X		X	X			X		Virtual manipulatives	Statistics	
Durdu & Dag, 2017	X						X		GeoGebra		Ten weeks
Koştur, 2018	X		X					X			
Saralar et al., 2018				X	X			X	GeoGebra	3D Shapes	
Mutlu et al., 2019	X		X	X		X	X		VuStat	Statistics	
Kafyulilo & Fisser, 2019	X		X	X		X	X				
Xu et al., 2019	X			X			X		3D Dynamic Geometry Software		
Açıkgül & Aslaner, 2020	X			X			X		GeoGebra	Polygons	14 weeks
Zambak & Tyminski, 2020			X				X		Geometer's Sketchpad	Geometry	13 weeks

[1]Videos that demonstrate using technology.
[2]Two weeks and technology use is encouraged.

question is as follows; 'What is the nature of the change in TPACK of PSTs who attended TPACK-based workshops and MLS?'

3. Method

We used a longitudinal two-phase multiple case study (Stake, 2006) to understand better how and why preservice teachers use technology in their naturalistic setting of a mathematics classroom (Yin, 2013). The reason for choosing the case study is that we focused on PSTs' experiences teaching with technology (Merriam, 2009). PSTs were considered individual cases. The units of analyses were TPACK components developed by Niess (2005) that are (i) an overarching conception about the purposes of incorporating technology in teaching mathematics, (ii) knowledge of students' understandings, thinking, and learning in mathematics with technology, (iii) knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics and (iv) knowledge of instructional strategies and representations for teaching and learning mathematics with technologies. In the first phase, we analyzed participants individually. In the second phase, we focused on the similarities and differences among participants.

3.1. The research context

In the context of mathematics teacher education in which this study was conducted, PSTs enroll in a four-year teacher preparation program. The first two years of the program are primarily content-based, and PSTs begin to take courses related to teaching mathematics since the second year. They start to learn GeoGebra and prepare GeoGebra-based materials. In the third year of the program, PSTs enroll in a two-semester teaching method course in which they learn methods, techniques, representations, and materials they can use to teach mathematics and analyze the national mathematics curriculum. In the final year, PSTs go to schools for student teaching that takes two semesters. They observe teachers, students, exams, textbooks, materials, and administrators in the first semester. Then, they teach mathematics under their cooperating teachers' supervision in the second semester. The teacher preparation program, in which the participants of this study enrolled, has a mathematics education laboratory that includes manipulatives and a smartboard with mathematical software (such as GeoGebra, GSP). The teaching method course was taught in this laboratory, and PSTs had the opportunity to use manipulatives and the smartboard in their microteaching. Besides, cooperating teachers in schools were using technology primarily for videos and representations.

3.2. Participants

Thirty-three junior PSTs who are expected to teach middle school mathematics participated in the teaching method course. At the beginning of the method course, we administered a TPACK survey, TPACK-SAS (Kartal et al., 2016), and a questionnaire regarding mathematical knowledge in polygons. The study is topic-specific; in other words, participants taught polygons with GeoGebra in their teaching practices. Researchers developed the polygon questionnaire to determine the mathematical knowledge of participants. The questionnaire includes 19 open-ended questionnaires related to the definitions (e.g. 'Please define a polygon'), mathematical relationships (e.g. 'Please demonstrate the relation that gives the area of the trapezoid by using the area relation of the parallelogram'), and mathematical processes about polygons (e.g. 'Find the sum of all the interior angles in the figure') (Kartal & Çinar, 2017). Data were analyzed using a rubric with a 4-point scale that focuses on the correct answers and detailed explanations regarding appropriateness for mathematical thinking. We used the questionnaire to see whether different content knowledge levels led to different TPACK levels and teaching practices.

We listed pre-test scores of the TPACK survey from the highest to the lowest and calculated the range on which PSTs' scores lay. Then, by dividing this range into three, PSTs were categorized into three groups (each consisting of 11 PSTs) as high, medium, and low based on their pre-test scores. On the other hand, the scores that PSTs gained from the polygon questionnaire were divided into two categories; high and low. First, PSTs in the TPACK-high group were checked up, one PST in the polygons-high group and one in the polygon-low group were determined. The same processes were conducted for both TPACK-medium and TPACK-low groups. Two PSTs from each TPACK group, a total of 6 PSTs, were selected to form the study group. One participant in each group (e.g. TPACK-high, TPACK-medium, TPACK-low) had a higher score than the other participant from the polygon questionnaire. It was attempted to provide maximum variation. Table 2 shows the participants' levels of TPACK and mathematical knowledge of polygons. PSTs were asked to participate in TPACK-based workshops and teach polygons with GeoGebra in the context of microteaching and MLS.

Participants were interviewed at the beginning of the study to get information about their views about mathematics, technology, and teaching mathematics with technology (Appendix). We divided participants into three MLS groups in their student teaching based on their assigned schools by faculty administers. We included participants who went to the same school for student teaching in the same MLS group. The first MLS group includes PST-1 and PST-2. PST-1 feels comfortable in teaching mathematics with technology. She was the only one who used technology in her presentation in the previous semester. She follows the recent activities of GeoGebra and applications on the internet. She refers to the importance of technology in mathematics education, so she insists on using technology. She is against rote mathematics learning. She reported that she paid attention not to give rules and formulas directly in her teaching practices and never memorized rules and formulas. PST-2 were in the TPACK-high group based on the TPACK-SAS scores, but it seemed that her survey scores did not represent her views. She insisted on the difficulties of teaching with technology. In the workshops, she seemed uninterested. PST-2 claimed that GeoGebra did not spark students' interest. So, she reported that she would

Table 2. Participants' Levels of TPACK and Knowledge of Polygons.

Participant	Gender	TPACK	Polygons	MLS Group
PST-1	Female	High	High	MLS-1
PST-2	Female	High	Low	MLS-1
PST-3	Female	Medium	High	MLS-2
PST-4	Female	Medium	Low	MLS-3
PST-5	Female	Low	High	MLS-3
PST-6	Female	Low	Low	MLS-2

use manipulatives rather than technology in her future classrooms. She emphasized using the technology after teaching concepts in a paper-pencil environment.

PST-3 and PST-6 were in the second MLS group. PST-3 is the quietest participant in the study group. She did not feel comfortable with technology in teaching mathematics. She was worried about technical problems she may encounter, and this fear is the most critical factor influencing her decisions about technology use. PST-3 has a more comprehensive understanding of mathematics than its peers. She defined mathematics as expressing, justifying, and evaluating ideas with mathematical symbols. PST-6 was the most self-confident PST even though she had the lowest scores in both TPACK-SAS and polygons questionnaire in the study group. Her communication with students was pleasant in her teaching practices. She believed that answers were more important than processes in mathematics education, so she thought she had to teach different solutions to problems quickly. Technology did not have an essential role in her teaching philosophy. She mentioned that she opposed using technology, especially in exploring mathematical concepts and ideas.

PST-4 and PST-5 were in the last MLS group. PST-4 is another participant who has a moderate TPACK self-assessment level in the study group. She emphasized the importance of mathematics calculations and defined mathematics as a computation tool ignoring mathematical thinking. The fear of losing control in the classroom seemed to hinder her from using technology. According to her, class management is the most crucial factor that makes a teacher effective. So, she did not lean towards using technology because of the concern of class management. The score of PST-5 in TPACK-SAS was in the low group. PST-5 mainly uses logical-mathematical intelligence, so she thought mathematics gives only calculation advantage. She reported that she did not feel comfortable when using technology because of the language of the software. PST-5 explained that the software in English compels her, especially in understanding errors and instructions.

3.3. Procedure

This study took three semesters. At the beginning of the study, TPACK-SAS and the polygon questionnaire were administered to 33 PSTs to identify those with different technological self-assessment levels and mathematical knowledge. PSTs also participated in a teaching method course, three mathematics courses, a pedagogy course related to measurement and evaluation in teaching and learning in the first semester of the study. The researcher also participated in the method course as a teaching assistant. The method course instructor did not apply any approach to develop TPACK, and digital contents such as representation tools and videos were the most used technological tools in the method course. Six PSTs were included in the study group due to their pre-test scores from TPACK-SAS and the polygon questionnaire.

Participants participated in TPACK-based workshops that were designed to model how to use different technological tools (e.g. calculators, applications, websites, dynamic geometry software, and computer algebra systems) in various mathematical content strands (e.g. geometry, algebra, calculus). We aimed to model technology use that enables students to explore mathematical ideas with these tools. Workshops occurred independently from their teacher preparation program and took fifteen hours in five sessions. The first researcher acted as a teacher and participants as students. Participants brought their computers with them. They have known GeoGebra, and they reported that they do not prefer to learn and use new software in their teaching practices because of language. GeoGebra serves in Turkish, and this feature makes GeoGebra practical and attractive in Turkey. So, they were encouraged to use GeoGebra constructively and fruitfully. The workshops' main aim was to encourage participants to use various technologies appropriately and actively to teach different subject matters with modeling technology use.

In the first workshop, we introduced the calculator as a learning tool. Participants addressed their previous teachers' strict rules related to using calculators. They reported their fears about the potential of calculators in making calculations for students impeding their number sense development. We attempted to demonstrate how calculators could promote students' skills in finding patterns and developing number sense. In the second workshop, the focus was on Web-based mathematical applications and digital games. The aim was to make participants realize how to use apps and digital games to develop students' number sense and motivation. The third session was related to the median in a triangle modeled with GSP. The reason for using GSP was to improve participants' repertoire of mathematical software and to help them recognize the similar features between GeoGebra and GSP. Understanding the similarities between the software may encourage them to use different software with the same features. Constructing the median of a triangle, discussing what students might know about median, attempting to identify students' misconceptions about median, and reflecting upon how GSP might support the concept of the median were the descriptors of the third workshop. The last two workshops were related to learning and teaching algebra and calculus with technology because participants reported that they thought geometry was the most appropriate strand to use technology. GSP and spreadsheets were used to show how they supported the development of concepts such as linear equations, limits, and variables. Table 3 demonstrates which technology is used in which session and for which activity.

All PSTs enrolled in the teaching method course planned and conducted a microteaching lesson individually. They acted as teachers in a middle school and were evaluated by their peers via an observation checklist. The study participants were asked to plan a lesson related to polygons by using technology. Their lesson implementations were videotaped. Participants evaluated themselves after watching videos and reviewing the evaluation forms of the lecturer and their peers. Then they re-planned their microteaching lesson and implemented it to a different group.

Participants were divided into three groups at the beginning of student teaching within the context of MLS. They organized two technology-based research lessons in real classrooms. The research lessons' goal was to demonstrate and explore geometrical concepts related to polygons using GeoGebra as an explorative app. The researchers videotaped their first research lessons, and participants watched the videos of themselves and their peers individually. Then, they came together to discuss and evaluate the research lessons. The

Table 3. Technologies and activities used in the workshops.

Session	Technology	Activity
First	Calculator	Finding patterns and developing number sense
Second	Web-based applications	Developing number sense and motivation
Third	GSP	Constructing the median of a triangle
Fourth	GSP	Solving equations using GSP activities
Fifth	Spreadsheet	Multiple representations / Modelling limit

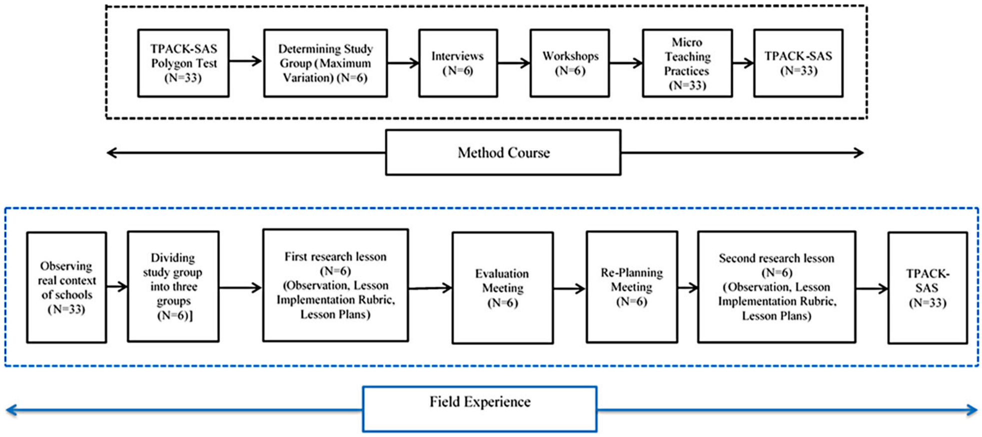

Figure 4. Research process.

prompting questions that guided the evaluation meeting are as follows: (i) How was the technology used in the research lesson? (ii) Does technology serve the purpose of research lessons? (iii) What are the affordances and constraints of the technology in the research lesson? (iv) How should it be used to serve the purpose of the research lesson better?

After the evaluation meeting, the first researcher came together with groups individually and guided them to plan the second research lesson. After re-planning, participants taught their re-planned research lessons to different groups. Participants did not work with a cooperating teacher who actively used technology for mathematical explorations. Figure 4 demonstrates the research process.

3.4. Data collection tools and data analysis

Interviews, lesson implementations, and lesson plans were data sources. At the beginning of the study, participants were interviewed to understand better their mathematical, technological, and pedagogical perceptions. Participants' lesson implementations were observed and videotaped, and the first author took observation notes about the learning environment that is difficult to see in videos. These records were transcribed, and transcripts were analyzed using the lesson implementation rubric (LIR) (Mudzimiri, 2012) to investigate TPACK components' development. Mudzimiri (2012) adapted this rubric from Lyublinskaya and Tournaki (2011), who reported that the rubric could be used for written artifacts, descriptions of the observed lessons, and teachers' self-reflections. In the original form of the rubric, items were particular to TI-Nspire. Mudzimiri (2012) rearranged items for preservice mathematics teachers' technology uses.

The rubric used to analyze lesson implementations is in a matrix form in which rows demonstrate TPACK components and columns demonstrate TPACK development levels. There are two indicators for each level in each component. For example, '(i) Technology is used for practice only, and all learning of new ideas was done through the teacher mostly without technology and (ii) Technology activities do not include inquiry tasks. Technology procedures concentrate on drills and practice only.' are the indicators of Recognizing level in the overarching conception about the purposes of incorporating technology in teaching mathematics. Every indicator met by PSTs is calculated as .5, as is in the Lyublinskaya and Tournaki (2011). Each component's level is calculated individually, and the TPACK level is determined due to the lowest score. For example, let us assume that a PST's scores for each component are as follows; 1.5, 2 (Accepting), .5 (Recognizing), and 1.5 (Accepting). We can see that PST's levels of three TPACK components are Accepting, but this does not mean her overall TPACK level is also Accepting. She must meet all the Accepting level indicators in the four central components to evaluate her overall TPACK level as Accepting. Although TPACK levels of three components are Accepting, she could not meet all the indicators in the Accepting level. So, the TPACK level is determined as Recognizing, in which PST has the lowest score.

The first step in analyzing the data was to transcript the videotapes of the teaching practices and the evaluating and re-planning meetings. Content analysis was applied to find evidence of TPACK components and to gain a comprehensive understanding of participants' knowledge about technology-supported mathematics teaching. PSTs' implementations were also analyzed using Lesson Implementation Rubric to determine their TPACK level for each TPACK component. We began data analysis with the first MLS group and the first participant in this group. We first watched the microteaching video and read the transcribed implementation and field notes. The analysis started with the first TPACK component (purpose) and continued with other components (student, curriculum, and strategy). We then continued analyzing the second microteaching, first research lesson, and second research lesson, respectively. The analyzing process (beginning with the first microteaching and ending with the second research lesson) was carried out for each participant by analyzing components individually and respectively. The researchers analyzed the implementations of the first MLS group independently and came together and discussed on analysis. For the first PST, inter-rater reliability was calculated as .86 and for the second as .89, indicating an acceptable level of interrater reliability between researchers (Creswell, 2012). Common descriptors were identified after the discussions (Table 4). Then, we analyzed all the implementations of participants using these descriptors.

Some common behaviors were observed in the development levels. For example, PSTs at the Recognizing level usually used GeoGebra only for visual representations and focused on using GeoGebra instead of mathematical processes, and mathematical knowledge was gained without GeoGebra. Those whose level is Accepting failed to guide students to make mathematical explorations, although they aimed to teach new knowledge with GeoGebra. Lastly, a PST who taught in a new and different way from teaching without GeoGebra and helped students explore mathematical ideas with GeoGebra was at the Adapting level. Also, PSTs' lesson plans were used to investigate the extent to which lesson plans represent their teaching practices.

3.5. Trustworthiness

In the first semester of the method course, the first author attended the class to enable PSTs to feel comfortable. Thus, the researcher was part of the world where she performed the research. Besides, triangulation of the source (collecting data from different participants) and the methods (collecting data with various tools such as questionnaires and observation) were other ways of producing trustworthiness. Detailed information about

Table 4. Descriptors of the TPACK levels for each TPACK component.

	Recognizing	Accepting	Adapting	Exploring	Advancing
Overarching conception about the purposes of incorporating technology in teaching mathematics	PST uses technology for practice and teaches new knowledge without technology	PST uses technology for the demonstration that also includes presenting new knowledge	PST teaches in a way that was different from teaching without technology, and her students used technology for inquiry tasks that concentrated on mathematical connections	PST allows students to use technology to construct new knowledge and do mathematics	Technology tasks deepen students' conceptual understanding and promote higher-order thinking level
Knowledge of students' understandings, thinking, and learning in mathematics with technology	Students primarily use technology for practice	Students' work with technology follows teacher demonstration. Technology document of PST presents mathematics textbook without active exploration	Students use technology for both learning new knowledge and reviewing prior knowledge	PST guides students to use technology for conceptual understanding and to perform mathematically meaningful actions	PST promotes students' higher-order thinking with technology
Knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics	PST does not use technology for mathematics learning	PST uses a standard approach to curriculum and uses technology as an add-on partially aligned with one or more curriculum goals	PST replaces technology for non-technology-based tasks. PST's technology used is aligned with the curriculum	Technology-based problem-solving tasks are given to students to expand their mathematical ideas. PST's technology use is aligned with curriculum goals and the chosen topics	Technology use is constructively and effectively that deepens students' conceptual understanding
Knowledge of instructional strategies and representations for teaching and learning mathematics with technologies	PST focuses on how to use technology rather than using technology for mathematical explorations	PST plans to teach with limited student' explorations with technology	PST uses teacher-led technology instructions to focus on mathematical explorations rather than traditional curriculum	PST's technology use is beyond traditional teaching. PST use various instructional strategies to promote students' thinking mathematics with technology	PST focuses on students' conceptual understanding related to new mathematical ideas and promotes students' question-posing with the aim of sense-making and reasoning

participants was given to ensure the transferability of results to another context of participants (Lincoln & Guba, 1985). The last step for trustworthiness is analyzing data individually and discussing it together until reaching a consensus.

4. Results

After workshops, participants were asked to plan and enact four technology-based lessons related to polygons. Two of them took place within the method course (individually) and others within the student teaching (collaboratively). The first two teaching practices conducted in the method course will be called 'microteaching,' and the last two teaching practices in the student teaching will be called 'research lesson,' as Cavin (2007) did. TPACK development of participants is discussed within the context of MLS groups.

4.1. TPACK development of the first MLS group

PST-1 and PST-2 were in the first MLS group. They had different beliefs about teaching with technology. PST-1 is aware of the affordances that technology provides for teaching mathematics, and PST-2 had resistant beliefs that insisted on using the technology after mastering concepts in a paper-pencil environment. There was no efficient interaction within this group.

PST-1 planned a lesson related to the angles and sides of the regular polygons in her first microteaching. She used GeoGebra to show the measurements of interior angles and the lengths of the sides of the regular polygons with three, four, five, and six sides (Figure 5). She showed the invariance of the equality of the sides and the angles by the dragging option. There was a limited exploration with GeoGebra even though her teaching was different from traditional. Peer evaluators and the lecturer approved her microteaching, so she did not change her lesson plan in the second microteaching. TPACK level of PST-1 was 'Accepting' during her two microteaching. PST-1 used GeoGebra primarily for demonstration, and peer-students did not need GeoGebra for inquiry.

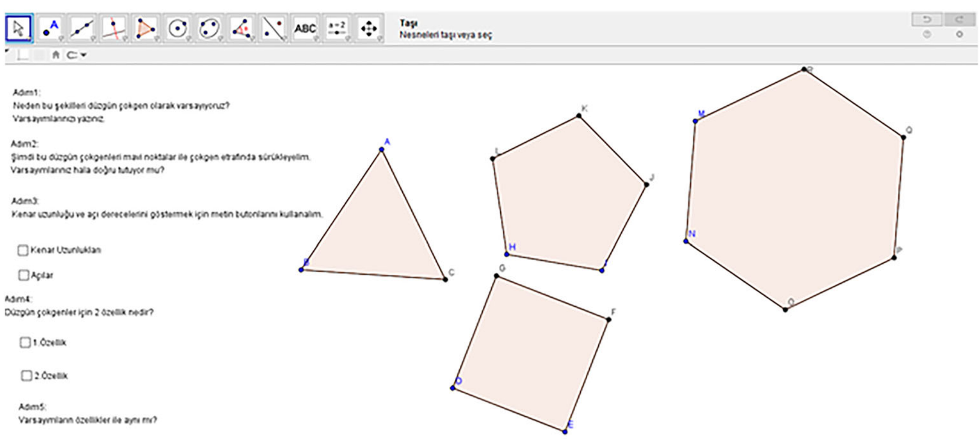

Figure 5. GeoGebra screenshot prepared by PST-1 for her lesson implementations.

In her first research lesson in student teaching, she taught the same topic, properties of regular polygons and polygons' interior and exterior angles. She used GeoGebra to draw accurate diagonals of quadrilateral, pentagon, and hexagon. Drawing the correct diagonals allowed her to show how to calculate the interior and exterior angles of polygons. However, she could not engage students in exploring concepts with technology, and she addressed important ideas on the blackboard. Her TPACK level drew back to the level of Recognizing. In the evaluation meeting, PST-1 insisted that their use of GeoGebra did not lead to students' mathematical inquiry, and they had to use GeoGebra differently from their prior teaching practices. She gave the most critical part of her teaching to technology in her second research lesson. Her teaching style was different from teaching without technology, and students could make mathematical explorations only with GeoGebra. She guided students to calculate the area of regular polygons with the help of equilateral triangles. Students explored the mathematical ideas with GeoGebra, and the TPACK level of PST-1 progressed to Adapting. Rubric scores of PST-1 and PST-2 are given in Figure 6.

PST-2 taught the relationship between the sides and the angles of a triangle. She enhanced her teaching with worksheets during microteaching. She only drew triangles on GeoGebra; peer-students performed all the inquiry tasks with worksheets. She used GeoGebra only for her practice, and students were not compelled to make inquiries with GeoGebra. However, she used the dragging option to demonstrate that the relationship between the sides and angles remained unchanged. Eventually, her TPACK level remained as Recognizing during her first three teaching practices. After the evaluation meeting, she attempted to change her teaching style. She had not prepared any material or worksheets before the second research lesson. She aimed to verify some statements with GeoGebra. She tried to make students participate in discussions by letting them use GeoGebra. However, she usually did not give them enough time to perform the task and answer questions. She failed to guide students to explore mathematical ideas with technology. PST-2 was determined as being at the Accepting level at the end of the study (Figure 6).

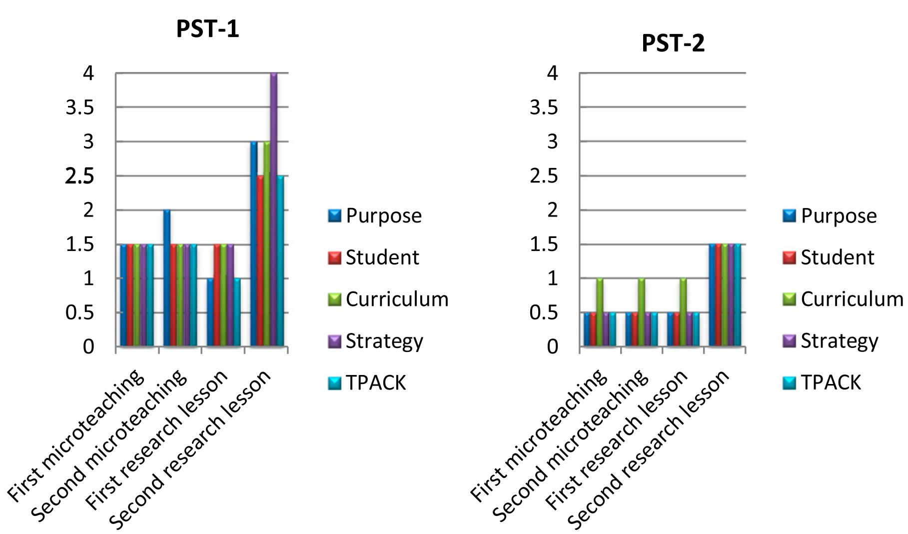

Figure 6. TPACK development of PST-1 and PST-2.

As seen from Figure 6, there is almost no change in PSTs' development of TPACK components during the microteaching sessions. PST-1 and PST-2 planned their research lessons together and taught different parts of their planned research lessons. It is possible to conclude that they made the most considerable improvement in the second research lesson, even though the development was different. Researchers argued that the underlying reason why their TPACK levels differed is their core beliefs about mathematics and technology. PST-1 reported the importance of technology, and she always seemed willing to incorporate technology. On the other hand, PST-2 suggested using the technology after mastering mathematical concepts by hand in a paper-pencil environment, indicating post-mastery beliefs defined by Hanzsek-Brill (1997).

4.2. TPACK development of the second MLS group

PST-3 and PST-6 were in the second MLS group. These PSTs were not so willing to use technology. PST-3 did not feel confident about her skills in teaching mathematics with technology. However, PST- 6 did not believe that technology could improve teaching ways of mathematics. She regarded mathematics as a calculation tool and gave answers more attention than processes.

In her microteaching sessions, PST-3 taught the area of a triangle. She did not prefer to use GeoGebra; instead, she prepared paper-based materials. She showed three questions from a website on the interactive board. She was the only one who chose not to use GeoGebra in the first microteaching lesson. She looked anxious, did not give enough time to peer-students for inquiry, and did not use technology effectively. Her technology use was as presenting a textbook and was only for demonstration. Her TPACK level was 'Recognizing.' In the second microteaching, she used GeoGebra. She used the dragging option of GeoGebra and drew all geometric shapes that she showed with manipulatives in her previous lesson. She failed to show the relationship between the areas of a triangle and a parallelogram. However, she accurately showed different positions of the height in a triangle- this type of use aligned with her lesson objectives. She used GeoGebra for practice, and her technology use did not include learning mathematics. Her TPACK level remained at Recognizing even though the type of her technology use has changed.

She demonstrated the properties of a rectangle and a parallelogram in the first research lesson. She had drawn a rectangle and parallelogram with GeoGebra before the lesson and gave students dotted paper. Students drew rectangles and parallelograms based on the coordinates of points in the GeoGebra, and then they found the pattern in the sides, angles, and diagonals of the rectangle and parallelogram. PST-3 used the dragging tool to construct various rectangles and parallelograms, but she could also have shown the invariance of the properties (such as the equality of opposite sides). She used GeoGebra for demonstration (Figure 7) and accurate drawing and measurements. Students explored ideas with worksheets. PST-3 had problems with time; therefore, her performance was affected negatively, and she failed to guide students. Her TPACK level did not change again. In the second research lesson, she used the same GeoGebra activity but changed her teaching style and overcame time problems. She only introduced the properties of the parallelogram. She firstly talked about GeoGebra. She used the dragging tool to draw different parallelograms and demonstrated that the drawing is still a parallelogram even if she dragged it. She aimed to guide students to reach patterns in a parallelogram with GeoGebra. Students

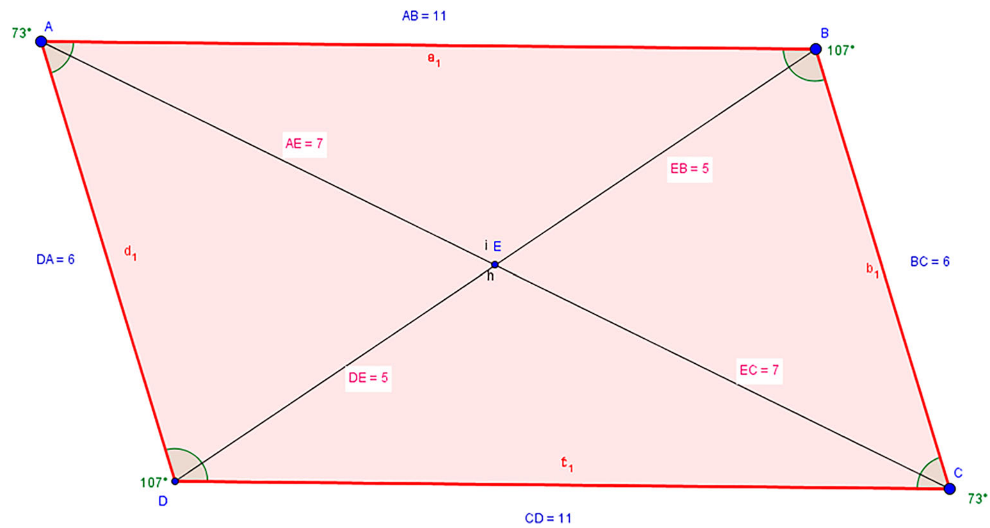

Figure 7. GeoGebra activity that PST-3 used.

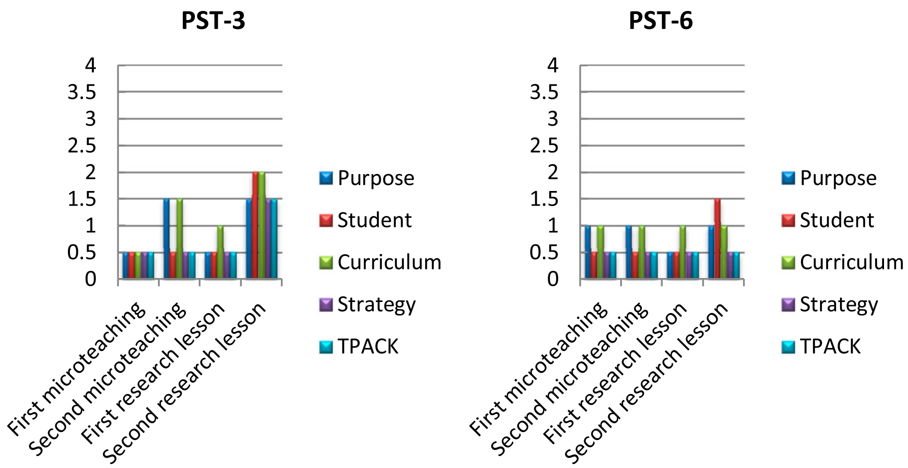

Figure 8. TPACK development of PST-3 and PST-6.

rarely explored mathematics individually with GeoGebra. She seemed more comfortable and confident, and her TPACK level progressed to Accepting.

PST-6 was the only participant who hardly ever made progress. She had the most resistant beliefs to change, and these resistant beliefs seemed not to allow her to use GeoGebra effectively and efficiently. She used GeoGebra to construct accurate drawings. She used the measurements that she calculated with GeoGebra. However, her communication with students was excellent, and she often guided students to explore ideas without GeoGebra. She had drawn accurate shapes before the lesson and made students discuss the properties of shapes and see the relationships in all lessons. Students did not engage in inquiry with GeoGebra in her implementations. Figure 8 shows the development of the second group consisting of PST-3 and PST-6.

PST-3 changed the technology used in the second microteaching and the teaching style with technology in the first research lesson. These changes led to a slight improvement in her conception about the purpose of integrating technology and her knowledge about planning a lesson that is both technology-based and aligned with curriculum goals. When she felt more confident in teaching with technology, her TPACK level progressed in the second research lesson. Her case underpins the findings that confidence is the main factor affecting effective technology integration (Ertmer & Ottenbreit-Leftwich, 2010; Moore-Hayes, 2011). PST-6 is a prospective teacher who believes being a successful mathematics teacher requires solving problems differently and quickly. She rejected using technology because she considered technology inefficient in achieving her professional goals. As seen in Figure 8, her resistant beliefs restrained her TPACK level from progressing.

4.3. TPACK development of the third MLS group

The third MLS group consists of PST-4 and PST-5, with moderate TPACK-SAS scores. They had effective interaction in planning and implementing research lessons.

PST-4 had the most increases and decreases in her rubric scores. In her first microteaching, she demonstrated the classification of triangles by the side and by the angle. She had prepared worksheets for peer students, and they calculated the length of sides with the ruler and the angles with a protractor. PST-4 had the same triangles on her GeoGebra worksheet. Peer students classified the triangles based on the data they obtained in their worksheets. In the second microteaching lesson, she used GeoGebra more actively than in her previous lesson. She guided peer-students to classify triangles with GeoGebra. However, in the first research lesson, GeoGebra was only an add-on. She presented knowledge related to the rhombus and the trapezoid on the blackboard with the help of worksheets. She used GeoGebra to provide a brief and visual presentation at the end of the course. Her GeoGebra worksheet consisted of the shapes and properties. She did not do anything on the GeoGebra worksheet. Therefore, her rubric scores decreased.

After the evaluation meeting, she was one of the participants who significantly improved. Before the second research lesson, she did not prepare anything with GeoGebra (Figure 9). Her purpose was to make students think about what they needed to know about the parallelogram to construct it with GeoGebra. She used GeoGebra as a construction tool. She posed questions about the parallelogram and drew the parallelogram in the direction of students' answers. Students had the opportunity to verify their answers. She used GeoGebra to control students' prior knowledge and present new knowledge. Students inquired with GeoGebra. PST-4 has demonstrated the instantaneous feedback that GeoGebra gave students for their propositions. Her teaching was wholly different from teaching without technology. She has guided students to gain knowledge with GeoGebra and was at the Adapting level. Also, she and her groupmate, PST-5, was the only group who prepared a GeoGebra-based worksheet to evaluate students' understanding.

PST-5 taught the triangle inequality theorem in her microteaching sessions. She predominantly used manipulatives to show the relationships among the length of the sides to draw a triangle. She used GeoGebra after peer-students had realized the relationship among the sides. She demonstrated the change in the lengths of sides in a triangle using a slide bar. This GeoGebra applet showed that if one of the sides of a triangle changed,

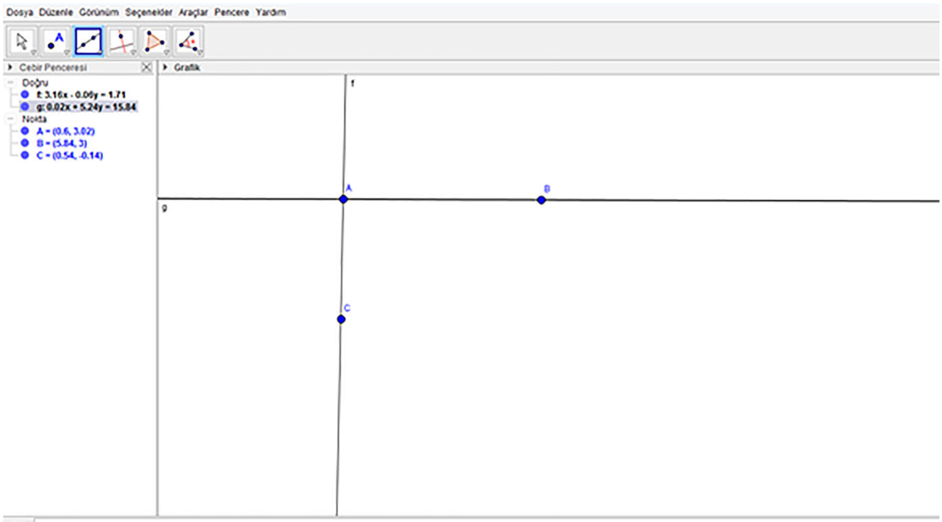

Figure 9. Screenshot from the second research lesson of PST-4.

the other two sides would change. PST-5 used a well-prepared GeoGebra activity but only for practice and demonstration and did not teach content with GeoGebra. Therefore, her TPACK level was 'Accepting' after her microteaching sessions.

Her first research lesson was not different from her groupmate. Students drew rectangles and parallelograms with the help of a ruler and a protractor. After the measurement, PST-5 guided them to discuss the properties of the rectangle (such as all angles are 90°) and the parallelogram (such as opposite sides are parallel and equal) based on the data from their worksheets. At the end of the course, she opened a GeoGebra worksheet demonstrating the accurate drawing and measurements of a rectangle and a parallelogram. Her use of GeoGebra was a brief and visual representation, so her scores did not change.

As occurred in PST-4, she improved her teaching in the second research lesson. She drew a rectangle on the blackboard and took it as a reference to construct the rectangle with GeoGebra. PST-4 helped a student to draw the rectangle on GeoGebra. It was the first time that a participant allowed a student to engage with GeoGebra actively. However, she still seemed to feel uncomfortable with GeoGebra. When a student asked an unexpected question, she immediately turned to traditional teaching by answering on the blackboard. She used a GeoGebra activity to assess student learning too. The TPACK levels of the third MLS group are in Figure 10.

We also examined participants' lesson plans to determine how they referred to technology. PSTs mentioned GeoGebra as a representation tool in all lesson plans only one or two times. Participants' intention to use GeoGebra was for the demonstration in their lesson plans.

5. Discussion

We aimed to investigate the development of preservice mathematics teachers' TPACK. Six PSTs with different mathematical and technological backgrounds were selected, and they

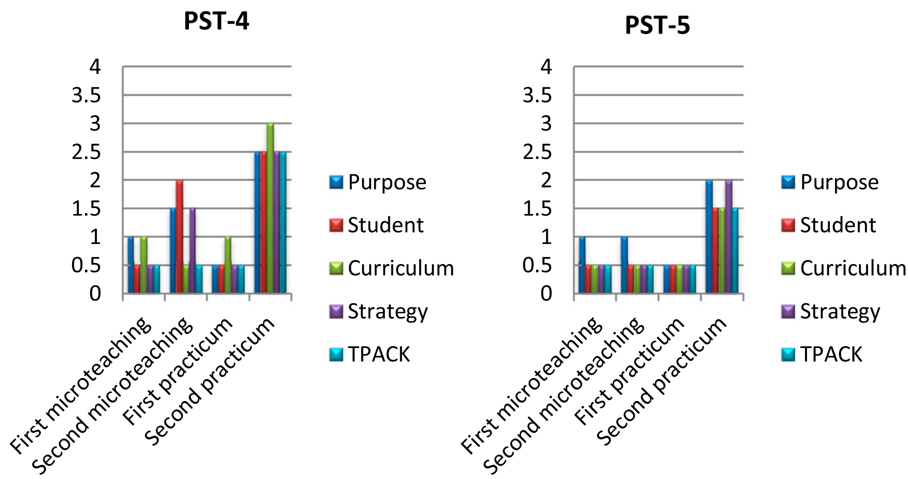

Figure 10. TPACK development of PST-4 and PST-5.

participated in technology-based workshops; planned and implemented four technology-based lessons. They conducted two microteaching in the method course and two research lessons in student teaching. Their teaching practices were videotaped and analyzed using Lesson Implementation Rubric (Mudzimiri, 2012) to identify their TPACK levels. Table 5 presents the TPACK levels of the participants throughout the study.

Most of the participants' TPACK levels were 'Recognizing' until the second research lesson. We can say that participants were aware of the positive effect of GeoGebra on teaching polygons, but they were not ready to integrate GeoGebra in their teaching. They could not have perceived whether their microteaching performance with technology impacted student learning or not because teaching did not occur in real classrooms (Pierce et al., 2009). When they self-critiqued their teaching, five of them developed a favorable attitude towards teaching and learning polygons with GeoGebra (Accepting and Adapting). Only one participant, PST-6, seemed to reject using GeoGebra in her future teaching. Her TPACK level remained at the Recognizing level throughout the study. Two participants who had positive attitudes about teaching with technology engaged with activities that helped them choose whether they used technology in their second research lesson. They reached the Adapting level at the end of the study. PSTs usually used GeoGebra to construct accurate geometrical shapes, and they attempted to guide students to discuss properties of the shapes on GeoGebra. They tended to conduct a student-centered learning environment, but they could not use GeoGebra as an inquiry tool.

Table 5. Participants' TPACK Levels during the study.

		First micro-teaching	Second micro-teaching	First research lesson	Second research lesson
First MLS Group	PST-1	Accepting	Accepting	Recognizing	Adapting
	PST-2	Recognizing	Recognizing	Recognizing	Accepting
Second MLS Group	PST-3	Recognizing	Recognizing	Recognizing	Accepting
	PST-6	Recognizing	Recognizing	Recognizing	Recognizing
Third MLS Group	PST-4	Recognizing	Recognizing	Recognizing	Adapting
	PST-5	Recognizing	Recognizing	Recognizing	Accepting

They used GeoGebra for demonstration, and GeoGebra was a presentation tool in their first research lessons. Using technologies for practice and demonstration is frequent in the findings of similar research (Inan & Lowther, 2010; Ozgun-Koca et al., 2010). They have constructed the shapes, angles, and sides that they need before the lesson. GeoGebra provided them with accurate drawings and measurements. Students could not realize the unique characteristics of GeoGebra. As a result of this type of GeoGebra use, participants remained or went back to the first TPACK level, Recognizing, in their first research lessons. Student teachers can have problems applying their learning to the real context of schools (Hofer & Grandgenett, 2012) because of the mismatch between teacher preparation programs and schools (Mouza et al., 2014; Mudzimiri, 2010; Polly et al., 2010). The first research lesson was participants' first experience teaching mathematics with technology in classrooms. So, the difficulty they experienced in both pedagogy and teaching with technology decreased their TPACK levels. They also used technology in a teacher-centered way because they did not feel confident in classrooms (Polly & Orrill, 2012; Zelkowski et al., 2013). The research addresses the importance of scaffolding provided for PSTs about lesson planning (Figg & Jaipal, 2012; Hofer & Grandgenett, 2012). The first research lesson was the only implementation that participants did not take any scaffolding about planning their lesson. So, it may lead to a decrease in the TPACK levels.

Perkmen and Pamuk (2011) suggested that PSTs can plan and design technology-supported learning environments, but this does not mean they can show the same performance in real classrooms. This finding occurred when participants conducted technology-based research lessons in the real context of schools. Participants spent more time preparing their GeoGebra activities before their first research lessons and were shocked when they implemented technology-based lessons in schools for the first time. They had problems with time (Agyei & Voogt, 2012). They had difficulties predicting student knowledge and interest and guiding students to explore mathematical ideas with technology.

In the evaluation meeting, they mentioned that overworking with GeoGebra before lessons led them to use GeoGebra as a presentation tool of a textbook or any source. So, they intended to enact their last research lesson to guide students to do mathematics with GeoGebra. However, only PST-1 and PST-4 shifted their uses of GeoGebra from a representation tool to a teaching tool that allows students to embody mathematical concepts, helped them to explore mathematical relations and properties, and replaced traditional approaches with GeoGebra as in similar research (Lyublinskaya & Tournaki, 2014; Ozgun-Koca et al., 2010). Participants' second research lessons were different from their previous teaching try-outs. They constructed all the drawings in the class with students. Thus, students had seen what GeoGebra could do and how GeoGebra could give instantaneous feedback for their propositions. However, some participants avoided allowing students to construct drawings. For example, PST-4 constructed a parallelogram based on students' comments, and PST-5 helped students build a rectangle. PST-4 usually mentioned the fear of using the control in class when teaching with technology, and it seems that this fear hindered her from allowing students to use GeoGebra actively. In similar research conducted by authors, it is seen that classroom management is one of the most prominent concerns influencing PSTs' beliefs of TPACK (Kartal & Çinar, 2018).

Participants improved their last teaching practice after the evaluation meeting right after the first research lesson. There may be many reasons for this finding. Firstly, the evaluation meeting made them reflect on what they did with GeoGebra, whether everything went right as they planned or not, and how they could improve their teaching with technology. It is known that the cycle of design-implement-reflect had a positive effect on TPACK (Kartal & Dilek, 2021; Mouza et al., 2014). Secondly, PSTs should not be expected to suddenly show TPACK in their actions (Niess et al., 2007). The value of technology in their belief systems increases as they teach mathematics in the real context of schools (Mouza et al., 2014; Mudzimiri, 2012). TPACK development is a progressive process, and it is required a long time for PSTs to apply their knowledge of technology, pedagogy, and content in their actions. Second research lessons were performed at the end of the study, in the third semester of the study. This long time may allow PSTs to assimilate their knowledge of technology, pedagogy, and mathematics into the center component, TPACK, and to feel prepared to utilize this knowledge in their teaching. Lastly, the evaluation meeting focused on the teaching and learning that guides students to do mathematics with technology. This aspect of the meeting is consistent with the descriptors of the Exploring level. It can be implied that the evaluation meeting helps PSTs attempt to teach in a way that they would reach the Exploring level. Watching and reflecting on video recording also impact improving instructional strategies that employed technology (Figg & Jaipal, 2012; Hofer & Grandgenett, 2012; Kartal & Dilek, 2021). Video analysis of their implementations and feedback made PSTs reflect on their practices (Fernandez, 2005), and they made significant improvements in the last research lesson. The positive effect of video analysis is also previously addressed by Baek and Ham (2009).

As seen in the table, participants in the same group did not show similar changes. For example, PST-1 made the most improvement, although her groupmate, PST-2, made a little. The context in which they teach, the technical conditions, and the students' backgrounds were similar. According to expected-value theory, individuals were supposed to believe the usefulness of activity or believe that they can achieve to act (Feather, 1982). It may be implied that PST-2 and PST-6 did not believe the affordances of GeoGebra, and PST-3 and PST-5 did not believe that they could teach polygons with GeoGebra. Therefore, they could not make a significant improvement in their TPACK levels. It is supposed that PSTs' beliefs and dispositions about what mathematics is and how mathematics should be taught shape their behavioral intentions to use GeoGebra. Those who thought mathematics was more than calculation and should not be memorized (PST-1 and PST-3) developed their TPACK level. However, the second and third MLS groups had good interaction, and the change in TPACK levels in each group was similar. The first MLS group did not interact constructively. It may be because their beliefs about teaching with technology were different. Interaction between peers seems essential to develop TPACK, but we can suggest that peers' beliefs and backgrounds should be similar to interact efficiently.

Hofer and Grandgenett (2012) proposed that longitudinal studies employing surveys and performance assessment help researchers understand the contextual factors. Students' and teachers' demographics, cognitive, physical, psychological, and social characteristics, teacher knowledge, skills, disposition, and physical features of the classrooms are in the knowledge of the context in the TPACK framework (Kelly, 2008). Student assessments were based on a central examination in Turkey. Teachers feel responsible for teaching the intensive curriculum to make their students succeed in these examinations. Teachers and faculty mostly do not feel confident teaching with technology in Turkey. We can conclude that preservice teachers need more experience teaching with technology in classrooms instead of being overloaded with information in contexts like Turkey.

6. Implications

In brief, this study revealed that the more PSTs experience teaching with technology, the more their TPACK levels increase. PSTs should have the opportunity to use their theoretical knowledge in schools' real context and evaluate and redefine these try-outs. Second, we see that PSTs' beliefs and confidence affect how they use technology in their teaching. The more PSTs felt confident with GeoGebra, the more inclined to use it as an inquiry tool. PSTs' confidence in using technology should be promoted with more educational technology courses in which they might learn which mathematics-specific technologies they may use and how they should use these technologies. Lastly, PSTs' core beliefs about the definition of mathematics, teaching styles of mathematics, and role models should be investigated by teacher educators to relate these beliefs to beliefs about teaching with technology.

The results obtained from this study may have important implications for the community of mathematics teacher education. The narratives about participants and their technology-based implementations would give an insight for researchers who might choose to use Niess' TPACK development model and the rubric of Lyublinskaya and Tournaki (2011). It is worth suggesting that employing the development model and the rubric would be beneficial if PSTs could teach with technology in real classrooms. A teaching environment without students (microteaching) may not support PSTs to reflect on the effectiveness of their teaching practice on students' learning.

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Appendix

(1) What is the role of technology in mathematics? How does technology affect mathematics?
(2) What do you think about teaching mathematics with technology? What are the cons and pros?
(3) What do you think about how mathematics should be thought? Can you define mathematics?