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Şebnem Yıldız

A new generalization on absolute Riesz summability

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Kırşehir Ahi Evran University, Kırşehir,Turkey

sebnem.yildiz82@gmail.com

Abstract. In this paper, we have extended a theorem dealing with absolute Riesz summability.
Keywords: Summability factors, absolute matrix summability, infinite series, Hölder's inequality, Minkowski's inequality
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INTRODUCTION

Let $\sum a_n$ be a given infinite series with partial sums (s_n).Let (p_n) be a sequence of positive numbers such that $P_n = \sum_{v=0}^{n} p_v \to \infty$ as $n \to \infty$, $(P_{-i} = p_{-i} = 0, \quad i \geq 1)$. The sequence-to-sequence transformation

$$w_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v \tag{1}$$

defines the sequence (w_n) of the weighted arithmetic mean or simply the $\left(\bar{N}, p_n\right)$ mean of the sequence (s_n), generated by the sequence of coefficients (p_n) (see [4]). Let (θ_n) be any sequence of positive constants. The series $\sum a_n$ is said to be summable $|\bar{N}, p_n, \theta_n|_k$, $k \geq 1$, if (see [7])

$$\sum_{n=1}^{\infty} \theta_n^{k-1} \mid t_n - t_{n-1} \mid^k < \infty. \tag{2}$$

Theorem 1 *Let (X_n) be an almost increasing sequence and let $(\theta_n a_{nn})$ be a non-increasing sequence. Suppose that there exists sequences (β_n) and (λ_n) such that*

$$|\Delta\lambda_n| \leq \beta_n, \tag{3}$$

$$\beta_n \to 0 \quad as \quad n \to \infty, \tag{4}$$

$$\sum_{n=1}^{\infty} n|\Delta\beta_n| X_n < \infty, \tag{5}$$

$$|\lambda_n| X_n = 0(1). \tag{6}$$

If

$$\sum_{n=1}^{m} \theta_n^{k-1} \frac{|t_n|^k}{n^k X_n^{k-1}} = O(X_m) \quad as \quad m \to \tag{7}$$

and (p_n) is a sequence such that

$$P_n = O(np_n), \tag{8}$$

$$P_n \Delta p_n = O(p_n p_{n+1}), \tag{9}$$

then the series $\sum_{n=1}^{\infty} a_n \lambda_n P_n / np_n$ is summable $|\bar{N}, p_n, \theta_n|_k$, $k \geq 1$ (see ([3])).

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Main Results

Given a normal matrix $A = (a_{nv})$, we associate two lower semimatrices $\bar{A} = (\bar{a}_{nv})$ and $\hat{A} = (\hat{a}_{nv})$ as follows:

$$\bar{a}_{nv} = \sum_{i=v}^{n} a_{ni}, \quad n, v = 0, 1, \ldots \quad \bar{\Delta} a_{nv} = a_{nv} - a_{n-1,v}, \quad a_{-1,0} = 0 \tag{10}$$

and

$$\hat{a}_{00} = \bar{a}_{00} = a_{00}, \quad \hat{a}_{nv} = \bar{\Delta}\bar{a}_{nv}, \quad n = 1, 2, \ldots \tag{11}$$

It may be noted that $\bar{A}$ and $\hat{A}$ are the well-known matrices of series-to-sequence and series-to-series transformations, respectively. Then, we have

$$A_n(s) \quad = \sum_{v=0}^{n} a_{nv} s_v = \sum_{v=0}^{n} \bar{a}_{nv} a_v \tag{12}$$

and

$$\bar{\Delta} A_n(s) \quad = \sum_{v=0}^{n} \hat{a}_{nv} a_v. \tag{13}$$

In this study, we extend Theorem 1 to $|A, \theta_n|_k$ summability method (see [5] and [6]) with above notation as follow:

Theorem 2 *Let $A = (a_{nv})$ be a positive normal matrix such that*

$$\bar{a}_{n0} \quad = 1, \ n = 0, 1, \ldots, \tag{14}$$

$$a_{n-1,v} \quad \geq a_{nv}, \ \text{for} \ n \geq v + 1, \tag{15}$$

$$a_{nn} \quad = O\left(\frac{p_n}{P_n}\right), \tag{16}$$

$$n a_{nn} \quad = O(1), \tag{17}$$

$$\hat{a}_{n,v+1} \quad = O(v|\Delta_v \hat{a}_{nv}|). \tag{18}$$

Let $(\theta_n a_{nn})$ be a non-increasing sequence and let (X_n) be an almost increasing sequence. If all the conditions of Theorem 1 are satisfied then the series $\sum_{n=1}^{\infty} a_n \lambda_n P_n / n p_n$ is summable $|A, \theta_n|_k$, $k \geq 1$.

Conclusion

If we take $\theta_n = \frac{P_n}{p_n}$ in Theorem 2, then we have a result concerning the $|A, p_n|_k$ summability factors of infinite series, and if we take $a_{nv} = \frac{p_v}{P_n}$ Theorem 2, then we have another result dealing with $\left|\bar{N}, p_n, \theta_n\right|_k$ summability factors of infinite series. Also, if we put $a_{nv} = \frac{p_v}{P_n}$ and $p_n = 1$ for all n in Theorem 2, then we obtain a result concerning $|C, 1, \theta_n|_k$ summability factors of infinite series. Moreover, if we take $\theta_n = \frac{P_n}{p_n}$, $k = 1$ and $a_{nv} = \frac{p_v}{P_n}$ in Theorem 2, then we have a result dealing with $\left|\bar{N}, p_n\right|$ summability factors of infinite series ,and if we take $\theta_n = n$, $a_{nv} = \frac{p_v}{P_n}$ and $p_n = 1$ for all n in Theorem 2, then we obtain a result concerning the $|C, 1|_k$ summability factors of infinite series.

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