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Cite as: AIP Conference Proceedings **2183**, 050016 (2019); https://doi.org/10.1063/1.5136154
Published Online: 06 December 2019

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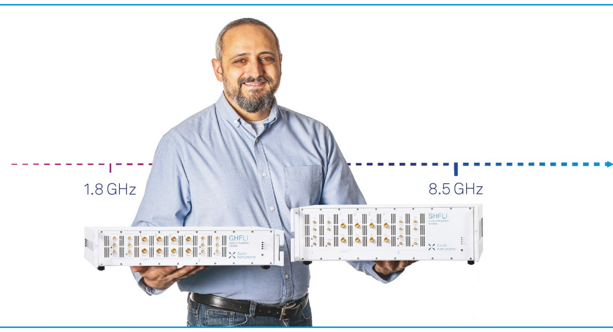

A new extension on absolute matrix summability factors of infinite series

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Abstract. In this paper, we have generalized a new summability factor theorem for infinite series involving quasi power increasing sequences. Some new results are also deduced.
Keywords: Summability factors, absolute matrix summability, infinite series, Hölder's inequality, Minkowski's inequality
PACS: 02.30.Lt, 02.30.Sa

INTRODUCTION

Let $A = (a_{nv})$ be a normal matrix, i.e., a lower triangular matrix of nonzero diagonal entries. Then A defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $As = (A_n(s))$, where

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v, \quad n = 0, 1, ... \tag{1}$$

The series $\sum a_n$ is said to be summable $|A, p_n|_k$, $k \geq 1$, if (see [13])

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} |A_n(s) - A_{n-1}(s)|^k < \infty. \tag{2}$$

If we take $p_n = 1$ for all n, then we have $|A|_k$ summability (see [15]). And also if we take $a_{nv} = \frac{p_v}{P_n}$, then we have $\left|\bar{N}, p_n\right|_k$ summability (see [2]). Furthermore, if we take $a_{nv} = \frac{p_v}{P_n}$ and $p_n = 1$ for all n, then $|A, p_n|_k$ summability reduces to $|C, 1|_k$ summability (see [10]). Let ω be the class of all matrices $A = (a_{nv})$ and A be a normal matrix satisfying:

$$\overline{a}_{n0} = 1, \; n = 0, 1, ..., \tag{3}$$

$$a_{n-1,v} \geq a_{nv}, \; \text{for} \; n \geq v + 1, \tag{4}$$

$$a_{nn} = O\left(\frac{p_n}{P_n}\right). \tag{5}$$

A positive sequence (b_n) is said to be an almost increasing sequence if there exists a positive increasing sequence (c_n) and two positive constants M and N such that $Mc_n \leq b_n \leq Nc_n$ (see [1]).
Every increasing sequence is almost increasing, but the converse need not to be true, by taking for example $b_n = e^{(-1)^n} n$ (see [12]). The concept of a quasi $\beta-$ power increasing sequence (see [8]) is that a positive sequence $\alpha = (\alpha_n)$ is said to be a quasi-β-power increasing sequence if there exists a constant $K = K(\alpha,\beta)$ such that $Kn^{\beta}\alpha_n \geq m^{\beta}\alpha_m$ holds for $n \geq m \geq 1$. Sulaiman generalizes this definition by giving [14]. A positive sequence $\alpha = (\alpha_n)$ is said to be quasi-f-power increasing sequence, $f = (f_n)$, if there exists a constant $K = K(\alpha, f)$ such that $Kf_n\alpha_n \geq f_m\alpha_m$ for all $n \geq m \geq 1$. It may be mentioned that every almost increasing is quasi-β-power increasing sequence for any nonnegative β, but the converse need not to be true, by taking, for example, $\alpha_n = n^{-\beta}$, $\beta > 0$ (see [8]).

Third International Conference of Mathematical Sciences (ICMS 2019)
AIP Conf. Proc. 2183, 050016-1–050016-3; https://doi.org/10.1063/1.5136154
Published by AIP Publishing. 978-0-7354-1930-8/$30.00

For any sequence (λ_n) we write that $\Delta^2\lambda_n = \Delta\lambda_n - \Delta\lambda_{n+1}$ and $\Delta\lambda_n = \lambda_n - \lambda_{n+1}$. The sequence (λ_n) is said to be of bounded variation, denoted by $(\lambda_n) \in \mathcal{BV}$, if $\sum_{n=1}^{\infty} |\Delta\lambda_n| < \infty$.

Mazhar has proved the main theorem concerning $|C,1|_k$ summability (see [11]), and this theorem has extended by Bor to $\left|\bar{N}, p_n\right|_k$ summability (see [6]). Sulaiman has obtained a further generalization of Mazhar's Theorem (see [12]) by taking quasi-f-power increasing sequence as follows:

Theorem 1 [14] Let (X_n) be a quasi-f-increasing sequence, where $f = (f_n) = (n^\beta log^\gamma n)$, $\gamma > 0$, $0 < \beta < 1$ and

$$|\lambda_m|X_m = O(1) \quad as \quad m \to \infty, \tag{6}$$

$$\sum_{n=1}^{m} nX_n|\Delta^2\lambda_n| = O(1) \quad as \quad m \to \infty, \tag{7}$$

$$\sum_{n=1}^{m} \frac{P_n}{n} = O(P_m) \quad as \quad m \to \infty, \tag{8}$$

and

$$\sum_{n=1}^{m} \frac{p_n}{P_n}|t_n|^k|\lambda_n|^k = O(1), \quad as \quad m \to \infty \tag{9}$$

$$\sum_{n=1}^{m} \frac{|t_n|_k}{nX_n^{k-1}} = O(X_m) \quad as \quad m \to \infty \tag{10}$$

hold, then the series $\sum a_n\lambda_n$ is summable $\left|\bar{N}, p_n\right|_k$, $k \geq 1$.

Theorem 2[14] If the condition (8) is replaced with

$$\lambda_n = O\left(n|\Delta\lambda_n|\right), \quad n \to \infty, \tag{11}$$

in Theorem 1 and all the other conditions are satisfied, then the series $\sum a_n\lambda_n$ is summable $\left|\bar{N}, p_n\right|_k$, $k \geq 1$.

The Main Results

The aim of this paper is to generalize Theorem 1 and Theorem 2 for $|A, p_n|_k$ summability method by concerning quasi-f-power increasing sequence. Recently, some studies have been done concerning absolute matrix summability factors of infinite series (see [3]-[7], [16]-[18]). Using the above matrix notations, we have the following theorems.

Theorem 3 Let $A \in \omega$ satisfy

$$\sum_{v=1}^{n-1} \frac{1}{v}\hat{a}_{n,v+1} = O(a_{nn}), \tag{12}$$

and (X_n) be a quasi-f-power increasing sequence, where $f = (f_n) = (n^\beta log^\gamma n)$, $\gamma > 0$, $0 < \beta < 1$. If all the other conditions of Theorem 1 are satisfied, then the series $\sum a_n\lambda_n$ is summable $|A, p_n|_k$, $k \geq 1$.

Theorem 4 Let $A \in \omega$ and (X_n) be a quasi-f-power increasing sequence, where $f = (f_n) = (n^\beta log^\gamma n)$, $\gamma > 0$, $0 < \beta < 1$. If all the other conditions of Theorem 2 are satisfied, then the series $\sum a_n\lambda_n$ is summable $|A, p_n|_k$, $k \geq 1$.

APPLICATIONS

By applying Theorem 3, Theorem 4 to weighted mean so, the following results can be easily verified.

1. If we take $a_{nv} = \frac{p_v}{P_n}$ in Theorem 3, Theorem 4, then we have Theorem 1 and Theorem 2.
2. If we take $p_n = 1$ for all n in Theorem 3 and Theorem 4, then we have a new result dealing with $|A|_k$ summability.
3. If we take $a_{nv} = \frac{p_v}{P_n}$ and $p_n = 1$ for all n in Theorem 3 and Theorem 4, then we have a new result concerning $|C,1|_k$ summability.

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