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Novel Complex Wave Solutions of the (2+1)-Dimensional Hyperbolic Nonlinear Schrödinger Equation

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Abstract: This manuscript focuses on the application of the $(m + 1/G')$-expansion method to the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation. With the help of projected method, the periodic and singular complex wave solutions to the considered model are derived. Various figures such as 3D and 2D surfaces with the selecting the suitable of parameter values are plotted.

Keywords: the $(m + 1/G')$-expansion method; the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation; periodic and singular complex wave solutions; traveling waves solutions

1. Introduction

Most of the properties of nature and science explained by using nonlinear partial differential equations (NPDEs) are closely associated with the basic properties of applied sciences. Recently, NPDEs have been used to investigate properties of many real-world problems arising in fluid mechanics, population ecology, shallow-water wave propagation, plasma physics, solid-state physics, heat, quantum mechanics, optical fibers and biology. Moreover, their mathematical structures have also been presented to literature. Therefore, many effective methods such as $(m + G'/G)$-expansion method [1,2], $(1/G')$-expansion method [3–5], rational sine–cosine function method [6], F-expansion method [7], Clarkson–Kruskal (CK) direct method [8], (G'/G)-expansion method [9], Bäcklund transformation method [10], modified $\exp(-\Omega(\xi))$-expansion function [11], the Painlevé analysis [12], $(G'/G, 1/G)$-expansion method [13], modified Laplace decomposition method [14], Hirota bilinear method [15,16], homotopy analysis method [17], modified Kudryashov method [18], etc. [19–47] have been presented to the literature for observing of deeper properties of these models. In this sense, many detailed explanations of some methods with the regards of physical and mathematical properties have been presented by R. Conte and his team [48,49].

In this work, we consider the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation (HNSE) [19]:

$$ih_y(x, y, t) + \frac{1}{2}[h_{xx}(x, y, t) - h_{tt}(x, y, t)] + \left|h(x, y, t)\right|^2 h(x, y, t) = 0, \tag{1}$$

where $h(x, y, t)$ is used to describe the complex field, x, y and t denote spatial and temporal variables, respectively. Nonlinear Schrödinger equations are mathematical models that correspond to basic physical phenomena that define the dynamics of optical strength propagation in single-mode optical fibers [43–46]. Many scientists have observed various properties of this model. Analytical properties to the Equation (1) have been obtained in [20], exact solutions Equation (1) using extended sinh–Gordon equation expansion method [21], via Adomian decomposition method [22], with the help of the first

integral method [23] and many other properties such as instabilities of Schrödinger equation in [24] and also group-invariant solutions and conservation laws in [36].

In second section, we present the general properties of the $(m+1/G')$-expansion method. This method is an extended version of the classic $(1/G')$-expansion method. Specifically, when $m=0$, solutions produced in $(1/G')$-expansion method can be obtained. In third section, we apply the $(m+1/G')$-expansion method to the governing model to find many new periodic and singular complex wave solutions. In fourth section, we discuss some important properties of new findings. In fifth section, we introduce a conclusion about the findings and figures.

2. General Properties of $(m+1/G')$-Expansion Method

Consider the general form of NPDEs as:

$$P\Big(u, u_x, u_y, u_z, u_t, u_{xyzt}, \cdots\Big) = 0, \tag{2}$$

and using wave transformation given as:

$$\phi(x,y,t) = U(\xi),\ \xi = c_1x + c_2y + c_3z + c_4t, \tag{3}$$

where $c_i \neq 0,\ (i=1,2,3,4)$. Using Equation (3) into Equation (2) yields a nonlinear ODE as following:

$$N\Big(U, U', U'', U^2, \cdots\Big) = 0. \tag{4}$$

The solution of Equation (4) may assumed in the following form according to projected method:

$$U(\xi) = \sum_{i=-n}^{n} a_i(m+F)^i = ma_0 + a_1(m+F) + a_2(m+F)^2 + \ldots + a_n(m+F)^n, \tag{5}$$

where $a_i, (\ i=0,1,\cdots,n)$ are constants, m is nonzero and real constant. With the balancing principle, we find the value of n. Moreover, F is defined as following:

$$F = \frac{1}{G'(\xi)}, \tag{6}$$

and $G' = G'(\xi)$ provides the following second order linear ordinary differential equation:

$$G'' + (\lambda + 2m\mu)G' + \mu = 0, \tag{7}$$

where λ and μ are real constants and non zero to be determined later. Putting the Equation (5) into Equation (4) and using Equation (6), then collect all terms with the same order of the $(m+F)^n$, we obtain a system of algebraic equations for $c_i \neq 0,\ (i=1,2,3,4)$, $a_i, (\ i=0,1,\cdots,n)$, μ and λ. Finally, when we solve the system to find the value of $c_i \neq 0,\ (i=1,2,3,4)$ and $a_i, (\ i=0,1,\cdots,n)$, and inserting them into Equation (5), we can extract the periodic and singular complex wave solutions to the Equation (2).

3. Application of Projected Method

In this section, we apply the considered method to the Equation (1). Applying the following wave transformation defined as:

$$h(x,y,t) = \mathrm{e}^{\mathrm{i}\phi(x,y,t)}U(\xi),\ \xi = x - t\rho + y\tau,\ \phi(x,y,t) = ax + by + dt + \theta_0, \tag{8}$$

where $a,\ b,\ d,\ \rho,\ \tau,\ \theta_0$ are real constants with not zero. ρ is velocity, τ is the slope of the connector between the two stable states of the solution, a is the frequency, d is the phase, b is wavenumber, θ_0 is the center of phase. Considering Equation (8) into Equation (1), we have follows:

$$\left(-a^2-2b+d^2\right)U+2U^3-\left(-1+\rho^2\right)U''=0, \tag{9}$$

$$2\mathrm{i}(a+d\rho+\tau)U'=0. \tag{10}$$

From imaginary part, we get the following strain condition as:

$$a=-\rho-\tau. \tag{11}$$

Balancing in Equation (9), we get $n=1$. Taking this into Equation (5), we get the following solution form

$$U(\xi)=+a_{-1}(m+F)^{-1}+ma_0+a_1(m+F)^1. \tag{12}$$

Substituting Equation (12) into Equation (9), we get the following system of equations:

$$\begin{aligned}
&\left(m+\tfrac{1}{G'}\right)^0: \quad 2m^2\lambda^2a_{-1}-2m^2\lambda^2\rho^2a_{-1}+4m^3\lambda\mu a_{-1}-4m^3\lambda\rho^2\mu a_{-1}+2m^4\mu^2a_{-1}\\
&\qquad -2m^4\rho^2\mu^2a_{-1}+2a_{-1}^3=0,\\
&\left(m+\tfrac{1}{G'}\right)^1: \quad -3m\lambda^2a_{-1}+3m\lambda^2\rho^2a_{-1}-3m^2\lambda\mu a_{-1}+3m^2\lambda\rho^2\mu a_{-1}+6a_{-1}^2a_0=0,\\
&\left(m+\tfrac{1}{G'}\right)^2: \quad -2ba_{-1}+d^2a_{-1}+\lambda^2a_{-1}-\lambda^2\rho^2a_{-1}-2m\lambda\mu a_{-1}+2m\lambda\rho^2\mu a_{-1}-2m^2\mu^2a_{-1}\\
&\qquad +2m^2\rho^2\mu^2a_{-1}-(-d\rho-\tau)^2a_{-1}+6a_{-1}a_0^2+6a_{-1}^2a_1=0,\\
&\left(m+\tfrac{1}{G'}\right)^3: \quad \lambda\mu a_{-1}-\lambda\rho^2\mu a_{-1}-2ba_0+d^2a_0-(-d\rho-\tau)^2a_0+2a_0^3-m\lambda^2a_1+m\lambda^2\rho^2a_1\\
&\qquad -m^2\lambda\mu a_1+\ m^2\lambda\rho^2\mu a_1+12a_{-1}a_0a_1=0,\\
&\left(m+\tfrac{1}{G'}\right)^4: \quad -2ba_1+d^2a_1+\lambda^2a_1-\lambda^2\rho^2a_1-2m\lambda\mu a_1+2m\lambda\rho^2\mu a_1-2m^2\mu^2a_1\\
&\qquad +2m^2\rho^2\mu^2a_1\ -(-d\rho-\tau)^2a_1+6a_0^2a_1+6a_{-1}a_1^2=0,\\
&\left(m+\tfrac{1}{G'}\right)^5: \quad 3\lambda\mu a_1-3\lambda\rho^2\mu a_1+6a_0a_1^2=0,\\
&\left(m+\tfrac{1}{G'}\right)^6: \quad 2\mu^2a_1-2\rho^2\mu^2a_1+2a_1^3=0.
\end{aligned} \tag{13}$$

Solving this system, we can find the following cases of the solutions to the Equation (1).

Case 1. Selecting the following coefficients

$$\begin{gathered}
a_{-1}=0, a_1=\sqrt{-1+\rho^2}\mu, b=\tfrac{1}{4}\left(\left(1-\rho^2\right)\left(2d^2-(\lambda+2m\mu)^2\right)-4d\rho\tau-2\tau^2\right),\\
a_0=\tfrac{1}{2}\lambda\sqrt{\rho^2-1},
\end{gathered} \tag{14}$$

we have the following singular complex wave solution to the Equation (1):

$$h_1=\frac{1}{2}\mathrm{e}^{\mathrm{i}\left(dt-x(d\rho+\mu1)+\frac{1}{4}y(\kappa-4\rho\tau-2\tau^2)+\theta_0\right)}\sqrt{\rho^2-1}\left(\lambda+2\mu\left(m+\frac{\lambda+2m\mu}{-\mu+\varpi\tau\mathrm{e}^{-w(x-t\rho+y\tau)}}\right)\right), \tag{15}$$

where $w=\lambda+2m\mu,\ \ \kappa=\left(1-\rho^2\right)\left(2d^2-w^2\right)$.

Case 2. When we consider another coefficient to the Equation (1) given as:

$$\begin{gathered}
a_{-1}=-m\sqrt{-1+\rho^2}(\lambda+m\mu), a_0=\tfrac{1}{2}\lambda\sqrt{-1+\rho^2}, a_1=0,\\
b=\tfrac{1}{4}\left(-\left(-1+\rho^2\right)\left(2d^2-(\lambda+2m\mu)^2\right)-4d\rho\tau-2\tau^2\right),
\end{gathered} \tag{16}$$

it gives another singular complex wave solution to the governing model as:

$$h_2=\frac{\sqrt{\rho^2-1}}{2}\left(\lambda-\frac{2m\gamma}{m+\frac{\gamma}{-\mu+\varpi e^{-(\lambda+2m\mu)(x-t\rho+y\tau)}\gamma}}\right)e^{i\left(dt-x(\rho+\tau)+\frac{1}{4}y(v-4d\rho\tau-2\tau^2)+\theta_0\right)}, \tag{17}$$

in which $\gamma = \lambda + 2m\mu, v = \left(1-\rho^2\right)\left(2d^2-\gamma^2\right)$.

Case 3. Choosing as:

$$a_{-1} = \frac{\left(-1-\mathrm{i}\sqrt{2}\right)m}{2\sqrt{d^2+\lambda^2}}\sqrt{-2b\lambda^2 + \frac{\lambda^2\tau\left(d^2\tau-\lambda^2\tau-2d\sqrt{-2b(d^2+\lambda^2)+(d^2+\lambda^2)^2-\lambda^2\tau^2}\right)}{d^2+\lambda^2}},$$

$$a_0 = \frac{1}{2\sqrt{d^2+\lambda^2}}\sqrt{-2b\lambda^2 + \frac{\lambda^2\tau\left(d^2\tau-\lambda^2\tau-2d\sqrt{-2b(d^2+\lambda^2)+(d^2+\lambda^2)^2-\lambda^2\tau^2}\right)}{d^2+\lambda^2}}, \tag{18}$$

$$a_1 = 0,\ \rho = \frac{-d\tau+\sqrt{-2b(d^2+\lambda^2)+(d^2+\lambda^2)^2-\lambda^2\tau^2}}{d^2+\lambda^2},\ \mu = \frac{\mathrm{i}\left(\mathrm{i}+\sqrt{2}\right)\lambda}{2m},$$

we extract the following periodic complex wave solution to the Equation (1):

$$h_3 = \frac{\left(2i+\sqrt{2}-4i\omega e^{-i\sqrt{2}\lambda\left(x+y\tau+\frac{t(d\tau-\sqrt{\beta})}{d^2+\lambda^2}\right)m}\right)e^{i\left(dt+by-x\tau+\frac{dx(d\tau-\sqrt{\beta})}{d^2+\lambda^2}+\theta_0\right)}}{2\left(-i+\sqrt{2}+2\sqrt{2}\omega e^{-i\sqrt{2}\lambda\left(x+y\tau+\frac{t(d\tau-\sqrt{\beta})}{d^2+\lambda^2}\right)m}\right)\sqrt{d^2+\lambda^2}}\sqrt{-2b\lambda^2 + \frac{\lambda^2\tau\left(d^2\tau-\lambda^2\tau-2d\sqrt{\beta}\right)}{d^2+\lambda^2}}, \tag{19}$$

where $\beta = d^4 + 2d^2\lambda^2 + \lambda^4 - 2b\left(d^2+\lambda^2\right) - \lambda^2\tau^2 > 0$, with strain condition.

Case 4. If it is taken as following form:

$$a_{-1} = 0,\ a_0 = -\frac{\sqrt{\frac{\lambda^2\left((d-\lambda)(d+\lambda)\tau^2-2b\left(d^2+\lambda^2\right)+2d\tau\sqrt{\left(d^2+\lambda^2\right)^2-2b\left(d^2+\lambda^2\right)-\lambda^2\tau^2}\right)}{d^2+\lambda^2}}}{2\sqrt{d^2+\lambda^2}},$$

$$a_1 = \frac{\left(1-\mathrm{i}\sqrt{2}\right)\sqrt{\frac{\lambda^2\left(-2b\left(d^2+\lambda^2\right)+(d-\lambda)(d+\lambda)\tau^2+2d\tau\sqrt{-2b\left(d^2+\lambda^2\right)+\left(d^2+\lambda^2\right)^2-\lambda^2\tau^2}\right)}{d^2+\lambda^2}}}{2m\sqrt{d^2+\lambda^2}}, \tag{20}$$

$$\rho = -\frac{d\tau+\sqrt{-2b(d^2+\lambda^2)+(d^2+\lambda^2)^2-\lambda^2\tau^2}}{d^2+\lambda^2}, \mu = \frac{\mathrm{i}\left(\mathrm{i}+\sqrt{2}\right)\lambda}{2m},$$

produces following new complex traveling wave solution given as

$$h_4 = -\frac{e^{i\left(dt+by-x\tau+\frac{dx(d\tau+\sqrt{\beta})}{d^2+\lambda^2}+\theta_0\right)}\left(\sqrt{2}-2i-4i\omega e^{-i\sqrt{2}\lambda\left(x+y\tau+\frac{t(d\tau+\sqrt{\beta})}{d^2+\lambda^2}\right)m}\right)\sqrt{-2b\lambda^2+\frac{\lambda^2\tau\left(d^2\tau-\lambda^2\tau+2d\sqrt{\beta}\right)}{d^2+\lambda^2}}}{2\left(i+\sqrt{2}-2\sqrt{2}\omega e^{-i\sqrt{2}\lambda\left(x+y\tau+\frac{t(d\tau+\sqrt{\beta})}{d^2+\lambda^2}\right)m}\right)\sqrt{d^2+\lambda^2}}, \tag{21}$$

where $-2b\lambda^2 + \frac{\lambda^2\tau\left(d^2\tau-\lambda^2\tau+2d\sqrt{\beta}\right)}{d^2+\lambda^2} > 0$, with strain condition.

4. Results and Discussions

First, it may be observed that Figures 1 and 2 are singular complex wave solutions to the governing model, Figures 3 and 4 are periodic complex wave solutions for the Equation (1).Unlike many analytical methods, we offer different solutions from the $(1/G')$-expansion method [25–28] which produces hyperbolic type traveling wave solution. What is interesting here is the idea that at the beginning, if $m = 0$, the solutions produced by the $(1/G')$-expansion method are obtained. However, if $m = 0$ is taken in Equation (20), μ is undefined. Therefore, we offered different solutions from the solution produced by the classic $(1/G')$-expansion method. Such solutions include singular points. Solutions containing single points are important for the shock wave structure. Moreover, the solutions that

provide the equation due to the structure of the Schrödinger equation are of the complex wave solution. These solutions are in hyperbolic form and are different from the solutions produced in other analytical solutions. Appropriate values are given so that the structure of the functions created by the parameters is not disrupted. The special values given to these constants have rendered to draw the shape of the wave at any given moment.

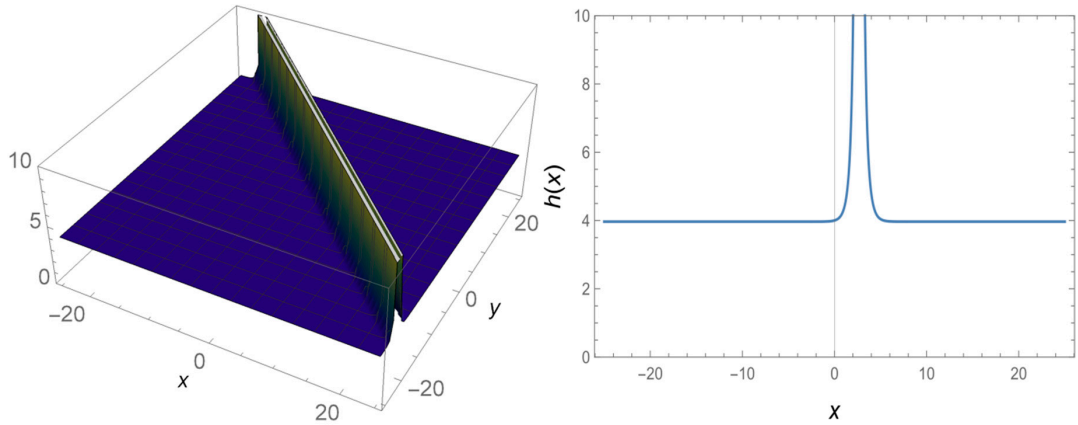

Figure 1. Three-dimensional and 2D graphs of Equation (15) for $t = 2, \theta_0 = 1, \lambda = 0.3, m = 2, \mu = 0.5, d = 2, \rho = 2, \tau = 1, b = 1, \varpi = 1$ values and $y = 2$ for 2D.

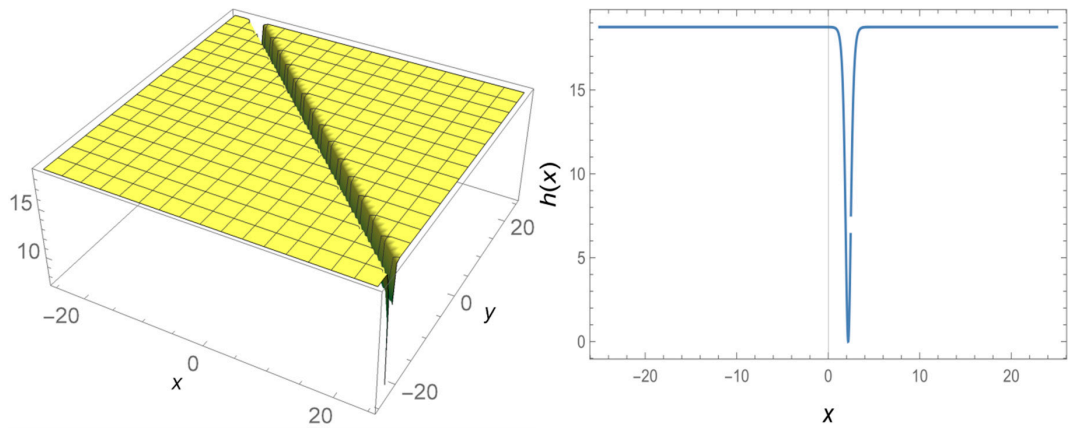

Figure 2. Three-dimensional and 2D graphs for values $t = 2, \theta_0 = 1, \lambda = 3, m = 2, \mu = 0.5, d = 0.4, \rho = 2, \tau = 1, \varpi = 1$ of Equation (17) and $y = 2$ for 2D.

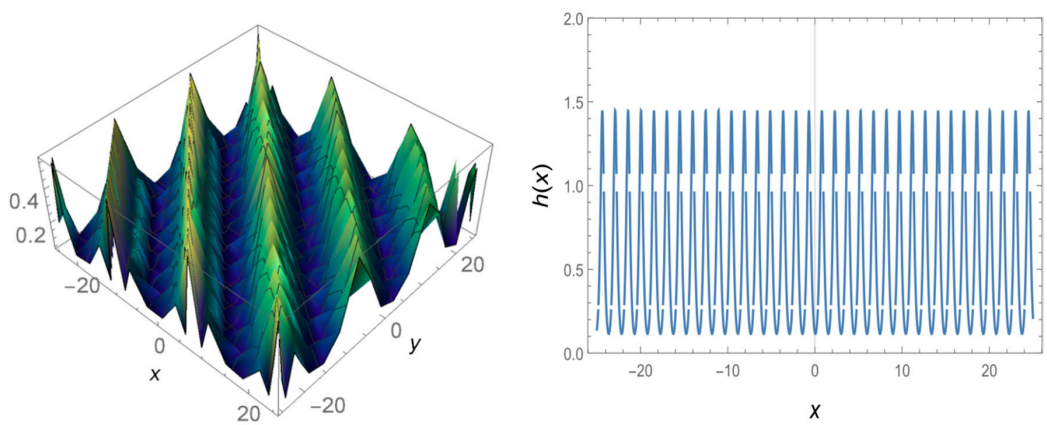

Figure 3. Three-dimensional and 2D graphs for values $t = 2, \theta_0 = 1, \lambda = 3, m = 2, d = 1, \tau = 1, b = -1, \varpi = 1$ of Equation (19) and $y = 2$ for 2D.

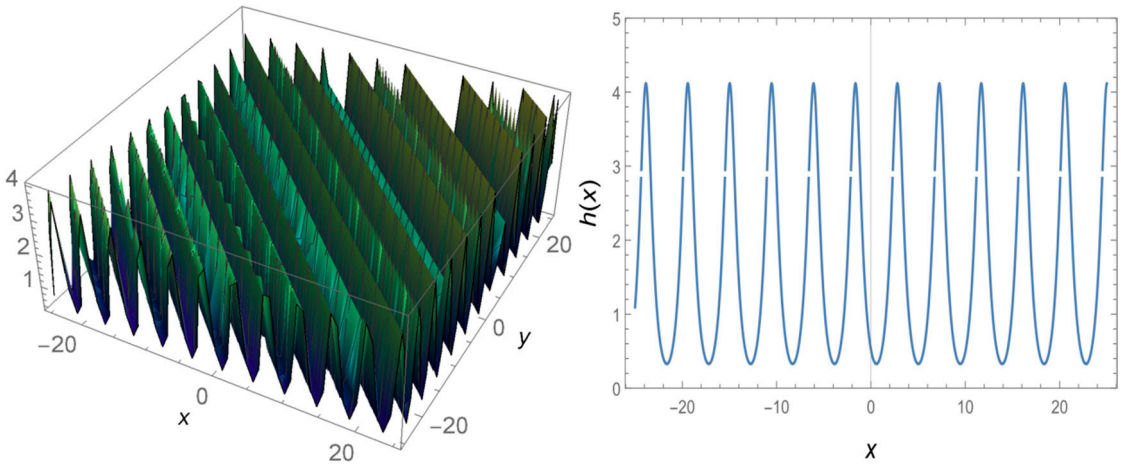

Figure 4. Three-dimensional and 2D graphs for values $t = 2, \theta_0 = 1, \lambda = 3, m = 2, \mu = 0.5, d = 1, \rho = 1, \tau = 1, b = -1, \varpi = 1$ of Equation (21) and $y = 2$ for 2D.

5. Conclusions

In this article, entirely new singular and periodic wave solutions to the governing model were successfully extracted via projected method. Strain conditions are also reported in this paper for validating the results. It may be also observed that these results satisfied the governing models. Figures of these solutions were plotted in 3D and 2D with the help of computational programs.

Comparing these results with the results obtained exp function solutions in [19], it may be seen that these results are entirely different solutions to the governing model. In this sense, these solutions may be also used to explain the nonlinear wave properties in defined intervals. Although it is quite difficult to obtain the solutions of NPDEs, the constructions of these solutions are facilitated with the help of some developed and modified methods. It can be also used that this method can be recommended in investigations of many other mathematical models with high nonlinearity. Considering these figures drawn in this paper and using symbolic calculation, the $(m + 1/G')$-expansion method was to be an effective, powerful and reliable mathematical tool for governing models.

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