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# ESTIMATION OF THE RANDOMIZED COMPLETE BLOCK DESIGN PARAMETERS WITH FUZZY GOAL PROGRAMMING

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**Abstract**

Since goal programming was introduced by Charnes, Cooper and Ferguson (1955), goal programming has been widely studied and applied in various areas. Parameter estimation is quite important in many areas. Recently, many researches have been studied in fuzzy estimation. In this study, fuzzy goal programming was proposed by Hannan (1981) adapted to estimation of randomized complete block design parameters. Suggested fuzzy goal programming is used for estimation of randomized complete block design parameters. Two numerical examples are used to illustrate the applicability of the suggested method. Then, the results obtained from classical method and suggested methods have been compared.

**Keywords:** Goal programming, Fuzzy goal programming, Randomized complete block design.

## 1. INTRODUCTION

Goal programming (GP) is a common tool used in decision making, but providing crisp goals can be a problem for a decision maker(DM). Since Zadeh proposed the concept of fuzzy sets, Bellman and Zadeh have developed a basic framework for decision making in a fuzzy environment. Thereafter, research extended the fuzzy set theory to the field of goal programming [15]

Fuzzy goal programming (FGP) methods is a common technique for many fields. There are many studies on the use of the FGP. Whang and Fu (1996) suggested the generalization of FGP with preemptive structure. Kim and Whang (1998) proposed the tolerance approach to the FGP problems with unbalanced triangular membership function. Lin (2004) suggested the weighted min-max model for FGP. Yaghoobi ve Tamiz (2005) criticized Kim and Whang (1998). Güneş and Umarosman (2005) defined the FGP for computational of the fuzzy arithmetic mean. Ciptomulyono (2008) used FGP for deriving priority weights in the analytical hierarchy process method.

In this study, we have proposed a new fuzzy goal programming approach for estimations of randomized complete block design (RCBD) parameters. In the second part of the study, GP and FGP methods are discussed first, followed by discussion of the RCBD. In the fourth part, we propose a new FGP in estimation of the RCBD parameters. In the last part, estimation of the RCBD parameters is obtained via the least square (LS) method and the suggested fuzzy goal programming (SFGP) method, and numerical results of the estimation of RCBD parameters are compared.

## 2. GOAL PROGRAMMING

In linear programming problems there is a single objective function to be maximized or minimized (subject to constraints). In some problems there may be more than one competing objective (or goal) and we need to trade-off objectives against each other. One way of handling problems with multiple objectives is to choose one of the goals as the supreme goal and to treat the others as constraints to ensure that some minimal 'satisfying' level of the other goals is achieved [16].

GP is a branch of multi-objective optimization, which in turn is a branch of multi-criteria decision analysis, also known as multiple-criteria decision making. It can be thought of as an extension or generalization of linear programming to handle multiple, normally conflicting objective measures. Each of these measures is given a goal or target value to be achieved. Unwanted deviations from this set of target values are then minimized in an achievement function [17]. Goal programming was introduced by Charnes, Cooper and Ferguson (1955).

### 2.1. FUZZY GOAL PROGRAMMING

The use of fuzzy set theory in goal programming was first discussed by Narasimhan, Hanan and Ignizio. Rubin and Narasimhan [9] and Tiwari et. al.[13] have presented various aspect of decision problem using FGP. The applications FGP in real world decision are found in numerous publications [2].

The FGP model is defined as follows

$$OPT \quad \begin{array}{ll} (AX)_i \lesssim b_i, & i = 1,\dots,i_0, \\ (AX)_i \gtrsim b_i, & i = i_0+1,\dots,j_0, \\ (AX)_i \cong b_i, & i = j_0+1,\dots,K, \\ X \geq 0, & \end{array}$$

where $(AX)_i = \sum_{j=1}^{n} a_{ij}x_j$, $i = 1,\dots,K$, "~" is a fuzzifier representing the imprecise fashion in which the goals are stated, $b_i$ is the aspiration level for the $i$th goal and OPT means finding an optimal decision $X$ such that all fuzzy goals are satisfied [14].

There is a quite difference between GP and FGP such as follows; GP requires the DM to set definite aspiration values for each objective that he/she wishes to achieve, whereas the latter is specified in an imprecise manner. A fuzzy goal is considered here as a goal with an imprecise aspiration level. Consideration of different relative importance and priorities of the goals in the fuzzy goal are proper than others [3].

In Narasimhan approach, there are $2^{m_l}$ sub-problem for $m_l$ fuzzy constraints. Hannan (1981) proposed a new model which is equivalent to Narasimhan (1980) model [8].

The Hannan model for solving an FGP problem is as follows,

$$\begin{array}{lll} \max & \lambda & \\ s.t. & \dfrac{(AX)_i}{\Delta_i} + d_i^- - d_i^+ = \dfrac{b_i}{\Delta_i}, & i = 1,\dots,K \\ & \lambda + d_i^- + d_i^+ \le 1, & i = 1,\dots,K \\ & \lambda, d_i^-, d_i^+ \ge 0, \quad X \ge 0 & i = 1,\dots,K \end{array}$$

where $\lambda$ is degree of membership value, $\Delta_i$ are subjectively chosen constant of admissible violations, $d_k^-$ and $d_k^-$ are negative deviation and positive deviation respectively [8].

## 3. THE RANDOMIZED COMPLETE BLOCK DESIGN

In cases where experimental units are not fully homogenous, the design must be developed by dividing these units up into more homogenous sub-units. This will eliminate the heterogeneity of experimental units to a certain extent. These relatively more homogenous sub-unit are called “blocks”. Differences between blocks are permitted to be large, but are not of major concern in the analysis, since the comparisons of treatments and the computation of experimental error is done within blocks. Blocking will be effective only if the error variance among units within blocks is smaller than the error variance over all units. Since data in the design of a random blocks experiment are designed with respect to two criteria as “block” and “treatment”, the process is also called “double classification” [12].
The statistical model for the RCBD is

$$Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \qquad \begin{array}{l} i = 1,2,\dots,a \\ j = 1,2,\dots,b \end{array} \tag{1}$$

where $Y_{ij}$ is any observation, $\mu$ is an overall mean, $\tau_i$ is the effect of the $i$th treatment, $\beta_j$ is the effect in the $j$th block, and $\varepsilon_{ij}$ is the usual $N\sim(0,\sigma^2)$ random error term. Treatments and blocks are considered initially as fixed factors. Furthermore, the treatment and block effect are defined as deviations from the overall mean so that

$$\sum_{i=1}^{a} \tau_i = 0 \text{ and } \sum_{j=1}^{b} \beta_j = 0 \text{ [7,10].}$$

Let $\hat{\mu}, \hat{\tau}_i$ and $\hat{\beta}_j$ denotes estimates of $\mu, \tau$ and $\beta$ made from the data. The predicted value of $Y_{ij}$ from the fitted model is then

$$\hat{Y}_{ij} = \hat{\mu} + \hat{\tau}_i + \hat{\beta}_j. \tag{2}$$

The least squares (LS) estimates are

$$\begin{aligned} \hat{\mu} &= \overline{Y}_{..} \\ \hat{\tau}_i &= \overline{Y}_{i.} - \overline{Y}_{..} \\ \hat{\beta}_j &= \overline{Y}_{.j} - \overline{Y}_{..} \end{aligned} \tag{3}$$

If we estimate any individual observation $Y_{ij}$ from the fitted model, the estimates is

$$\begin{aligned}\hat{Y}_{ij} &= \hat{\mu} + \hat{\tau}_i + \hat{\beta}_j \\ &= \overline{Y}_{..} + \left(\overline{Y}_{i.} - \overline{Y}_{..}\right) + \left(\overline{Y}_{.j} - \overline{Y}_{..}\right) \\ &= \overline{Y}_{i.} + \overline{Y}_{.j} - \overline{Y}_{..}\end{aligned} \tag{4}$$

[11].

## 4. ESTIMATION OF THE RANDOMIZED COMPLETE BLOCK DESIGN PARAMETERS WITH FUZZY GOAL PROGRAMMING

In this section, a new model is suggested for estimation of RCBD parameters. Two numerical examples are used to illustrate the applicability of the proposed method.

In this study, the FGP proposed by Hannan (1981) is adapted to estimation of RCBD parameters. So, the FGP proposed by Hannan (1981) are added to constraint $\sum_{i=1}^{a} \tau_i = 0, \sum_{j=1}^{b} \beta_i = 0$ and parameters are unrestricted in sign. And constraint $\lambda + d_i^- + d_i^+ \leq 1, \quad i = 1,....,K$ is modified to $\lambda + \dfrac{d_k^- + d_k^+}{\Delta_i} \leq 1, \quad k = 1,2,....,K$.

The SFHP model for estimation of the RCBD parameters is given as follows

$$\begin{aligned}&\max \quad \lambda \\ &s.t. \quad \frac{(X\theta)_k}{\Delta_i} + d_k^- - d_k^+ = \frac{Y_i}{\Delta_i}, \qquad k = 1,2,....,K \\ &\qquad \lambda + \frac{d_k^- + d_k^+}{\Delta_i} \leq 1, \qquad k = 1,2,....,K \\ &\qquad \sum_{i=1}^{a} \tau_i = 0, \\ &\qquad \sum_{j=1}^{b} \beta_j = 0, \\ &\qquad \lambda, d_k^-, d_k^+ \geq 0, \qquad k = 1,2,....,K \\ &\qquad \theta = \left(\mu, \tau_1,..., \tau_a; \beta_1,..., \beta_b\right)\end{aligned} \tag{5}$$

where $\theta$ is RCBD parameters.

### 4.1. Numerical Examples

In this section, two numerical examples are used to illustrate the applicability of the suggested method. First numerical example is given Sahai and Ageel (2000) [10] and second numerical example is given Montgomery (1997) [7].

**Example 1.** The data used for estimation of the RCBD parameters are given in Table 1. For the Example 1, $a$=4($i=1,2,3,4$) and $b$=3($j=1,2,3$) in the equation (1) are chosen. For this data set, model estimations are obtained via the LS defined in Section 3 and SFGP method in Section 4. The model estimation derived from the SFGP, which is related to the data set and the sum of squares of residuals ($\sum e^2$) these model, are displayed in Table 2. In addition, the model estimation that is obtained with LS and the sum of squares of residuals ($\sum e^2$) related with this model estimation is also displayed in the same table. The estimation of RCBD parameters is taken violations (10,20,….,80) for SFGP. The violations are admissible risk for decision marker. For the LS method and the SFGP method, WinQSB package program is used model estimation, is shown in Table 2.

**Table 1.** Data set for Example 1

| Material | Position 1 | 2 | 3 |
|---|---|---|---|
| 1 | 241 | 270 | 274 |
| 2 | 195 | 241 | 218 |
| 3 | 235 | 273 | 230 |
| 4 | 234 | 236 | 227 |

**Table 2.** The estimation of the RCBD parameters for Example 1

| **Method** | **Model Estimation** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\hat{\mu}$ | $\hat{\tau}_1$ | $\hat{\tau}_2$ | $\hat{\tau}_3$ | $\hat{\tau}_4$ | $\hat{\beta}_1$ | $\hat{\beta}_2$ | $\hat{\beta}_3$ | $\sum e^2$ |
| **LS** | 239.5 | 22.1667 | -21.5 | 6.5 | -7.1667 | -13.25 | 15.5 | -2.25 | 1173.833 |
| **SFGP** $\Delta=10$ | 240.75 | 24.25 | -21.75 | 4.9167 | -7.4167 | -11.667 | 15 | -3.333 | 12.29149 |
| $\Delta=20$ | 240.75 | 24.25 | -21.75 | 4.9167 | -7.4167 | -11.667 | 15 | -3.333 | 3.073351 |
| $\Delta=30$ | 240.0240 | 26.7027 | -25.8026 | 6.3006 | -7.2007 | -10.5010 | 15.0015 | -4.5005 | 1.2873 |
| $\Delta=40$ | 240.75 | 24.25 | -21.75 | 4.9167 | -7.4167 | -11.667 | 15 | -3.333 | 0.768116 |
| $\Delta=50$ | 240.75 | 24.25 | -21.75 | 4.9167 | -7.4167 | -11.667 | 15 | -3.333 | 0.491807 |
| $\Delta=60$ | 239.3688 | 25.5784 | -25.7447 | 6.2487 | -6.0823 | -11.6643 | 14.9970 | -3.3327 | 0.356419 |
| $\Delta=80$ | 240.75 | 24.25 | -21.75 | 4.9167 | -7.4167 | -11.667 | 15 | -3.333 | 0.19214 |

In table 2, for LS method and SFGP, estimations of parameters are given, respectively. When the table is examined, it is seen that estimation value obtained via the suggested method are the same in sign and nearly the same in magnitude as those obtained with LS. In additional, $\sum e^2$ obtained from the SFGP are lower than LS method.

**Example 2.** The data used for estimation of the RCBD parameters are given in Table 3. For the Example 2, $a$=4($i=1,2,3,4$) and $b$=3($j=1,2,3$) in the equation (1) are chosen. For this data set, model estimations are obtained via the LS defined in Section 3 and SFGP method in Section 4. The model estimation derived from the SFGP, which is related to the data set and the sum of squares of residuals these model, are displayed in Table 4. In addition, the model estimation that is obtained with LS and the sum of squares of residuals related with this model estimation is also displayed in the same table. The estimation of RCBD parameters is given violations (1,2,3,4) for SFGP. For SFGP method, WinQSB package program is used model estimation is shown in Table 4.

**Table 3:** Data set for Example 2

| Type of Tip | Test Coupon 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **1** | 9.3 | 9.4 | 9.6 | 10 |
| **2** | 9.4 | 9.3 | 9.8 | 9.9 |
| **3** | 9.2 | 9.4 | 9.5 | 9.7 |
| **4** | 9.7 | 9.6 | 10 | 10.2 |

**Table 4:** The estimation of the RCBD parameters for Example 2

| Method | Model Estimation | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{\mu}$ | $\hat{\tau}_1$ | $\hat{\tau}_2$ | $\hat{\tau}_3$ | $\hat{\tau}_4$ | $\hat{\beta}_1$ | $\hat{\beta}_2$ | $\hat{\beta}_3$ | $\hat{\beta}_4$ | $\sum e^2$ |
| **LS** | 9.625 | -0.05 | -0.025 | -0.175 | 0.25 | -0.225 | -0.2 | 0.1 | 0.325 | 0.08 |
| **SFGP** $\Delta=1$ | 9.65 | -0.05 | -0.05 | -0.15 | 0.25 | -0.2 | -0.2 | 0.1 | 0.3 | 0.1 |
| $\Delta=2$ | 9.65 | -0.05 | -0.05 | -0.15 | 0.25 | -0.2 | -0.2 | 0.1 | 0.3 | 0.025 |
| $\Delta=3$ | 9.626 | -0.025 | -0.025 | -0.125 | 0.175 | -0.2 | -0.2 | 0.1 | 0.3 | 0.013287 |
| $\Delta=4$ | 9.625 | -0.025 | -0.025 | -0.125 | 0.175 | -0.2 | -0.2 | 0.1 | 0.3 | 0.0075 |

In table 4, for LS method and SFGP, estimations of parameters are given, respectively. When the table is examined, it is seen that estimation value obtained via the suggested method are the same in sign and nearly the same in magnitude as those obtained with LS. In additional, $\sum e^2$ obtained from the SFGP are lower than LS method.

## 4. CONCLUSION

Parameter estimation is quite important in many areas. Recently, many researches have been studied in fuzzy estimation. In this study, a new model which based on the FGP is suggested for estimation of RCBD parameters. To illustrate how the proposed method is applied, two examples are discussed and compared in LS method. For Example 1, when Table 2 is examined, it is seen that estimation value obtained via the suggested method are the same in sign and nearly the same in magnitude as those obtained with LS. In additional, $\sum e^2$ obtained from the SFGP are lower than LS method. The interpretations are presented in Example 1 similar to Example 2. The SFGP is gives good solutions in the estimation of RCBD parameters.

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