OPEN ACCESS
Fractals, Vol. 30, No. 1 (2022) 2240018 (14 pages)

DOI: 10.1142/S0218348X22400187

ON THE COMPLEX MIXED DARK-BRIGHT WAVE DISTRIBUTIONS TO SOME CONFORMABLE NONLINEAR INTEGRABLE MODELS

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Received January 31, 2021
Accepted May 30, 2021
Published September 4, 2021

Abstract

In this research paper, we implement the sine-Gordon expansion method to two governing models which are the (2+1)-dimensional Nizhnik–Novikov–Veselov equation and the Caudrey–Dodd–Gibbon–Sawada–Kotera equation. We use conformable derivative to transform these nonlinear partial differential models to ordinary differential equations. We find some wave solutions having trigonometric function, hyperbolic function. Under the strain conditions of these solutions obtained in this paper, various simulations are plotted.

Keywords: Conformable Derivative; (2+1)-Dimensional Nizhnik–Novikov–Veselov Equation; Caudrey–Dodd–Gibbon–Sawada–Kotera Equation; Sine-Gordon Expansion Method; Wave Solutions.

1. INTRODUCTION

In the modern century, many real-world problems arising in physics, applied science, engineering and so on have been explained via nonlinear mathematical models (NMMs). Moreover, there is no single property of such NMMs. For a better understanding of such models, many new methods have been improved and studied to observe the exhaustive properties of them. In this sense, many powerful methods such as the cosh-sinh method,[1] the generalized Kudryashov method,[2] the Hirota's bilinear method,[4,5] the modified simple equation method,[6] the exp-function method,[7] the modified exp-$(\Phi(\xi))$ function method,[8] an auxiliary ordinary differential equation method,[9] the first integral method,[10] the Hirota method,[11] the extended sinh-Gordon equation expansion method,[12–14] the auto-Bäcklund transformation method,[15] the sin-cosine method[16] and many others[17–31] have been presented to the literature.

It is well known that many properties of the conformable derivatives are of much more importance compared with others. Thus, this field of science has attracted attention of experts from all over the world. Furthermore, it allows a better understanding of the physical behaviors of NMMs. In this paper, we consider two different conformable models which are the (2+1)-dimensional Nizhnik–Novikov–Veselov equation and the Caudrey–Dodd–Gibbon–Sawada–Kotera (CDGSK) equation with conformable derivative. First, we consider the (2+1)-dimensional conformable Nizhnik–Novikov–Veselov equation defined as follows:[1,7,9,32–35]

$$\begin{aligned} D_t^{\theta}\Phi &= a\Phi_{xxx} + b\Phi_{yyy} - 3a\Psi_x\Phi - 3a\Psi\Phi_x \\ &\quad - 3bw_y\Phi - 3bw\Phi_y, \\ \Phi_x &= \Psi_y, \\ \Phi_y &= w_x, \end{aligned} \tag{1}$$

where a, b are nonzero constants and also Φ, Ψ, w are the functions of x, y, t.

Second, we consider the conformable CDGSK model given by

$$\begin{aligned} D_t^{\alpha}\Phi + \Phi_{xxxxx} + 30\Phi\Phi_{xxx} + 30\Phi_x\Phi_{xx} \\ + 180\Phi^2\Phi_x = 0. \end{aligned} \tag{2}$$

Here, $\Phi = \Phi(x,t)$, α $(0 < \alpha \leq 1)$ is real constant.[3,36–38] With $\alpha = 1$, Eq. (2) has been first presented to the literature by Sawada and Kotera in 1974.[39] Ray has investigated Eq. (2) by considering extended Kudryashov's method,[38] and many others with the help of various methods.[16,40] Equation (2) is a special case of the general fifth-order Korteweg–de Vries arising in shallow water under gravity potential. Moreover, it also arises in quantum mechanics and nonlinear optics.[41]

The manuscript is organized into the following sections. The important facts of the sine-Gordon expansion method (SGEM) are introduced in Sec. 2. The projected method is applied to the governing models to find several analytical solutions in Secs. 3 and 4. This work ends with some conclusions in Sec. 5.

2. GENERAL FACTS OF SGEM

Here, SGEM is given in detailed manner. The nonlinear sine-Gordon differential equation is given by[11,33,36,42]

$$\Omega_{xx} - \Omega_{tt} = m^2 \sin(\Omega), \tag{3}$$

where $\Omega = \Omega(x,t)$ and m is nonzero constant. Considering $\Omega = \Omega(x,t) = V(\zeta)$, $\zeta = \mu(x-ct)$ transform into Eq. (3), it is converted as follows:

$$V'' = \frac{m^2}{\mu^2(1-c^2)} \sin(V), \tag{4}$$

where $V = V(\zeta)$ and $\zeta = \mu(x-ct)$ and $\mu \neq 0, c \neq 0$.

After integration of Eq. (4) and doing some mathematical operations, we can find

$$\left[\left(\frac{V}{2}\right)'\right]^2 = \frac{m^2}{\mu^2(1-c^2)} \sin^2\left(\frac{V}{2}\right) + T, \tag{5}$$

where T is integration real constant. Taking as $T = 0, \varpi(\zeta) = \frac{V}{2}$ and $\sigma^2 = \frac{m^2}{\mu^2(1-c^2)}$, then Eq. (5) is converted as follows:

$$\varpi' = \sigma \sin(\varpi). \tag{6}$$

Putting $\sigma = 1$, it yields

$$\varpi' = \sin(\varpi). \tag{7}$$

Using separation of variables method, it is found as follows:

$$\sin(\varpi) = \sin(\varpi(\zeta)) = \left.\frac{2pe^{\zeta}}{p^2e^{2\zeta}+1}\right|_{p=1} = \operatorname{sech}(\zeta), \tag{8}$$

$$\cos(\varpi) = \cos(\varpi(\zeta)) = \left.\frac{p^2e^{2\zeta}-1}{p^2e^{2\zeta}+1}\right|_{p=1} = \tanh(\zeta), \tag{9}$$

where p is the integral constant. In this stage, we take the general version of nonlinear partial differential model as

$$P(\Omega, \Omega_x, \Omega_t, \Omega_{xx}, \Omega_{tt}, \Omega^2, \ldots) = 0. \tag{10}$$

Considering the wave transformation as $\Omega = \Omega(x,t)= V(\zeta)$, $\zeta = \mu(x-ct)$, it gives

$$N(V, V', V'', V^2, \ldots) = 0.$$

In this equation, we may select the following trial equation function to be solution of Eq. (10) which is defined by

$$V(\zeta) = \sum_{i=1}^{n} \tanh^{i-1}(\zeta) \times [B_i \operatorname{sech}(\zeta) + A_1 \tanh(\zeta)] + A_0. \tag{11}$$

Taking Eqs. (8) and (9), Eq. (11) can be rewritten as follows:

$$V(\varpi) = \sum_{i=1}^{n} \cos^{i-1}(\varpi) \times [B_i \sin(\varpi) + A_1 \cos(\varpi)] + A_0. \tag{12}$$

Balancing in this equation, n may be found. As the coefficients of $\sin^i(\varpi)\cos^j(\varpi)$ are taken as zero, it yields an algebraic equations. By solving the algebraic equations system via computational package tools, the values of parameters are produced. Using these parameters in Eq. (11), we find new analytical solutions for governing models.

3. APPLICATION OF SGEM TO THE CNNVE

In this section, we will analyze the analytical solutions of the (2+1)-dimensional conformable Nizhnik–Novikov–Veselov equation via SGEM. First, Eq. (1) may be converted into NODEs by considering the traveling wave transform defined as follows:

$$\zeta = x + y - k\frac{t^{\theta}}{\theta}, \tag{13}$$

and

$$\begin{aligned} &\Phi(x,y,t) = \phi(\zeta), \quad \Psi(x,y,t) = \psi(\zeta), \\ &w(x,y,t) = \tau(\zeta), \end{aligned} \tag{14}$$

where k is a constant and θ $(0 < \theta \le 1)$. After some calculations into Eq. (1), the following nonlinear ordinary differential equations are obtained

$$(a+b)\phi''' - 3a(\psi\phi)' - 3b(\tau\phi)' - k\phi' = 0. \quad (15)$$

In this last equation, by doing some simplifications, it is rewritten as follows:

$$(a+b)\phi'' - 3a(\psi\phi) - 3b(\tau\phi) - k\phi = 0, \quad (16)$$

$$\phi' = \psi', \quad \phi' = \tau', \quad (17)$$

which produces the following

$$\psi = \phi + c, \quad \tau = \phi + d. \quad (18)$$

Considering Eqs. (16) and (18), it may be reached as follows:

$$(a+b)\phi'' - 3(a+b)\phi^2 + (3ac + 3bd + k)\phi = 0. \quad (19)$$

Via balance principle, it yields $n = 2$, which produces

$$\phi(\varpi) = B_1\sin(\varpi) + A_1\cos(\varpi) + B_2\cos(\varpi)\sin(\varpi) + A_2\cos^2(\varpi) + A_0 \quad (20)$$

and

$$\begin{aligned}\phi''(\varpi) &= B_1\cos^2(\varpi)\sin(\varpi) - B_1\sin^3(\varpi) \\ &\quad - 2A_1\sin^2(\varpi)\cos(\varpi) + B_2\cos^3\sin(\varpi) \\ &\quad - 5B_2\sin^3(\varpi)\cos(\varpi) - 4A_2\cos^2(\varpi) \\ &\quad \times \sin^2(\varpi) + 2A_2\sin^4(\varpi). \quad (21)\end{aligned}$$

Taking into account Eqs. (20) and (21) into Eq. (19), it produces many novel analytical solutions to the governing models as follows.

Case 1. Taking as $A_2 = 1, B_1 = 0, B_2 = -i, A_1 = 0, a = -b, d = c - \frac{k}{3b}$, gives

$$\begin{aligned}\Phi_1(x,y,t) &= A_0 - i\,\mathrm{sech}\left(x + y - k\frac{t^\theta}{\theta}\right) \\ &\quad \times \tanh\left(x + y - k\frac{t^\theta}{\theta}\right) \\ &\quad + \tanh^2\left(x + y - k\frac{t^\theta}{\theta}\right). \quad (22)\end{aligned}$$

With the suitable values, wave distribution of Eq. (22) can be seen in Figs. 1–3.

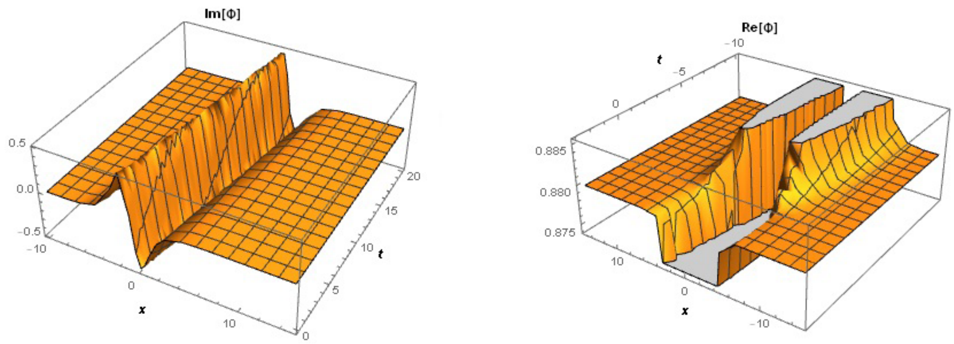

Fig. 1 Three-dimensional simulation of imaginary and real parts of Eq. (22).

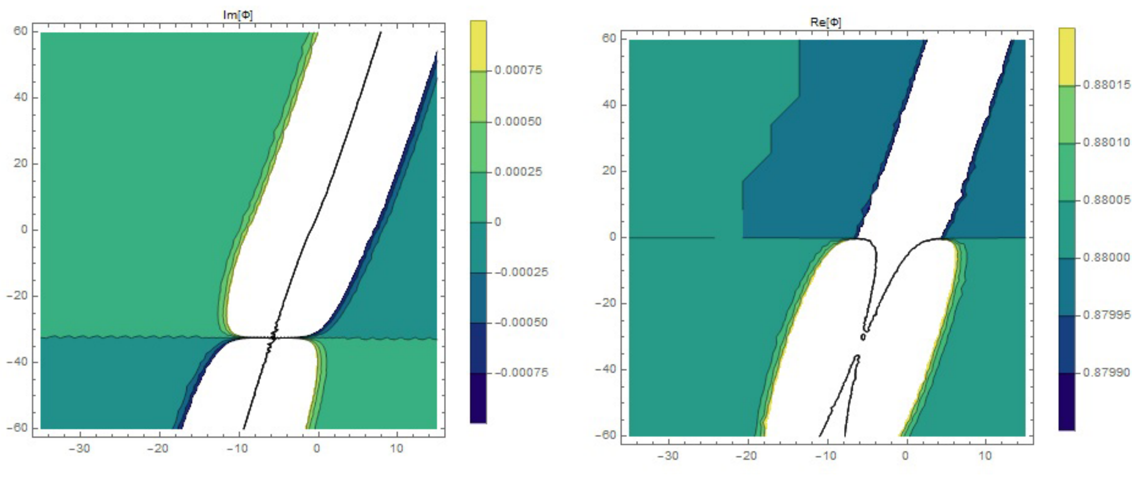

Fig. 2 Contour surfaces of Eq. (22).

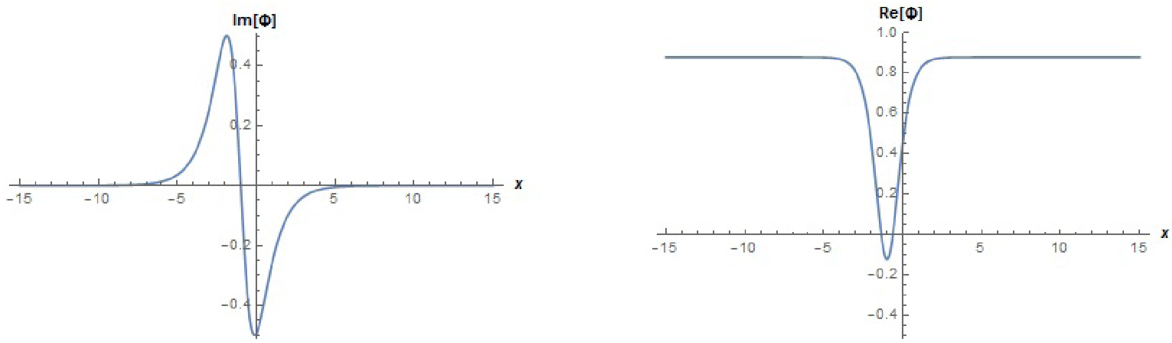

Fig. 3 Two-dimensional simulations of imaginary and real parts of Eq. (22).

Case 2. If we consider these coefficients as $A_0 = -1, A_1 = 0, A_2 = 1, B_1 = 0, B_2 = i, k = -a(1+3c) - b(1+3d)$, we extract another complex result as follows:

$$\Phi_2(x,y,t) = -1 + i\,\mathrm{sech}\left(x + y - \frac{(-a(1+3c) - b(1+3d))t^{\theta}}{\theta}\right) \times \tanh\left(x + y - \frac{(-a(1+3c) - b(1+3d))t^{\theta}}{\theta}\right) + \tanh^2\left(x + y - \frac{(-a(1+3c) - b(1+3d))t^{\theta}}{\theta}\right). \quad (23)$$

Various simulations of Eq. (23) can be observed in Figs. 4–6.

Case 3. When $A_0 = -2, A_1 = 0, A_2 = 2, B_1 = 0, B_2 = 0, k = -a(4+3c) - b(4+3d)$, it produces

$$\Phi_3(x,y,t) = -2 + 2\tanh^2\left(x + y - \frac{(-a(4+3c) - b(4+3d))t^{\theta}}{\theta}\right). \quad (24)$$

For Eq. (24), various graphs in Figs. 7 and 8 are presented.

Case 4. Taking these cases as for $A_1 = 0, A_2 = 1, B_1 = 0, B_2 = i, a = -\frac{k}{3(c-d)}, b = \frac{k}{(3c-3d)}$ reaches another solution for governing model

$$\phi_4(x,y,t) = A_0 + i\,\mathrm{sech}\left(x + y - k\frac{t^{\theta}}{\theta}\right) \times \tanh\left(x + y - k\frac{t^{\theta}}{\theta}\right) + \tanh^2\left(x + y - k\frac{t^{\theta}}{\theta}\right). \quad (25)$$

Considering the strain conditions, it may be observed that this solution is of the following surfaces as shown in Figs. 9–11.

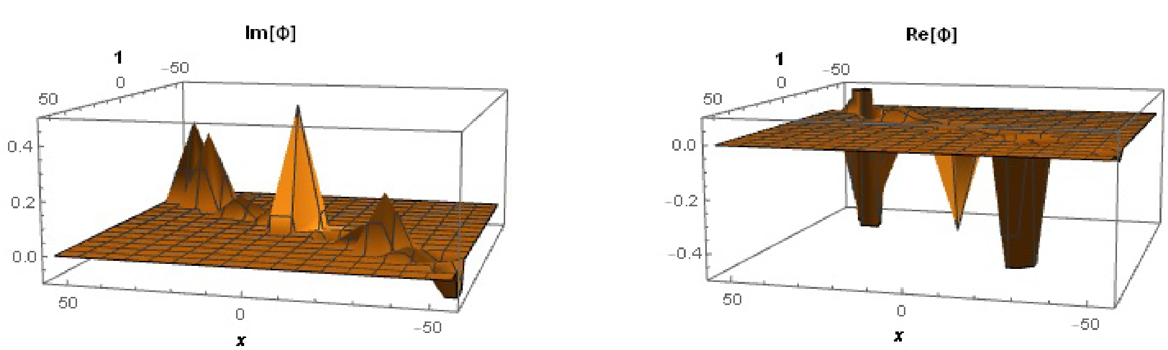

Fig. 4 Three-dimensional simulation of imaginary and real parts of Eq. (23).

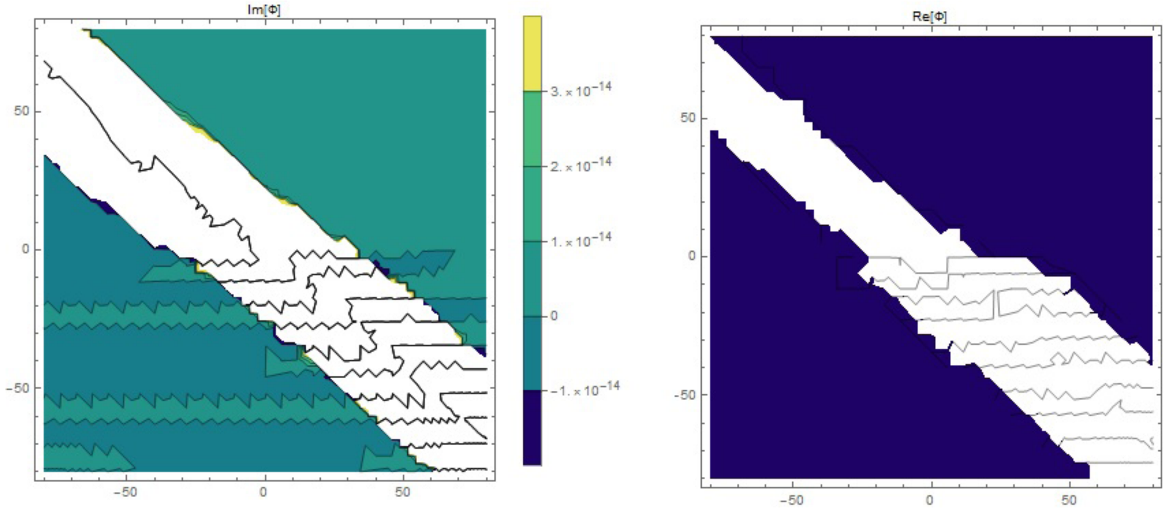

Fig. 5 Contour surfaces of Eq. (23).

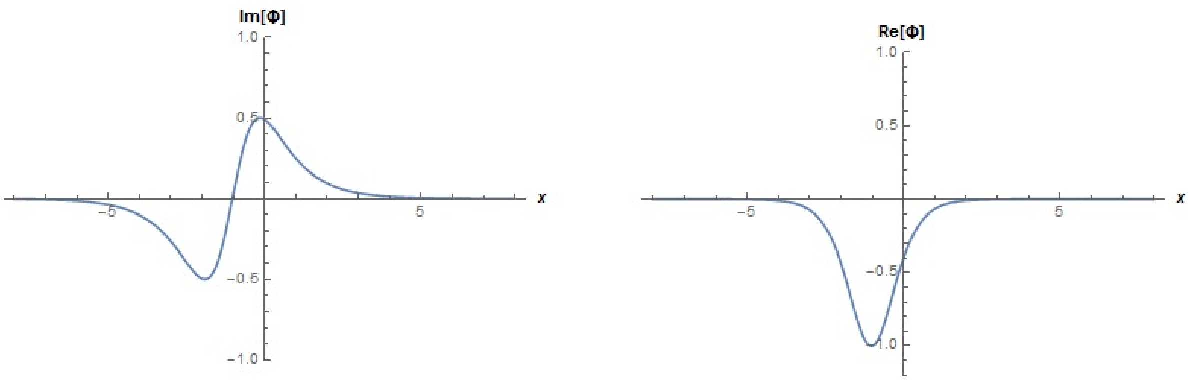

Fig. 6 Two-dimensional simulations of imaginary and real parts of Eq. (23).

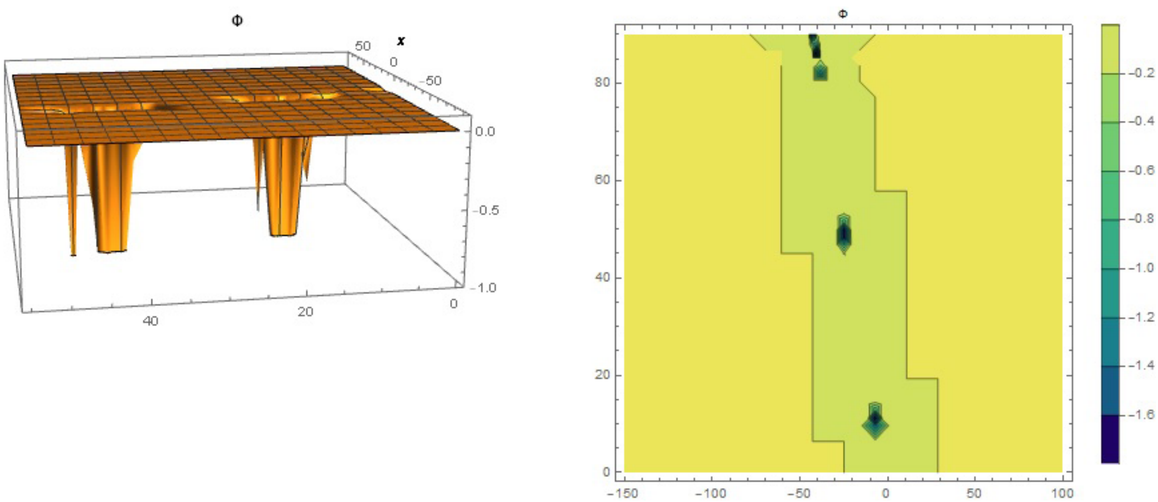

Fig. 7 Three-dimensional and contour simulations of Eq. (24).

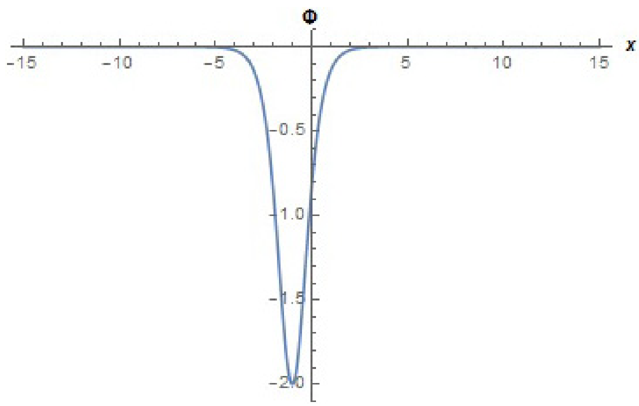

Fig. 8 Two-dimensional simulations of Eq. (24).

4. APPLICATION OF SGEM TO THE CDGSKE

In this subsection of paper, the application of the SGEM on the CDGSKE[35,38] is introduced. We, first start by transforming Eq. (2) to NODEs by considering the following traveling wave transformation

$$\phi(x,t) = U(\xi), \quad \xi = kx - c\frac{t^{\theta}}{\theta}. \tag{26}$$

Substituting Eq. (27) into Eq. (2), the following NODEs is obtained

$$-cU + k^5U^{(5)} + 30k^3(UU'')' + 60k(U^3)' = 0. \tag{27}$$

Integrating Eq. (28) once with respect to ζ and getting the integrate constant as zero results in

$$-cU + k^5U^{(4)} + 30k^3UU'' + 60kU^3 = 0. \tag{28}$$

Balancing $U^{(4)}$ with U^3 in Eq. (29) is obtained with $n = 2$. Using $n = 2$ into Eq. (12) produces

$$U(\varpi) = B_1\sin(\varpi) + A_1\cos(\varpi) + B_2\cos(\varpi)\sin(\varpi) + A_2\cos^2(\varpi) + A_0. \tag{29}$$

With the necessary integrations of Eq. (30) accordingly produces

$$\begin{aligned} U''(\varpi) &= B_1\cos^2(\varpi)\sin(\varpi) - B_1\sin^3(\varpi) \\ &\quad - 2A_1\sin^2(\varpi)\cos(\varpi) + B_2\cos^3\sin(\varpi) \\ &\quad - 5B_2\sin^3(w)\cos(\varpi) - 4A_2\cos^2(\varpi) \\ &\quad \times \sin^2(\varpi) + 2A_2\sin^4(\varpi), \end{aligned} \tag{30}$$

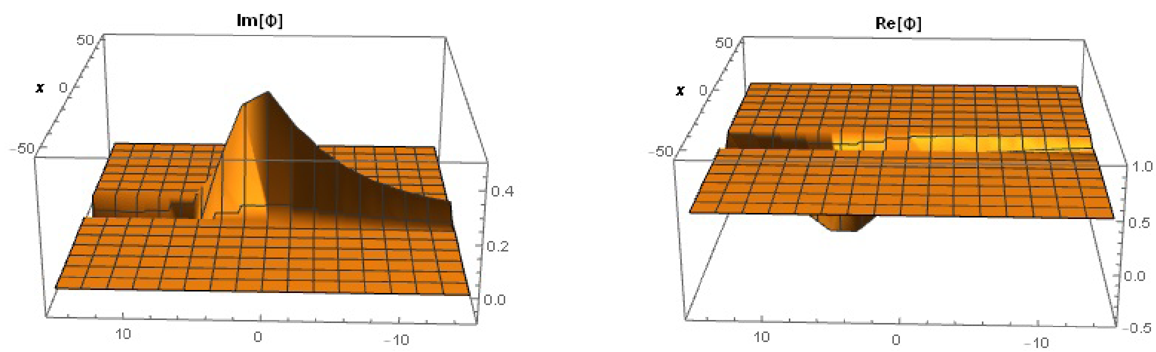

Fig. 9 Three-dimensional simulation of imaginary and real parts of Eq. (25).

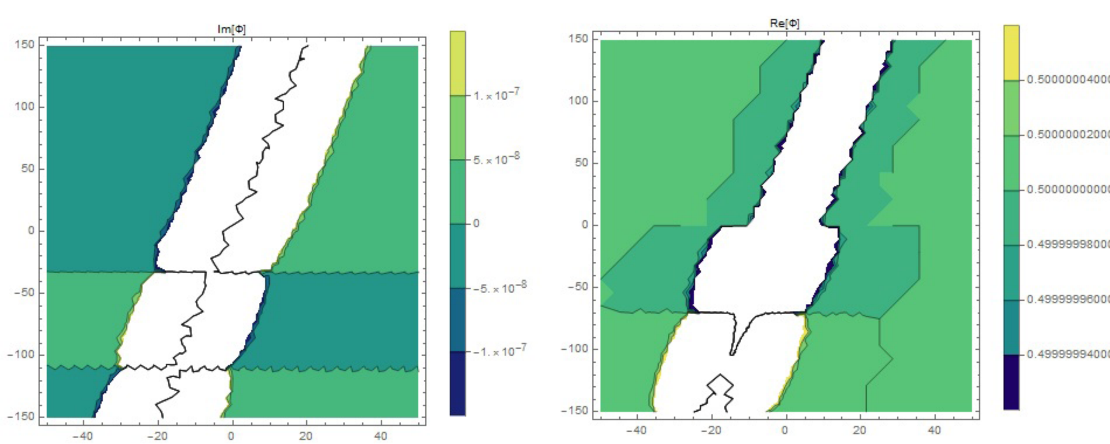

Fig. 10 Contour surfaces of Eq. (25).

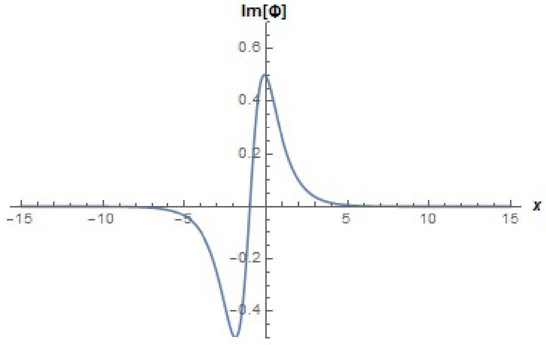

Fig. 11 Two-dimensional simulations of imaginary and real parts of Eq. (25).

$$U^{(4)}(w) = -8\cos^3(\varpi)\sin^2(\varpi)A_1 + 16\cos(\varpi) \times \sin^4(\varpi)A_1 - 16\cos^4(\varpi)\sin^2(\varpi)A_2 + 88\cos^2(\varpi)\sin^4(\varpi)A_2 - 16\sin^6(\varpi)A_2 + \cos^4(\varpi)\sin(\varpi)B_1 - 18\cos^2(\varpi) \times \sin^3(\varpi)B_1 + 5\sin^5(\varpi)B_1 + \cos^5(\varpi) \times \sin(\varpi)B_2 - 58\cos^3(\varpi)\sin^3(\varpi)B_2 + 61\cos(\varpi)\sin^5(\varpi)B_2. \tag{31}$$

Inserting Eqs. (30)–(32) into Eq. (29) gives an algebraic equation in trigonometric function including various forms of $\sin^i(\varpi)\cos^j(\varpi)$. Getting the coefficients of trigonometric terms in the same power to zero gives a system of equations. Solving this system with aid of symbolic software to obtain the values of the coefficients gives the traveling wave solutions to Eq. (2).

Case 1. When $A_0 = \frac{1}{120}(45+\sqrt{105})k^2, A_1 = 0, A_2 = -\frac{k^2}{2}, B_1 = 0, B_2 = \frac{ik^2}{2}, c = -\frac{1}{2}(-11+\sqrt{105})k^5$, then, we have the combined dark-bright solitary wave solutions to Eq. (2) as follows:

$$\Phi_1(x,t) = \frac{1}{120}(45+\sqrt{105})k^2 + \frac{1}{2}ik^2\text{sech}\left(kx + \frac{(-11+\sqrt{105})k^5t^\theta}{8\theta}\right) \times \tanh\left(kx + \frac{(-11+\sqrt{105})k^5t^\theta}{8\theta}\right) - \frac{1}{2}k^2\tanh^2\left(kx + \frac{(-11+\sqrt{105})k^5t^\theta}{8\theta}\right). \tag{32}$$

With the help of taking some values of parameters under the strain conditions, we plot its surfaces as Figs. 12–14.

Case 2. If taking as $A_0 = \frac{k^2}{2}, A_1 = 0, A_2 = -\frac{k^2}{2}, B_1 = 0, B_2 = -\frac{ik^2}{2}, c = k^5$, then, we have

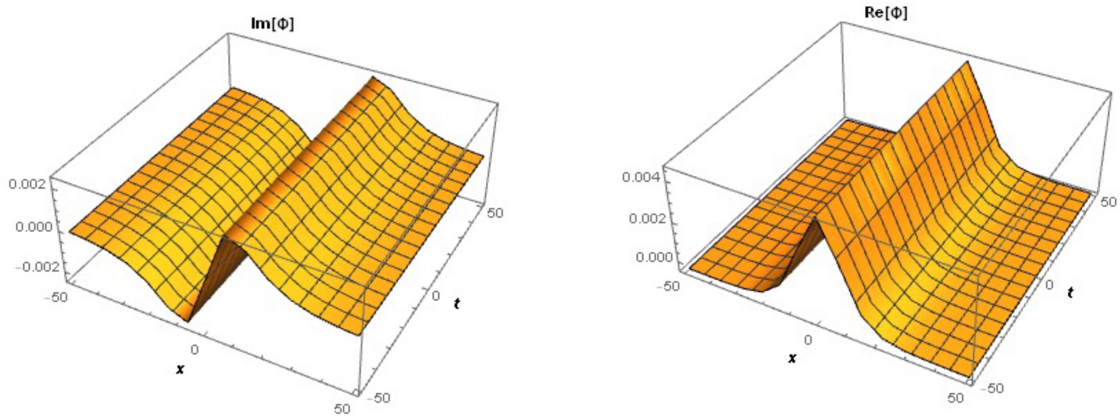

Fig. 12 Three-dimensional simulations of imaginary and real parts of Eq. (33).

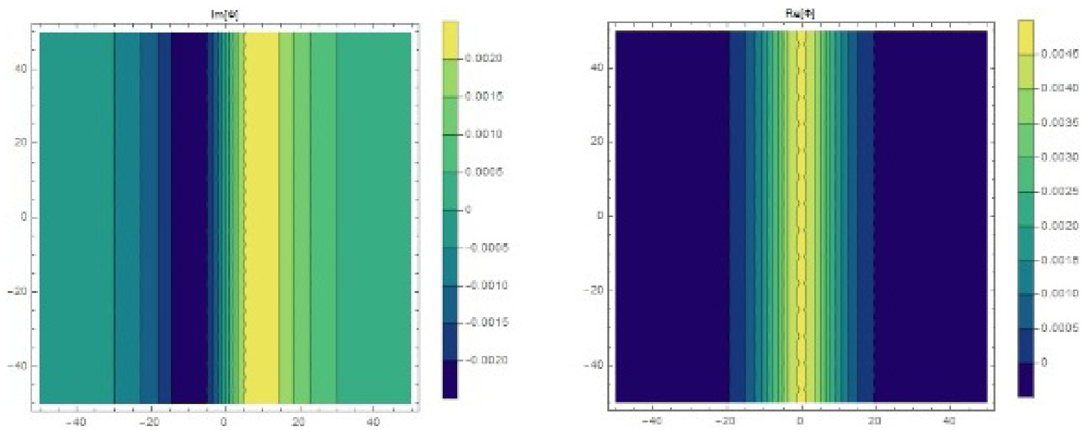

Fig. 13 Contour surfaces of Eq. (33).

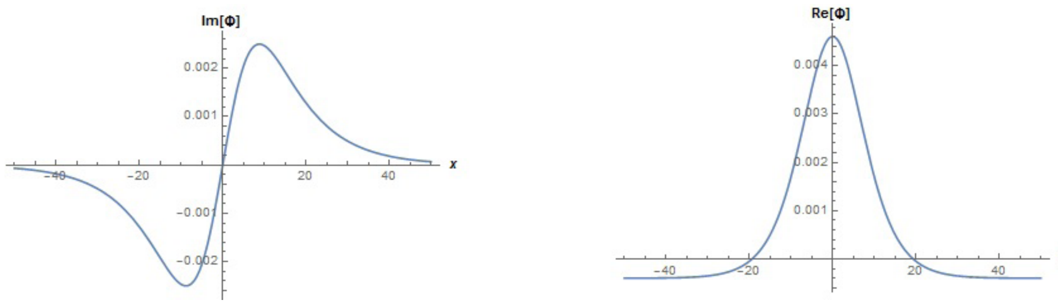

Fig. 14 Two-dimensional simulations of imaginary and real parts of Eq. (33).

hyperbolic function solutions to Eq. (2) as being

$$\Phi_2(x,t) = \frac{k^2}{2} - \frac{1}{2}ik^2\text{sech}\left(kx - \frac{k^5t^\theta}{\theta}\right) \times \tanh\left(kx - \frac{k^5t^\theta}{\theta}\right) - \frac{1}{2}k^2\tanh^2\left(kx - \frac{k^5t^\theta}{\theta}\right), \quad (33)$$

where k is real constant with nonzero. Choosing some suitable values of parameters under the strain conditions, we plot its surfaces as Figs. 15–17.

Case 3. Once $A_0 = -\frac{1}{2}(-1)^{\frac{3}{5}}c^{\frac{2}{5}}$, $A_1 = 0$, $A_2 = \frac{1}{2}(-1)^{\frac{3}{5}}c^{\frac{2}{5}}$, $B_1 = 0$, $B_2 = \frac{1}{2}(-1)^{\frac{1}{10}}c^{\frac{2}{5}}$, $k = (-1)^{\frac{4}{5}}c^{\frac{1}{5}}$, we find the bright soliton solution to Eq. (2)

$$\Phi_3(x,t) = \frac{(-1)^{\frac{1}{10}}c^{\frac{2}{5}}}{2i + 2\sinh((-1)^{\frac{4}{5}}c^{\frac{1}{5}}x - \frac{ct^\theta}{\theta})} \quad (34)$$

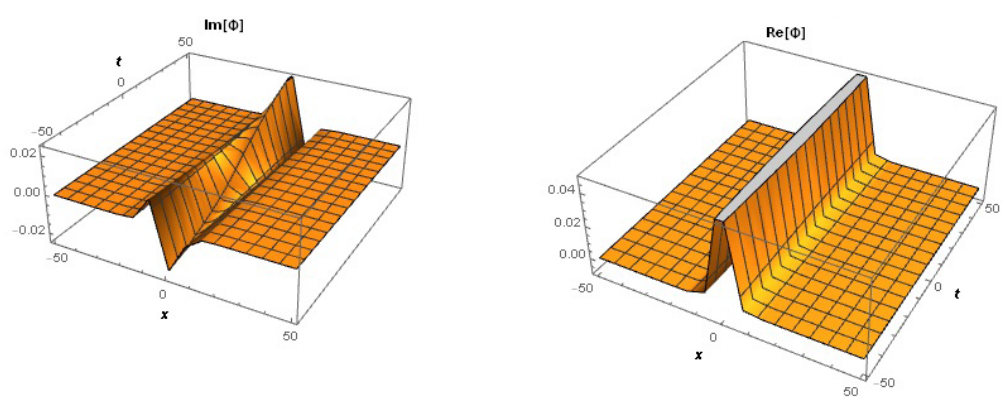

Fig. 15 Three-dimensional simulations of imaginary and real parts of Eq. (34).

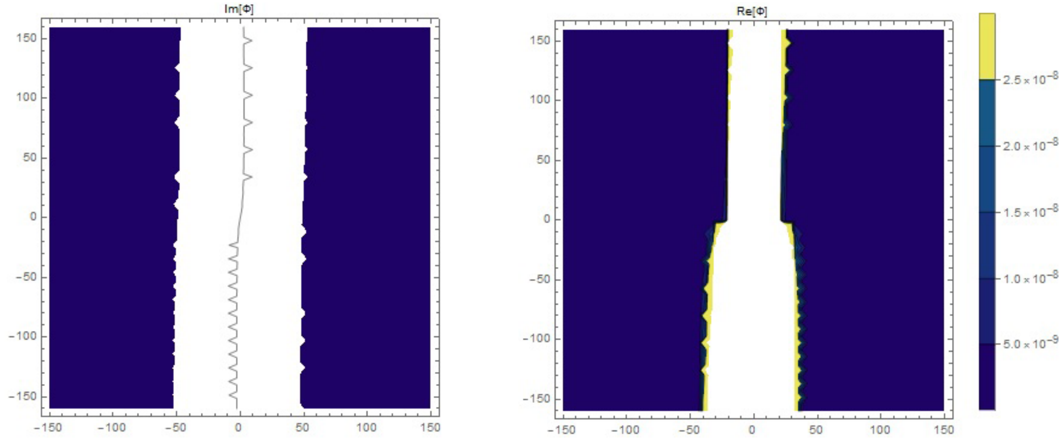

Fig. 16 Contour surfaces of Eq. (34).

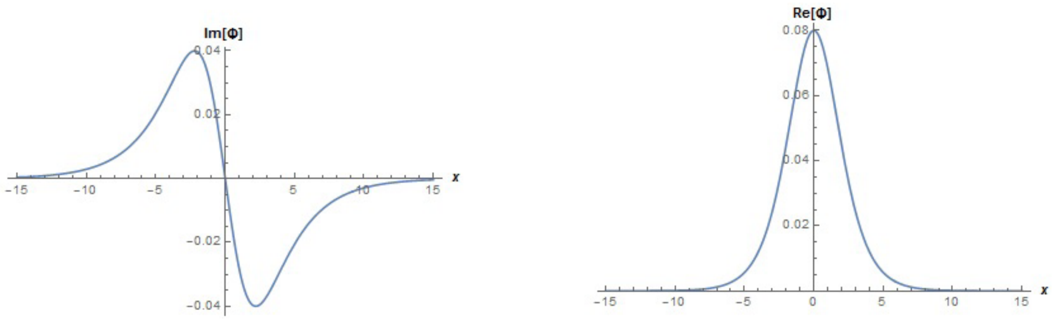

Fig. 17 Two-dimensional simulations of imaginary and real parts of Eq. (34).

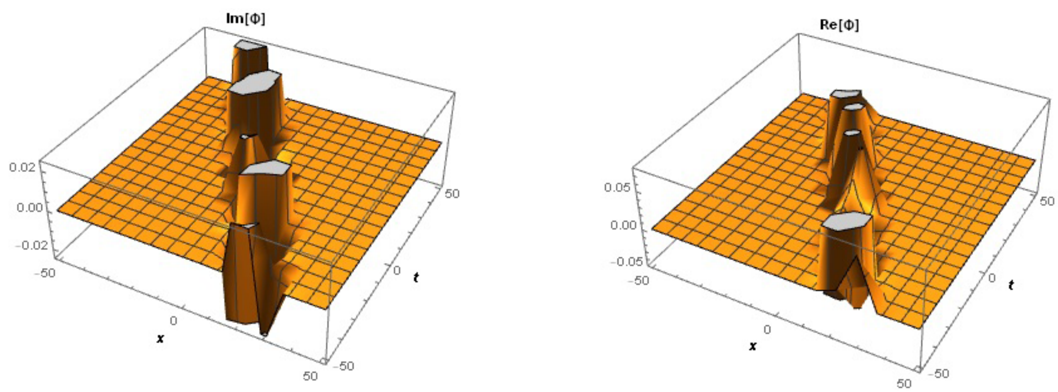

Fig. 18 Three-dimensional simulations of imaginary and real parts of Eq. (35).

in which c is real constant with nonzero. Handling suitable values of parameters with the strain conditions, we form its surfaces as Figs. 18–20.

Case 4. When it comes to these coefficients, $A_0 = \frac{c^{\frac{2}{5}}}{2}$, $A_1 = 0$, $A_2 = -\frac{c^{\frac{2}{5}}}{2}$, $B_1 = 0$, $B_2 = \frac{1}{2}ic^{\frac{2}{5}}$, $k = c^{\frac{1}{5}}$, it is introduced the mixed dark-bright soliton solution to Eq. (2) as follows:

$$\Phi_4(x,t) = \frac{c^{\frac{2}{5}}}{2} + \frac{1}{2}ic^{\frac{2}{5}}\operatorname{sech}\left(c^{\frac{1}{5}}x - \frac{ct^{\theta}}{\theta}\right) \times \tanh\left(c^{\frac{1}{5}}x - \frac{ct^{\theta}}{\theta}\right) - \frac{1}{2}c^{\frac{2}{5}}\tanh^2\left(c^{\frac{1}{5}}x - \frac{ct^{\theta}}{\theta}\right). \tag{35}$$

Various surfaces of Eq. (36) under the strain conditions can be observed in Figs. 21–23.

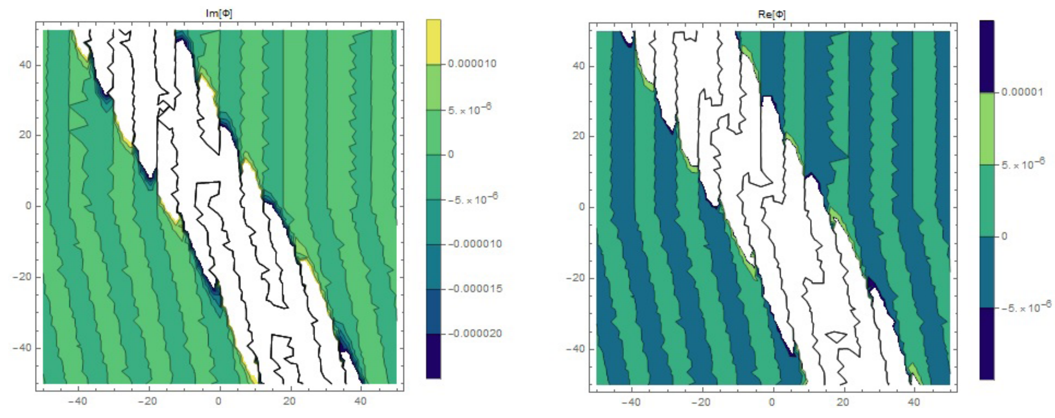

Fig. 19 Contour surfaces of Eq. (35).

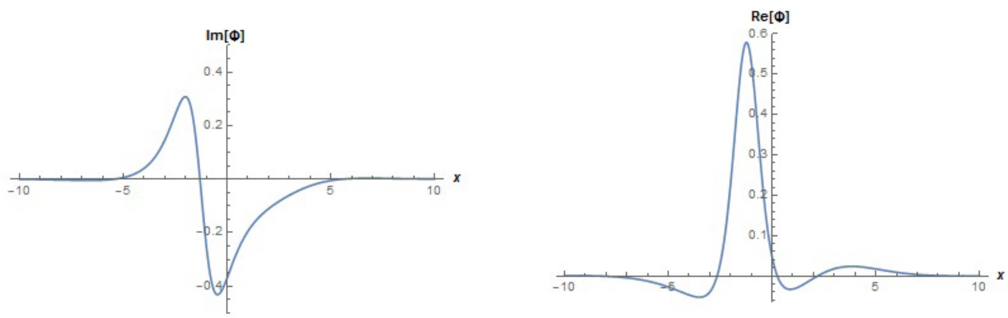

Fig. 20 Two-dimensional simulations of imaginary and real parts of Eq. (35).

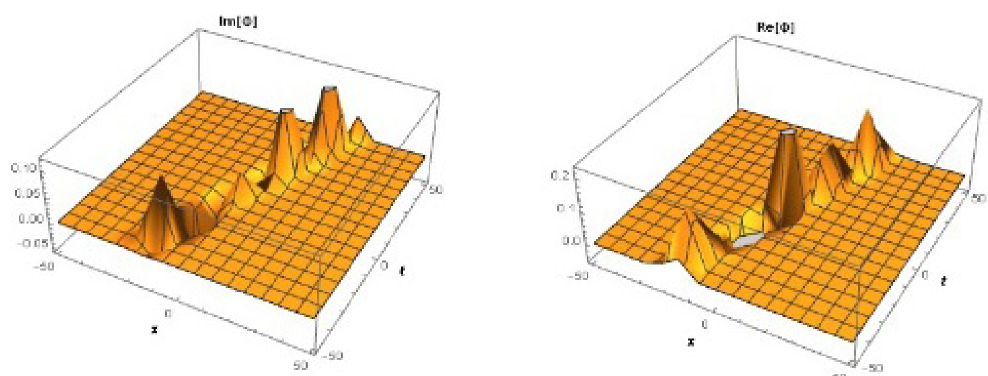

Fig. 21 Three-dimensional simulations of imaginary and real parts of Eq. (36).

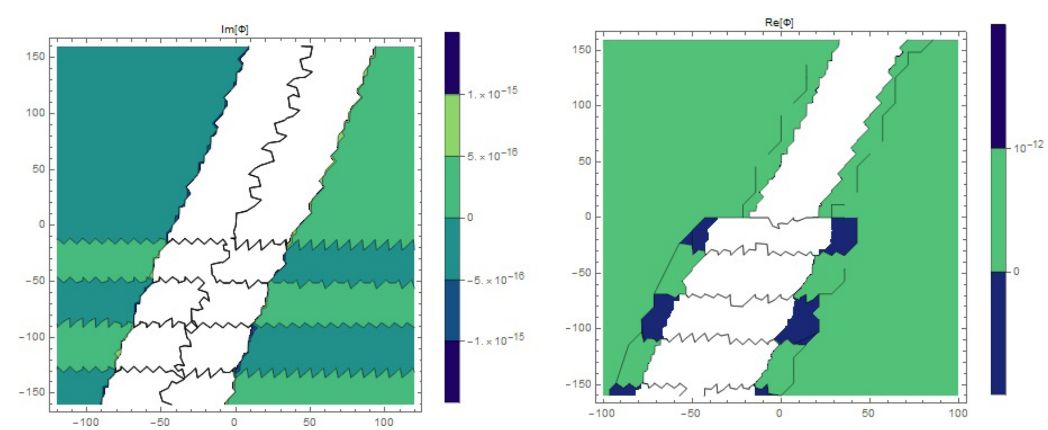

Fig. 22 Contour surfaces of Eq. (36).

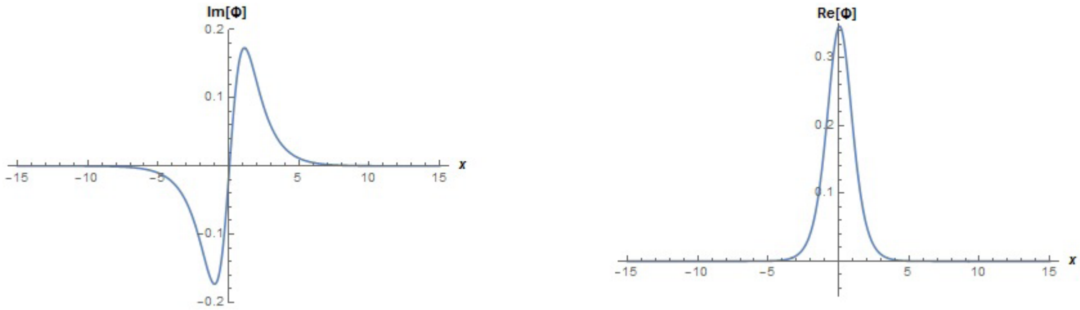

Fig. 23 Two-dimensional simulations of imaginary and real parts of Eq. (36).

5. CONCLUSION

In this paper, we have successfully applied SGEM to both conformable (2+1)-dimensional Nizhnik–Novikov–Veselov equation and conformable CDGSK equation. We have constructed many entirely new complex, dark, bright, mixed dark-bright soliton solutions to the governing models. To the better understanding of physical meanings of results, we have submitted various surfaces of wave solutions, as seen in Figs. 1–23, under the strain conditions and choosing suitable values of parameters with $\theta = 0.9$. We utilized the Mathematica software in all provided computations. Both the Nizhnik–Novikov–Veselov equation and the CDGSK equation are an integrable models derived from KdV equation. These models are generally described phenomenon such as in fluid mechanics, plasma physics, geometrical optics, etc. The submitted soliton solutions may play a key role in mathematical physics and engineering. The purposed method is very efficient and simple to construct various types of wave solutions of nonlinear partial differential models which are faced in every nonlinear phenomena.

ACKNOWLEDGMENT

This projected work was partially (not financial) supported by the Scientific Research Project Fund of Harran University with the project entitled "Investigation of New Analytical and Complex Solutions of Some Nonlinear Partial Differential Equations". All authors contributed equally to this paper.

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