Alexandria Engineering Journal (2022) 61, 1747–1756

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Alexandria Engineering Journal

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A new numerical investigation of fractional order susceptible-infected-recovered epidemic model of childhood disease

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Received 23 March 2021; revised 7 May 2021; accepted 9 July 2021
Available online 29 July 2021

KEYWORDS

Caputo fractional derivative;
Laplace transform;
Susceptible-infected-recovered epidemic model;
q-homotopy analysis method

Abstract The susceptible-infected-recovered (SIR) epidemic model of childhood disease is analyzed in the present framework with the help of *q*-homotopy analysis transform method (*q*-HATM). The considered model consists the system of three differential equations having fractional derivative, and the non-linear system exemplifies the evolution of childhood disease in a population and its influence on the community with susceptible, infected and recovered compartment. The projected method is a mixture of *q*-homotopy analysis method and Laplace transform. Two distinct explanatory cases are considered, and corresponding simulations have been demonstrated in terms of plots for different value of the order. The present investigation elucidates that the projected both derivative and technique play a vital role in the analysis and illustrate the behaviour of diverse mathematical models described with differential equations in human disease.

1. Introduction

Childhood diseases become the most deliberate infective diseases in recent years. In the eighteenth century, Swiss physicist and mathematician Bernoulli proposed and cultivated the concept of modelling for disease evolution [1] by considering mathematics as an essential tool, which provides the origin to the development of modern epidemiology. Later in the twentieth century, Ross [2] established the modelling of infectious disease and elucidates the behaviour of epidemic models with the law of mass action. These models are recently widely

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Peer review under responsibility of Faculty of Engineering, Alexandria University.

https://doi.org/10.1016/j.aej.2021.07.015

applied to analyse the epidemiological processes that contain the transmission of the contagious disease. The mathematical model can aid us in the transmission and dynamical behaviour of childhood diseases [3–8].

Rubella, poliomyelitis and measles are the more familiar childhood diseases [9,10]. These diseases usually influence the children, due to the population of the child is extremely large in prone to the disease as relate to the adults [9]. Particularly, measles is an extremely virulent disease and is instigated by respiratory infection of Morbilli-virus. Further, the population can be mainly distributed into two classes: mature and premature populations. The premature population consider the fixed duration to become mature, which is called a maturation delay. In the dynamics of disease, the diseases are doesn't spread rapidly but instead require some duration in the body, known as the latent period of the diseases. However, after WHO originated the Expanded Program on Immunization [11] the effort of vaccination is widely started to all children began in the year 1974. In this connection, mathematical models play an important role in sympathetic the nature of transmission of the diseases and help us analyse the behaviour of disease affecting children. Further, the mathematical models can help us capture the growth of the diseases, and these models can provide diverse methods to control its propagation.

The concept of differential calculus was begun in the 17th century in order to study the phenomena described with the rate of change. Later, it considered an essential tool to examine and predict all most natural phenomena. Within the frame of differential calculus, the epidemic models and their corresponding consequences are effectively examined. In order to study the evolution of the virus and its exponential growth through leaving beings are efficiently exemplified. However, recently many investigators pointed out that classical calculus is not an effective and accurate tool to investigate and model non-linear and complex phenomena.

Soon after the birth of classical calculus, the concept of fractional calculus (FC) was originated. But lately, it magnetized the attention of many researchers while analysing phenomena related to a long-range, random walk, non-Markovian processes and anomalous diffusion [12–16]. More preciously, from the last three decades due to the growth of new computational tools associate with computers and novel mathematical methods, many researchers hired FC to study and illustrate some interesting consequences of real-world models [17–26]. For instance, authors in [17] presented detained and interesting results of fractional calculus while investigating the models related to nanotechnology. For the Fokas-Lenells equation, the optical solitons and other solutions have been investigated by Esen et al. in [18] chaos analysis and asymptotic stability is derived in [19] with fractional order, authors in [20] analysed the mathematical model of cancer chemotherapy effect in Caputo fractional derivatives, the dynamics of a fractional epidemiological model with disease infection is studied with equilibrium analysis in [21]. Recently, FC is magnetizing the attention of the researchers and also study of diverse mathematical models in order to predict the corresponding consequences [27–29].

The biological models that modelled and described with the help of arbitrary order differential equations have demonstrated an appreciation in analysing SIR epidemic models and their corresponding behaviour. Many researchers developed and analysed these models using diverse techniques with the aid of classical and fractional order derivatives [30–32]. Moreover, researchers considered fractional-order derivatives to examine the distinct class of problems in comparison with integer-order, and they prove that arbitrary order derivatives are more effective while exemplifying the behaviour of the models [33–45].

In the present investigation, we considered an efficient and highly methodical scheme called q-HATM, which proposed by Singh et al. [46] with the assist of the homotopy analysis method (HAM) [47] and Laplace transform. The considered solution procedure is a mixture of two well-known algorithms to find the solution for non-linear differential and integral equations with conserving new polynomials, perturbatinga given system, discretising and extracting any basis. Forthe last three years, the considered scheme is effectively and widely employed by many researchers to find the solution for the differential equations exemplifying various models and phenomena associated with fractional calculus and presented the numerical simulation to confirm the methods accuracy [48–57].

The rest of the present investigation is presented as: the basic and fundamental notions are recalled in the next section. The discussion about the considered SIR model is presented in Section 3. The basic procedure of the considered method is presented in Section 4, and then it has been employed in Section 5 for the considered system. In Section 6, the results and discussion on achieved results and their corresponding consequences captured in figures are presented, and then finally, the concluding remarks of the present study are illustrated.

2. Preliminaries

Here, we present the basic notion of FC and Laplace transform.

Definition 1. The Riemann-Liouville integral of a function $f(t) \in C_{\delta}(\delta \geq -1)$ having fractional order $(\mu > 0)$ is defined as

$$\begin{aligned} J^{\mu} f(t) &= \tfrac{1}{\Gamma(\mu)} \int_0^t (t-\vartheta)^{\mu-1} f(\vartheta) d\vartheta, \\ J^{0} f(t) &= f(t). \end{aligned} \tag{1}$$

Definition 2. The derivative of $f \in C_{-1}^{n}$ in Caputo fractional-order is defined as

$$D_t^{\mu} f(t) = \begin{cases} \frac{d^n f(t)}{dt^n}, \mu = n \in N, \\ \frac{1}{\Gamma(n-\mu)} \int_0^t (t-\vartheta)^{n-\mu-1} f^{(n)}(\vartheta) d\vartheta, n-1 < \mu < n, n \in \mathbb{N}. \end{cases} \tag{2}$$

Definition 3. Let $D_t^{\mu} f(t)$ be a Caputo fractional derivative, then the Laplace transform (LT) is presented as

$$L\left[D_t^{\mu} f(t)\right] = s^{\mu} F(s) - \sum_{r=0}^{n-1} s^{\mu-r-1} f^{(r)}(0^+), (n-1 < \mu \leq n), \tag{3}$$

where $F(s)$ is LT of $f(t)$.

3. Mathematical model of the childhood disease in Caputo fractional derivatives

In this segment, we study the system of three fractional-order differential equations describing the epidemic model of childhood disease (see Fig. 1). Here, we consider $N(t)$ denotes the total population strength with time t. In the proposed model, susceptible group, an infected group, and quarantined or removed group are respectively symbolized by $S(t), I(t)$ and $R(t)$. Here, we assumed that the birth rate is not equal to the natural death charge μ in the population; hence the population size N is accurately not constant. Here, $p(0 < p < 1)$ designates the citizens vaccinated fraction at the birth of each year and citizens are born into the population at a fixed-birth rate α. An average contact rate is denoted by β, and the infected individual recovers rate is represented by δ. Now, we consider the system of differential equation elucidating the above phenomenon [58]

$$\frac{ds}{dt} = (1-p)\alpha - \beta SI - \alpha S, \tag{4}$$

$$\frac{di}{dt} = \beta SI - (\eta + \alpha)I, \tag{5}$$

$$\frac{dr}{dt} = p\alpha + \eta I - \alpha R. \tag{6}$$

The considered model is analysed by authors in [59] and presented some stimulating in terms of qualitative analysis. Particularly, they specify that the considered system is of the endemic or die out are two categories. Also, the disease-free equilibrium E_0 is derived with respect to lost two equations and the equilibrium with vaccination reproduction number, are respectively presented as follows

$$E_0 = (1-p,\ 0) \text{ and } R_v = \frac{\beta(1-p)}{\eta + \alpha}.$$

Further, the disease-free equilibrium is locally stable if $R_v < 1$. Most importantly, authors in [59] derived the conditions for thenumber of infectives initiates to surgeup to that process is earlier than the quantity of susceptible being added to the population.

In this paper, we modified the above system by including the effect of memory effect, nonlocality and hereditary by introducing the Caputo fractional derivative in the place of the time-derivative, and which are presented as follow

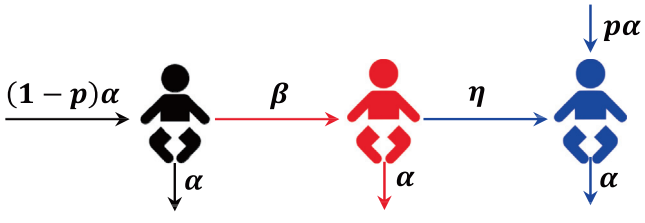


Fig. 1 Flow chart of the considered SIR model with S (black), I (Red) and R (blue). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$$\begin{aligned} D_t^{\mu} S &= (1-p)\alpha - \beta SI - \alpha S, \\ D_t^{\mu} I &= \beta SI - (\eta + \alpha)I, \quad 0 < \mu \le 1, \\ D_t^{\mu} R &= p\alpha + \eta I - \alpha R, \end{aligned} \tag{7}$$

with initial conditions

$$S(0) = N_1, I(0) = N_2, R(0) = N_3. \tag{8}$$

4. Fundamental idea of the proposed scheme

Here, we consider differential equation with fractional order to present the procedure of the considered method

$$D_t^{\mu} v(x,t) + \mathscr{R} v(x,t) + \mathscr{N} v(x,t) = f(x,t),\ 0 < \mu \le 1, \tag{9}$$

where $D_t^{\mu} v(x,t)$ symbolize the derivative with fractional derivative in Caputo sense for$v(x,t)$. By applying LT on Eq. (9), one can get

$$\begin{aligned} &s^{\mu}\mathscr{L}[v(x,t)] - \sum_{k=0}^{n-1} s^{\mu-k-1} v^{(k)}(x,0) + \mathscr{L}[\mathscr{R} v(x,t)] \\ &\quad + \mathscr{L}[\mathscr{N} v(x,t)] \\ &\quad = \mathscr{L}[f(x,t)]. \end{aligned} \tag{10}$$

After simplification, the Eq. (10) reduces to

$$\begin{aligned} &\mathscr{L}[v(x,t)] - \frac{1}{s^{\mu}} \sum_{k=0}^{n-1} s^{\mu-k-1} v^{k}(x,0) \\ &\quad + \frac{1}{s^{\mu}} \{\mathscr{L}[\mathscr{R} v(x,t)] + \mathscr{L}[\mathscr{N} v(x,t)] - \mathscr{L}[f(x,t)]\} \\ &\quad = 0. \end{aligned} \tag{11}$$

Now, with respect to$\varphi(x,t;q)$ the non-linear operator is presented as

$$\begin{aligned} \mathscr{N}[\varphi(x,t;q)] &= \mathscr{L}[\varphi(x,t;q)] - \tfrac{1}{s^{\mu}} \textstyle\sum_{k=0}^{n-1} s^{\mu-k-1} \varphi^{(k)}(x,t;q)(0^{+}) \\ &\quad + \tfrac{1}{s^{\mu}} \{\mathscr{L}[\mathscr{R}\varphi(x,t;q)] + L[\mathscr{N}\varphi(x,t;q)] - L[f(x,t)]\}, \end{aligned} \tag{12}$$

where $q \in [0, \frac{1}{n}]$. Then, we have

$$(1 - nq)\mathscr{L}[\varphi(x,t;q) - v_0(x,t)] = \hbar q \mathscr{N}[\varphi(x,t;q)], \tag{13}$$

where $L$is signifying LT. The following conditions are respectively satisfied for $q = 0$ and$q = \frac{1}{n}$

$$\varphi(x,t;0) = v_0(x,t), \varphi\left(x,t;\frac{1}{n}\right) = v(x,t). \tag{14}$$

Now, by rising$q$ from 0 to $\frac{1}{n}$, then $\varphi(x,t;q)$ varies from $v_0(x,t)$ to $v(x,t)$. By applying Taylor theorem near to$q$, we have

$$\varphi(x,t;q) = v_0(x,t) + \sum_{m=1}^{\infty} v_m(x,t) q^m, \tag{15}$$

where

$$v_m(x,t) = \frac{1}{m!} \frac{\partial^m \varphi(x,t;q)}{\partial q^m}|_{q=0}. \tag{16}$$

At the specific choice of$v_0(x,t)$,$n$and $\hbar$, the series (12) converges at$q = \frac{1}{n}$. Then

$$v(x,t) = v_0(x,t) + \sum_{m=1}^{\infty} v_m(x,t) \left(\frac{1}{n}\right)^m. \tag{17}$$

With the assist of Eq. (13), we have

$$\mathscr{L}[v_m(x,t) - k_m v_{m-1}(x,t)] = \hbar \Re_m\left(\overrightarrow{v}_{m-1}\right), \tag{18}$$

where the vectors are defined as

$$\overrightarrow{v}_m = \{v_0(x,t), v_1(x,t), \cdots, v_m(x,t)\}. \tag{19}$$

On applying inverse $LT$to Eq. (18), it reduces to

$$v_m(x,t) = k_m v_{m-1}(x,t) + \hbar \mathscr{L}^{-1}\left[\Re_m\left(\overrightarrow{v}_{m-1}\right)\right], \tag{20}$$

where

$$\Re_m\left(\overrightarrow{v}_{m-1}\right) = L[v_{m-1}(x,t)] - \left(1 - \tfrac{k_m}{n}\right)\left(\textstyle\sum_{k=0}^{n-1} s^{\mu-k-1} v^{(k)}(x,0) + \tfrac{1}{s^\mu} L[f(x,t)]\right) + \tfrac{1}{s^\mu} L[R v_{m-1} + \mathscr{H}_{m-1}], \tag{21}$$

and

$$\mathsf{k}_m = \begin{cases} 0, m \le 1, \\ n, m > 1. \end{cases} \tag{22}$$

In Eq. (21), $\mathscr{H}_m$is homotopy polynomial and which is defined as

$$\mathscr{H}_m = \frac{1}{m!}\left[\frac{\partial^m \varphi(x,t;q)}{\partial q^m}\right]_{q=0} \text{and} \varphi(x,t;q) = \varphi_0 + q\varphi_1 + q^2\varphi_2 + \cdots. \tag{23}$$

By the help of Eqs. (20) and (21), we have

$$v_m(x,t) = (k_m + \hbar) v_{m-1}(x,t) - \left(1 - \tfrac{k_m}{n}\right)\mathscr{L}^{-1}\left(\textstyle\sum_{k=0}^{n-1} s^{\mu-k-1} v^{(k)}(x,0) + \tfrac{1}{s^\mu}\mathscr{L}[f(x,t)]\right) + \hbar \mathscr{L}^{-1}\left\{\tfrac{1}{s^\mu} L[R v_{m-1} + \mathscr{H}_{m-1}]\right\}. \tag{24}$$

Using Eq. (24), we can find the terms of $v_m(x,t)$. The q-HATM solution is presented as

$$v(x,t) = \sum_{m=0}^{\infty} v_m(x,t). \tag{25}$$

5. q-HATM solution for the system of fractional-order epidemic model of childhood disease

Here, we demonstratethe solutions for model (7) with distinct parameters. Consider the system of the equation describing the fractional-order SIR epidemic model of childhood disease

$$\begin{aligned} D_t^\mu S &= (1-p)\alpha - \beta SI - \alpha S, \\ D_t^\mu I &= \beta SI - (\eta + \alpha) I, \\ D_t^\mu R &= p\alpha + \eta I - \alpha R, \end{aligned} \tag{26}$$

with initial conditions

$$S(0) = S_0, I(0) = I_0, R(0) = R_0. \tag{27}$$

Taking LT on Eq. (26) and then using the Eq. (27), we get

$$\begin{aligned} \mathscr{L}[S(t)] - \tfrac{1}{s}(S_0) - \tfrac{1}{s^\mu}\mathscr{L}\{(1-p)\alpha - \beta SI - \alpha S\} = 0, \\ \mathscr{L}[I(t)] - \tfrac{1}{s}(I_0) - \tfrac{1}{s^\mu}\mathscr{L}\{\beta SI - (\eta + \alpha) I\}, \\ \mathscr{L}[R(t)] - \tfrac{1}{s}(R_0) - \tfrac{1}{s^\mu}\mathscr{L}\{p\alpha + \eta I - \alpha R\}. \end{aligned} \tag{28}$$

Now, we present the non-linear operator Nas

$$\begin{aligned} &N^1[\varphi_1(t;q), \varphi_2(t;q), \varphi_3(t;q)] \\ &= \mathscr{L}[\varphi_1(t;q)] - \tfrac{1}{s}(S_0) - \tfrac{1}{s^\mu}\mathscr{L}\{(1-p)\alpha - \beta\varphi_1(t;q)\varphi_2(t;q) - \alpha\varphi_1(t;q)\}, \\ &N^2[\varphi_1(t;q), \varphi_2(t;q), \varphi_3(t;q)] \\ &= \mathscr{L}[\varphi_2(t;q)] - \tfrac{1}{s}(I_0) - \tfrac{1}{s^\mu}\mathscr{L}\{\beta\varphi_1(t;q)\varphi_2(t;q) - (\eta + \alpha)\varphi_2(t;q)\}, \\ &N^3[\varphi_1(t;q), \varphi_2(t;q), \varphi_3(t;q)] \\ &= \mathscr{L}[\varphi_3(t;q)] - \tfrac{1}{s}(R_0) - \tfrac{1}{s^\mu}\mathscr{L}\{p\alpha + \eta\varphi_2(t;q) - \alpha\varphi_3(t;q)\}. \end{aligned} \tag{29}$$

At $\mathscr{H}(t) = 1$, the m-th order deformation equation is defined as

$$\begin{aligned} \mathscr{L}[S_m(t) - k_m S_{m-1}(t)] &= \hbar \mathscr{L}^{-1}\left\{\Re_{1,m}\left[\overrightarrow{S}_{m-1}, \overrightarrow{I}_{m-1}, \overrightarrow{R}_{m-1}\right]\right\}, \\ \mathscr{L}[I_m(t) - k_m I_{m-1}(t)] &= \hbar \mathscr{L}^{-1}\left\{\Re_{2,m}\left[\overrightarrow{S}_{m-1}, \overrightarrow{I}_{m-1}, \overrightarrow{R}_{m-1}\right]\right\}, \\ \mathscr{L}[R_m(t) - k_m R_{m-1}(t)] &= \hbar \mathscr{L}^{-1}\left\{\Re_{3,m}\left[\overrightarrow{S}_{m-1}, \overrightarrow{I}_{m-1}, \overrightarrow{R}_{m-1}\right]\right\}, \end{aligned} \tag{30}$$

where

$$\begin{aligned} &\Re_{1,m}\left[\overrightarrow{S}_{m-1}, \overrightarrow{I}_{m-1}, \overrightarrow{R}_{m-1}\right] \\ &= \mathscr{L}[S_{m-1}(t)] - \left(1 - \tfrac{k_m}{n}\right)\tfrac{1}{s}(S_0) - \tfrac{1}{s^\mu}\mathscr{L}\left\{(1-p)\alpha - \beta\textstyle\sum_{i=0}^{m-1} S_i I_{m-1-i} - \alpha S_{m-1}\right\}, \\ &\Re_{2,m}\left[\overrightarrow{S}_{m-1}, \overrightarrow{I}_{m-1}, \overrightarrow{R}_{m-1}\right] \\ &= \mathscr{L}[I_{m-1}(t)] - \left(1 - \tfrac{k_m}{n}\right)\tfrac{1}{s}(I_0) - \tfrac{1}{s^\mu}\mathscr{L}\left\{\beta\textstyle\sum_{i=0}^{m-1} S_i I_{m-1-i} - (\eta + \alpha) I_{m-1}\right\}, \\ &\Re_{3,m}\left[\overrightarrow{S}_{m-1}, \overrightarrow{I}_{m-1}, \overrightarrow{R}_{m-1}\right] \\ &= \mathscr{L}[R_{m-1}(t)] - \left(1 - \tfrac{k_m}{n}\right)\tfrac{1}{s}(R_0) - \tfrac{1}{s^\mu}\mathscr{L}\{p\alpha + \eta I_{m-1} - \alpha R_{m-1}\}. \end{aligned} \tag{31}$$

By employing inverse LT on Eq. (30), we get

$$\begin{aligned} S_m(t) &= k_m S_{m-1}(t) + \hbar \mathscr{L}^{-1}\left\{\Re_{1,m}\left[\overrightarrow{S}_{m-1}, \overrightarrow{I}_{m-1}, \overrightarrow{R}_{m-1}\right]\right\}, \\ I_m(t) &= k_m I_{m-1}(t) + \hbar \mathscr{L}^{-1}\left\{\Re_{2,m}\left[\overrightarrow{S}_{m-1}, \overrightarrow{I}_{m-1}, \overrightarrow{R}_{m-1}\right]\right\}, \\ R_m(t) &= k_m R_{m-1}(t) + \hbar \mathscr{L}^{-1}\left\{\Re_{3,m}\left[\overrightarrow{S}_{m-1}, \overrightarrow{I}_{m-1}, \overrightarrow{R}_{m-1}\right]\right\}. \end{aligned} \tag{32}$$

Using the initial conditions and the above system, we have

$$S_1(t) = (\alpha - p\alpha - (\alpha + \beta Q_0) S_0)\frac{\hbar t^\mu}{\Gamma[\mu+1]},$$

$$I_1(t) = (-Q_0(\alpha + \eta - \beta S_0))\frac{\hbar t^\mu}{\Gamma[\mu+1]},$$

$$R_1(t) = (\eta Q_0 + \alpha(p - R_0))\frac{\hbar t^\mu}{\Gamma[\mu+1]},$$

$$S_2(t) = (\alpha - p\alpha - (\alpha + \beta Q_0) S_0)\frac{\hbar(n+\hbar)t^\mu}{\Gamma[\mu+1]} + \frac{\hbar t^\mu}{\Gamma[2\mu+1]} \times (t^\mu \beta^2 \hbar Q_0^2 S_0 + \alpha((-1+p)(t^\mu \alpha\hbar - \Gamma[1+\mu]) + t^\mu \alpha \hbar S_0) + t^\mu \beta \hbar Q_0\left((-1+p)\alpha + (3\alpha + \eta) S_0 - \beta S_0^2\right)),$$

$$I_2(t) = (-Q_0(\alpha + \eta - \beta S_0))\frac{\hbar(n+\hbar)t^\mu}{\Gamma[\mu+1]} - \frac{\hbar^2 Q_0 t^{2\mu}}{\Gamma[2\mu+1]}(-\alpha^2 + \alpha((-1+p)\beta - 2\eta) - \eta^2 + \beta(3\alpha + 2\eta + \beta Q_0) S_0 - \beta^2 S_0^2),$$

Table 1 The effect of the vaccination for distinct parameters [44].

Case	Comments	S_0	I_0	R_0	α	β	η	p
I	Disease eradication	1	0	0	0.8	0.003	0.4	0.9
II	No eradication	0.8	0.2	0	0.8	0.03	0.4	0.0

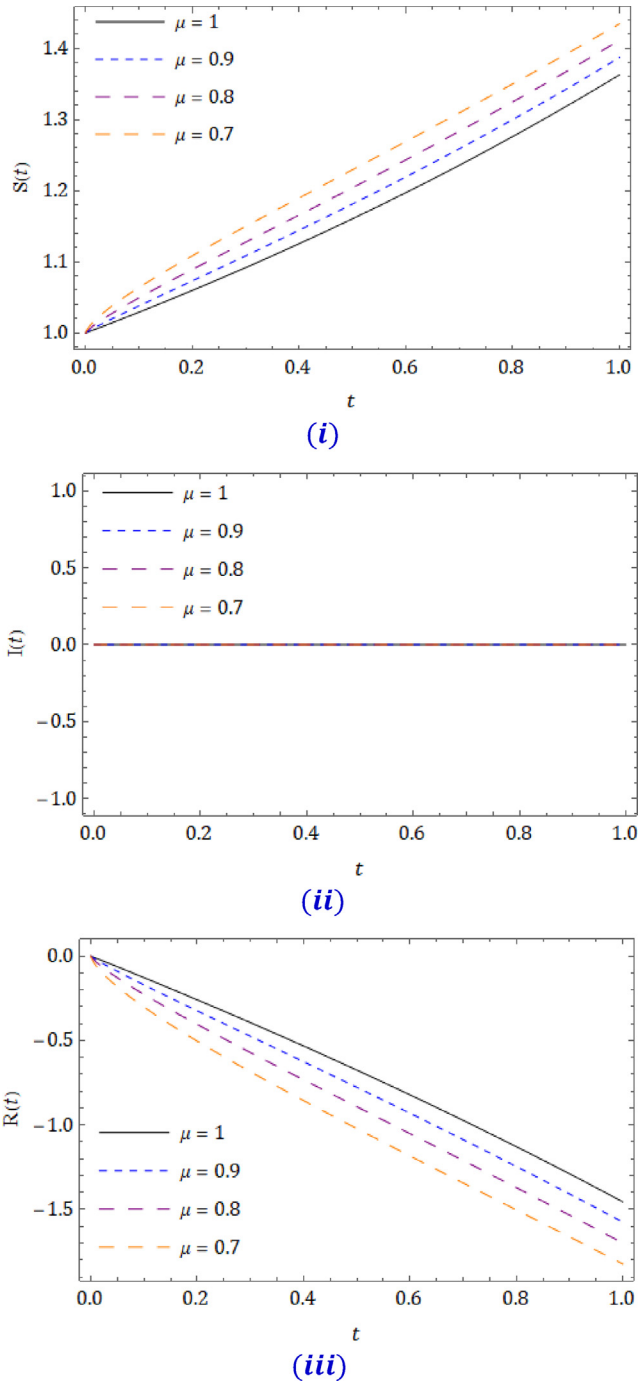


Fig. 2 Response of q-HATM solution with respect to t for (***i***) $S(t)$, (***ii***) $I(t)$, (***iii***) $R(t)$ at $n = 1$ and $\hbar = -1$ for the **Case I** with diverseμusing Table 1.

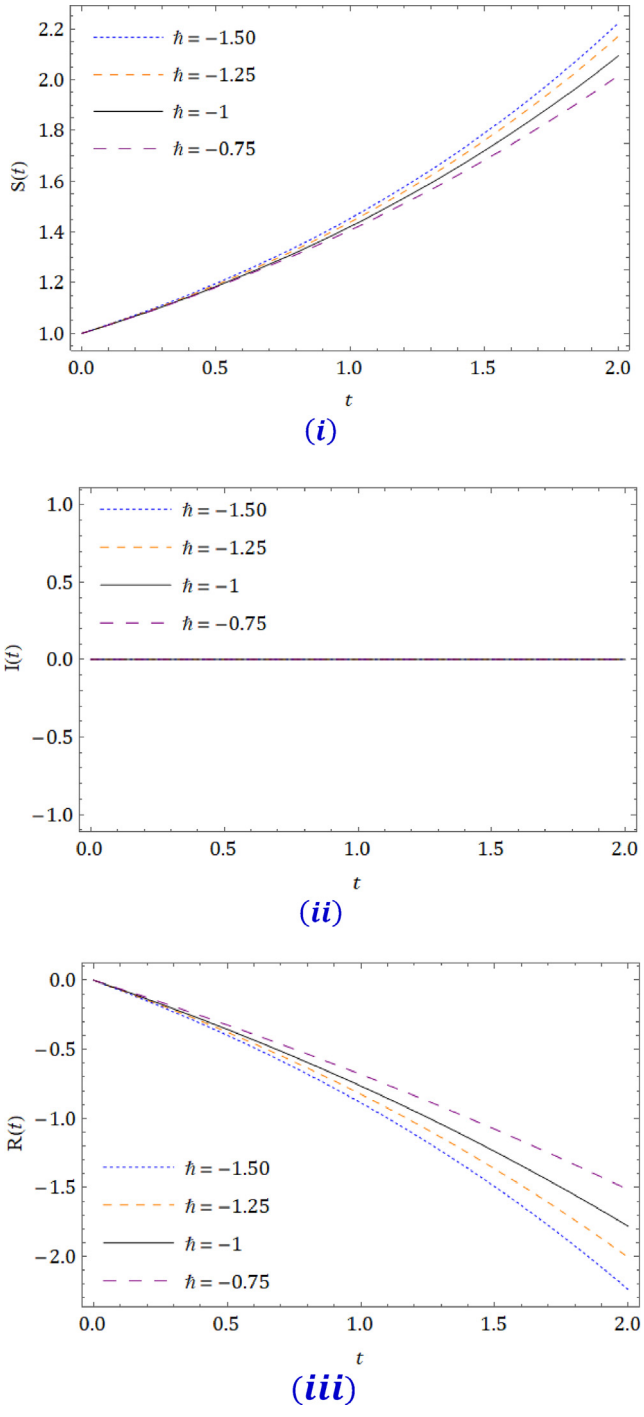
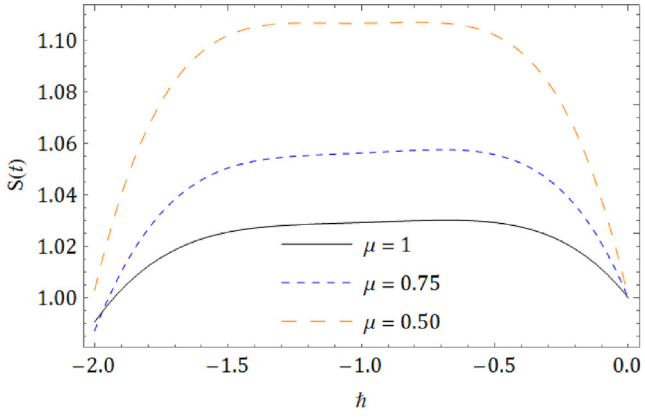


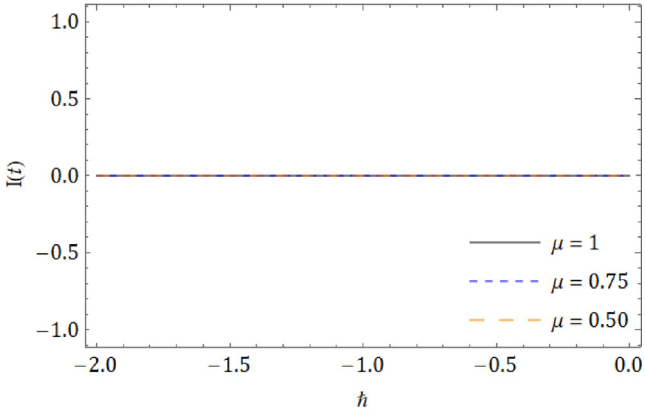
Fig. 3 Nature of obtained solution for (***i***) $S(t)$, (***ii***) $I(t)$, (***iii***) $R(t)$ at $n = 1$ and $\mu = 1$ for the **Case I** with distinct$\hbar$ using Table 1.

$$R_2(t) = (\eta Q_0 + \alpha(p - R_0))\frac{\hbar(n+\hbar)t^{\mu}}{\Gamma[\mu+1]} + (\alpha(p(-t^{\mu}\alpha\hbar + \Gamma[1+\mu]) + t^{\mu}\alpha\hbar R_0) - t^{\mu}\eta\hbar Q_0(2\alpha + \eta - \beta S_0))\frac{\hbar t^{\mu}}{\Gamma[2\mu+1]},$$

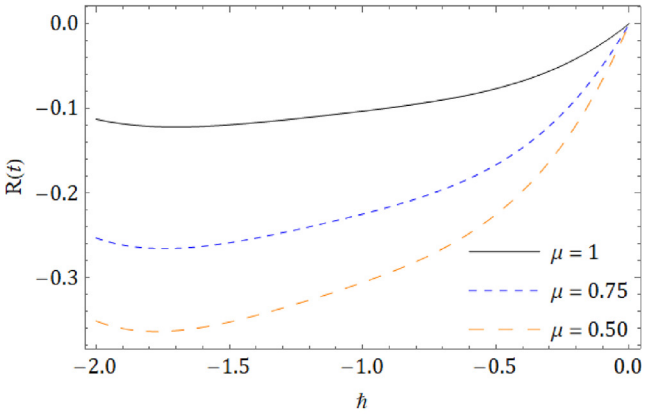
$$S_3(t) = (\alpha - p\alpha - (\alpha + \beta Q_0)S_0)\frac{\hbar(n+\hbar)^2 t^{\mu}}{\Gamma[\mu+1]} + \frac{\hbar^2(n+\hbar)t^{\mu}}{\Gamma[2\mu+1]} \times (t^{\mu}\beta^2\hbar Q_0^2 S_0 + \alpha((-1+p)(t^{\mu}\alpha\hbar$$



(*i*)

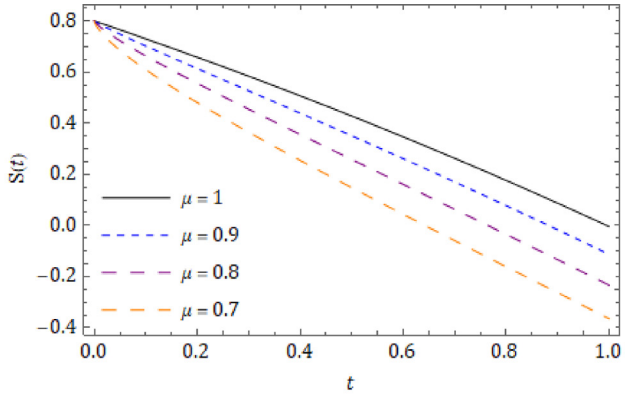


(*ii*)

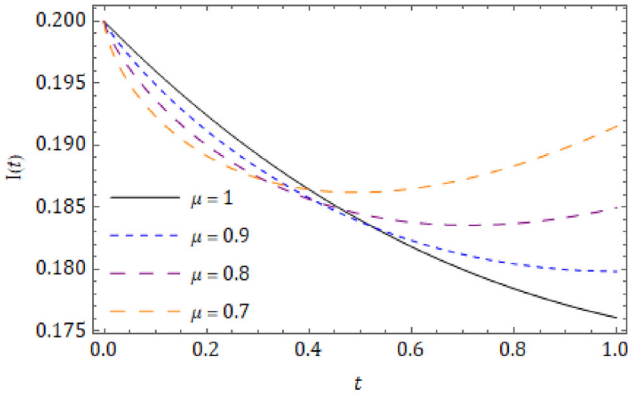


(*iii*)

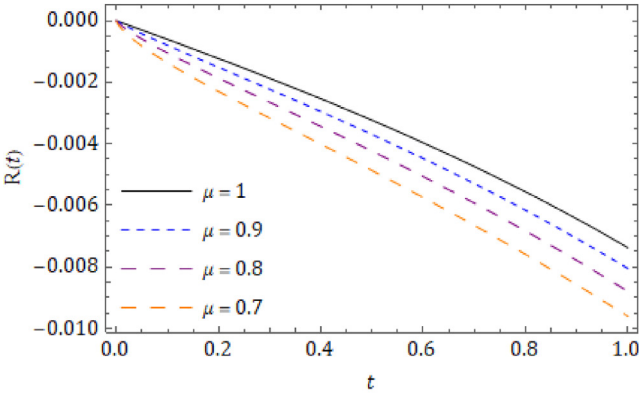
Fig. 4 $\hbar$-curves drawn for (*i*) $S(t)$, (*ii*) $I(t)$, (*iii*) $R(t)$ with $n = 1$ and $t = 0.01$ for **Case I** at differentμ using Table 1.



(*i*)

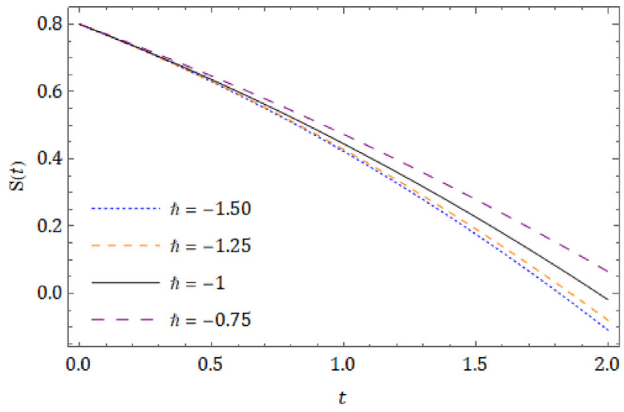


(*ii*)

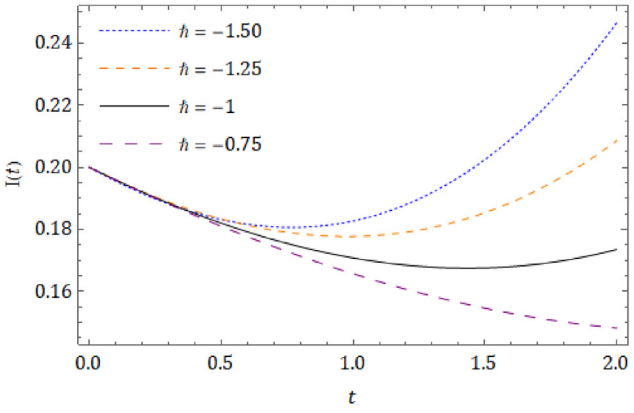


(*iii*)

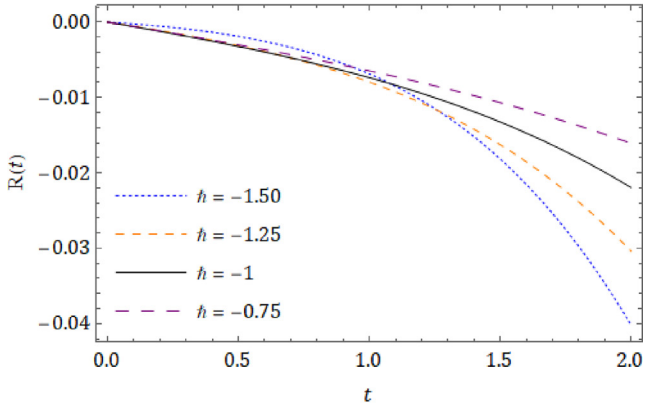
Fig. 5 Nature of q-HATM solution with t for (*i*) $S(t)$, (*ii*) $I(t)$, (*iii*) $R(t)$ at $\hbar = -1, n = 1$ and using Table 1 for the **Case II** with diverseμ.



(*i*)

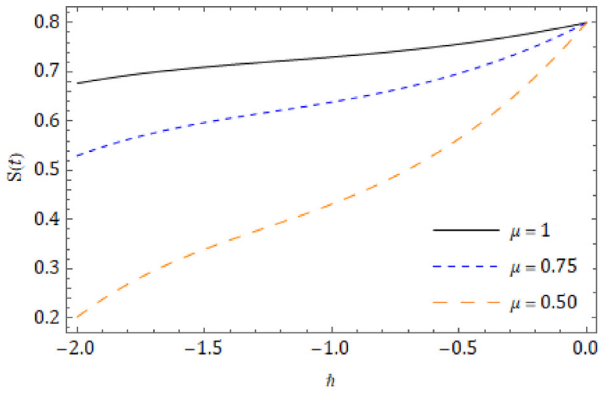


(*ii*)

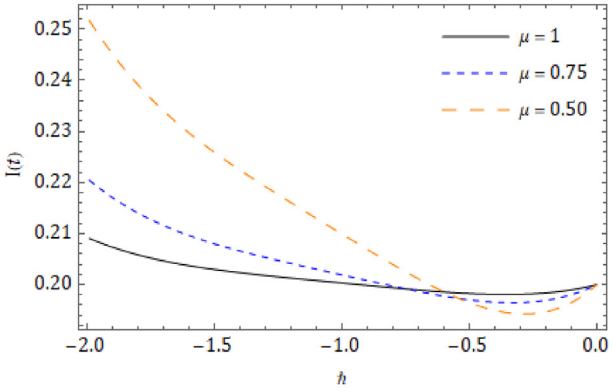


(*iii*)

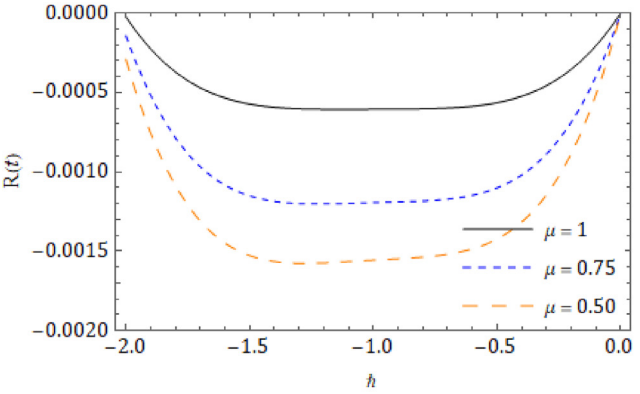
Fig. 6 Nature of obtained solution for (***i***) $S(t)$, (***ii***) $I(t)$, (***iii***) $R(t)$ at $n = 1, \mu = 1$ and using Table 1 for the **Case II** with diverse$\hbar$.



(*i*)



(*ii*)



(*iii*)

Fig. 7 $\hbar$-curves drown for (***i***) $S(t)$, (***ii***) $I(t)$, (***iii***) $R(t)$ at $n = 1$ and $t = 0.01$ for the **Case II** with distinctμ using Table 1.

$$-\Gamma[1+\mu]) + t^{\mu}\alpha\hbar S_0)$$
$$+ t^{\mu}\beta\hbar Q_0\big((-1+p)\alpha + (3\alpha+\eta)S_0 - \beta S_0^2\big)),$$

$$+\Big(\frac{1}{\Gamma[3\mu+1]}\big(-(-1+p)\alpha(t^{2\mu}\alpha^2\hbar^2 - t^{\mu}\alpha\hbar\Gamma[1+\mu] + \Gamma[1+2\mu]\big)$$
$$+\frac{1}{\Gamma[\mu+1]^2}t^{\mu}\hbar((-1$$

$$+p)\alpha\beta\Gamma[\mu+1]^3Q_0 + t^{\mu}\hbar(-\beta\Gamma[2\mu+1]Q_0(\alpha+\eta-\beta S_0)((-1$$
$$+p)\alpha + (\alpha+\beta Q_0)S_0)$$

$$+\Gamma[\mu+1]^2(-(-1+p)\alpha\beta Q_0(2\alpha+\beta Q_0)-(\alpha^3+\beta Q_0(\alpha(5\alpha+\beta -p\beta)+3\alpha\eta$$

$$+\eta^2+\beta Q_0(4\alpha+\eta+\beta Q_0)))S_0+2\beta^2 Q_0(2\alpha+\eta+\beta Q_0)S_0^2 -\beta^3 Q_0 S_0^3)))))\hbar t^\mu,$$

$$I_3(t)=(-Q_0(\alpha+\eta-\beta S_0))\frac{\hbar(n+\hbar)^2 t^\mu}{\Gamma[\mu+1]}-\frac{\hbar^2(n+\hbar)Q_0 t^{2\mu}}{\Gamma[2\mu+1]}(-\alpha^2 +\alpha((-1+p)\beta-2\eta)-\eta^2$$

$$+\beta(3\alpha+2\eta+\beta Q_0)S_0-\beta^2 S_0^2)(t^\mu\hbar Q_0(-\frac{t^\mu(\alpha+\eta)\hbar}{\Gamma[3\mu+1]} \times(\alpha(\alpha+\beta-p\beta)+2\alpha\eta+\eta^2$$

$$+\beta S_0(-3\alpha-2\eta-\beta Q_0+\beta S_0)) +\beta(\frac{t^\mu\hbar(\alpha+\eta-\beta S_0)((-1+p)\alpha+(\alpha+\beta Q_0)S_0)}{\Gamma[\mu+1]^2\Gamma[\mu+1]}$$

$$+\frac{t^\mu\hbar S_0}{\Gamma[3\mu+1]}(\alpha(\alpha+\beta-p\beta)+2\alpha\eta+\eta^2+\beta S_0(-3\alpha-2\eta-\beta Q_0+\beta S_0))$$

$$+\frac{1}{\Gamma[3\mu+1]}(-(-1+p)\alpha\Gamma[\mu+1]+t^\mu\hbar((-1+p)\alpha(\alpha+\beta Q_0) +(\alpha^2+\beta Q_0(3\alpha+\eta$$

$$+\beta Q_0))S_0-\beta^2 Q_0 S_0^2)))))\hbar t^\mu,$$

$$R_3(t)=(\eta Q_0+\alpha(p-R_0))\frac{\hbar(n+\hbar)^2 t^\mu}{\Gamma[\mu+1]} +(\alpha(p(-t^\mu\alpha\hbar+\Gamma[1+\mu])+t^\mu\alpha\hbar R_0)$$

$$-t^\mu\eta\hbar Q_0(2\alpha+\eta-\beta S_0))\frac{\hbar(n+\hbar)t^\mu}{\Gamma[2\mu+1]}+(\frac{1}{\Gamma[3\mu+1]}(\alpha(p(t^{2\mu}\alpha^2\hbar^2 -t^\mu\alpha\hbar\Gamma[1+\mu]+\Gamma[1+2\mu])$$

$$-t^{2\mu}\alpha^2\hbar^2 R_0)-t^{2\mu}\beta^2\eta\hbar^2 Q_0^2 S_0+t^{2\mu}\eta\hbar^2 Q_0(3\alpha^2+\eta^2+\alpha(\beta-p\beta +3\eta)$$

$$-2\beta(2\alpha+\eta)S_0+\beta^2 S_0^2)))\hbar t^\mu,$$

$$\vdots$$

We can get the rest of the term in a similar way. The q-HATM series solution for the FBM equation considered in Eq. (26) is given by

$$\begin{aligned} S(t)&=S_0(t)+\textstyle\sum_{m=1}^{\infty}S_m(t)\left(\frac{1}{n}\right)^m,\\ I(t)&=I_0(t)+\textstyle\sum_{m=1}^{\infty}I_m(t)\left(\frac{1}{n}\right)^m,\\ R(t)&=R_0(t)+\textstyle\sum_{m=1}^{\infty}R_m(t)\left(\frac{1}{n}\right)^m. \end{aligned} \tag{33}$$

6. Results and discussion

Here, the new series solutions have been evaluated in the present investigation for the SIR epidemic model of childhood disease in order to demonstrate the efficiency of the propped method. In Table 1, we present the two special cases which are considered in the present investigation with specific values of the parameters. Figs. 2 and 5 illustrated the nature of q-HATM solutions of the susceptible group $S(t)$, infected group $I(t)$, and removed or quarantined group $R(t)$with distinct Brownian motion and standard motion $(\mu=1)$ for Case I and Case II, respectively and from the plots, it is confirmed that the proposed fractional epidemic model has some interesting behaviour. The behaviour of q-HATM solution for diverse $\hbar$is cited in Figs. 3 and 6 respectively for Case I and II, and which help us to identify the effect of the homotopy parameter. The $\hbar$-curves are drowned for diverse α and which respectively presented in Figs. 4 and 7 to analyse the behaviour of the achieved results for the projected model considered in two cases with $(\hbar)$. The line segment in the horizontal positionof the $\hbar$-curves indicates the convergence region of the obtained solution.

In Case I, we can observe from the plots that as time increases the susceptible group $S(t)$ is increasing and removed group $R(t)$is decreases, but the infected group $(I(t))$ is remain constant due to our initial approach $(I_0(t)=0)$. Further, in Case II we get some of the interesting and also stimulating results as we can see in Fig. 5. Authors in [43] considered CFDTM to find the solution and analyse the behaviour of the obtained solution for the proposed model. However, as compared to the above-cited technique q-HATM is offers a simple algorithm to find the solution for the non-linear problem, it does not require any transformation of the functions like CFDTM. Moreover, the proposed scheme reduces massive computation and requires less time to evaluate the terms of the series solution.

7. Conclusion

In this paper, the solution for the fractional-order non-linear system of differential equations describing the SIR model of childhood diseaseis obtained with the aid of q-HATM. The achieved solutions are illustrated in the series form, which quickly converges to the analytical solution. From the obtained result, we can see that the dynamic nature of the projected model depends on time instant and time history. These properties can be proficiently demonstrated with the aid of the theory of FC. The graphical illustration confirms that the model conspicuously depends on the considered parameters and the arbitrary order, and which can stimulus the stability of the model. Lastly, we conclude that, the considered scheme is more effective as well as highly methodical, and hence it can applyto find the solution for non-linear differential equations exemplifying the various mechanisms. Particularly, to analyse the behaviour as well as predict the dynamical growth of human diseases. These findings may help to a better understanding of the dynamical growth of human diseases.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This projected work was partially (not financial) supported by the Scientific Research Project Fund of Harran University with the project ID: HUBAP-21132.

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