The Effect of Realistic Mathematics Education on Fourth Graders' Problem Posing/Problem-solving Skills and Academic Achievement[*]

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To cite this article::
Uskun Aytekin, K., Çil, O., & Kuzu, O. (s). (2021). The effect of realistic mathematics education on fourth graders' problem posing/problem-solving skills and academic achievement. *Journal of Qualitative Research in Education*, 28, 22-50. doi: 10.14689/enad.28.2

Abstract: This study investigated the effect of the Realistic Mathematics Education (RME) approach on fourth-grade students' achievement and problem posing and problem-solving skills based on four basic operations. "Academic Achievement Test towards Four Basic Operations" was administered to a total of 70 students. Case study model was used in the qualitative part of the research designed with a mixed model, and the obtained data were analyzed by content analysis. The quantitative part of the study was carried out using a quasi-experimental pretest-posttest design with the control group. After the 17-week experimental process, a significant difference was found in experimental group students' academic achievements and problem posing and problem-solving skills using four basic operations. Students who received instruction based on the RME approach comprehended the problem statements better, created more meaningful problems and developed their ability to select the appropriate data and the operation. Extending the classroom use of the RME approach will effectively improve students' problem posing and solving skills and increase student achievement in international exams.

Keywords: Realistic Mathematics Education, four operations, problem solving, problem posing.

Article Info

Received: 16 Sept. 2020
Revised: 13 Aug. 2021
Accepted: 23 Oct. 2021

Article Type

Research

[*] This article was produced based on the first author's master's thesis written under the supervision of the co-authors.

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Declaration of Conflicts of Interests: None

Introduction

Mathematics, which facilitates daily life, has been the basis for many studies in the past and present. In fact, mathematics, which arose from the basic needs of human beings and led to the development of simple counting and measuring operations to meet daily needs, is often used to solve problems in agriculture, commerce, astronomy and architecture (Bayam, 2014).Mathematics, used as a tool to understand and find solutions to the problems encountered in life, is concisely defined by Altun (2008) as the abstracted form of life. Hence, mathematics is not only the science of numbers, shapes, space, sizes and the relations between them, but a universal language based on symbols and shapes. Mathematics is not a static field of science containing exact, fait accompli truths, but a lively field of study based on the trial-and-error approach which is open to new investigations and inventions (Yıldırım, 2010). As defined by the Ministry of National Education (MoNE), mathematics includes producing and processing information, making predictions and solving problems using this language and the relations of all these skills with daily life should be presented to students effectively (MEB, 2009). According to these definitions, mathematics is learned most easily while solving problems encountered in daily life. It cannot be separated from both human and daily life; at the same time, mathematics cannot be taught uniformly.

To what extent individuals can acquire, produce, use information at all levels of the education system starting from pre-school education and to what extent individuals can guide society, science and technology is crucial and these aspects help measure the quality of education programs. In short, a quality education program is expected to ensure that people can "solve problems" (Alemdar-Coşkun, 2016). Problem-solving, the most important part of mathematics programs, is one of the significant components of mathematics in providing students with the necessary knowledge and skills (Baki, 2015). Problem-solving is a cognitive process of transforming a current situation into a desired outcome to solve a problem encountered for the first time (Keane & Eysenck, 2010). This process positively affects the learning process in mathematics, forms the basis of learning and enables individuals to reach a more original solution utilizing developing their creative thinking (Aksu, 1989). Problem-solving stages are defined as "identifying and defining the problem, generating realistic and achievable goals for the problem, producing alternative solutions, evaluating the pros and cons of the problem, deciding on a solution, developing and implementing an action plan and evaluating the outcome" (Rosen et al., 2011). All stages of the problem-solving process require thinking and problem solving Problem-solving is not solely regarded as the ability to obtain an outcome. The problem posing process is used as a tool in the stages of developing problem-solving skills, understanding mathematical concepts more accurately and associating mathematics with daily life (Stoyanova, 1998). Problem posing generates a new problem or reconstructs a given problem based on mathematical situations or models (Duncker, 1945). This process is regarded as an important stage in making sense of mathematics (Silver, 1994) and students will develop higher-order thinking skills in this process (Cankoy & Darbaz, 2010; Yuan & Sriraman, 2011). The investigation of previous studies shows that problem setting has a positive and significant relationship with problem solving (Arıkan & Ünal, 2013; Aytekin-Uskun, et al., 2020; English, 1997; Şengül & Kantarcı, 2014) and that the process of problem setting is specified as a component and stage in the problem solving process (Christou, et al., 2005; Kılıç, 2017).

It is evident that today, when mathematics is needed in every aspect of life, students should be guided in breaking down prejudices against mathematics and in transferring mathematics to their lives. In particular, the abstract structure of the mathematics course makes it difficult to understand the subjects related to mathematics and to establish the relationship between mathematical knowledge and real life. In this context, Çilingir et al. (2015) emphasize that the abstract structure of mathematics negatively affects student attitudes and that students regard mathematics as a process based on basic operational skills rather than as a part of real life. Mathematics anxiety embedded in society and difficulties in transferring mathematical knowledge to real life negatively affect achievement in international exams for Turkey. For example, the analyses of the results in PISA 2018, which assessed reading skills, science and mathematical literacy, demonstrated that while Turkey achieved higher scores in reading skills (466) and science literacy (468) than the average scores (reading skills: 453; science literacy: 459), it remained below the average score (459) in the field of mathematical literacy with an average score of 454 (MEB, 2019; Reiss et al., 2019). As Yücel et al. (2013) stated, the results of international examinations show that Turkish students cannot achieve the desired level of performance, they have difficulties, especially in applying the knowledge acquired in school to find solutions to the problems they encounter in real life, they have a high level of anxiety and therefore these negative conditions have a negative impact on their performance in mathematics. However, the 2018 mathematics program emphasizes the relationship between mathematics education and human life. Moreover, mathematics education is student-centered, focuses on conceptual understanding, is organized through concrete materials and situations, and defines mathematics as a process in which students should be constantly reminded that it is part of daily life (MEB, 2018a). In fact, it is found that designing an instructional process that is oriented towards students' understanding is important for understanding basic mathematical concepts (Aktaş et al., 2018; Güven & Karataş, 2004; Kuzu et al., 2018). Transferring mathematical knowledge and skills to daily life and using a process-based teaching approach are reported to be more important than traditional teaching in understanding concepts and creating a more durable learning environment (Çil et al., 2019). In this context, it is believed that the use of Realistic Mathematics Education (RME) student-centred approach will be very effective in the teaching process.

RME was developed in the "Institute for the Development of Mathematics Education (Instituut Ontwikkeling WiskundeOnderwijs-IOWO) established in Utrecht University in 1971 within the scope of the Wiskobas Project (Mathematics in Primary School Project), which was initiated by Edu Wijdeveld, Fred Goffree, Adri Treffers in 1968 and became more significant with the participation of Hans Freudenthal (Van den Heuvel-Panhuizen, 2003; Treffers, 1993; Van den Heuvel-Panhuizen & Drijvers, 2014). The project mainly aimed to protect Dutch mathematics education from the effects of the "New Mathematics" education that emerged in the United States and create a realistic mathematics education free from the conventionality of traditional arithmetic. The name of IOWO was changed to Freudenthal Institute (FI) since Freudenthal, who was the most important member of the project and determined the current principles of RME, had significant contributions in the field of mathematics (Robertson, 2000). This approach, developed by the Freudenthal Institute, has been adopted and accepted in the education systems of several countries such as Germany, America, Brazil, Denmark, South Africa, England, Spain, Japan, Malaysia and Portugal, especially the Netherlands (Arseven, 2010).

RME is a student-centered teaching and learning approach that allows students to think multidimensionally by helping their imaginations develop (Freudenthal, 1973). RME is based on didactic phenomenology and the teaching process is carried out through a context or a situation that mediates the construction of mathematical concepts as mathematical objects in students' minds (Freudenthal, 1973). According to the RME approach, the teaching process starts with real-life problems, and the students reach the desired information in the problem-solving process. A connection is established between daily life experiences and mathematical concepts in RME, a problem-solving process (Olkun & Toluk, 2007), and abstract mathematics becomes more permanent through concretization. In the RME approach, topics start with real-life problems and students are given opportunities to make sense of the information at every stage of the teaching process. Since the RME approach allows students to interact with each other and play an active role in the teaching process, it lays the groundwork for an increase in students' academic performance and ensures an efficient educational process (Kaylak, 2014).

According to Freudenthal (1991); mathematics begins with problems arising from real life and real life is mathematized and then formal mathematics appears. He defined this process as " mathematization/mathematizing". The mathematization process in RME has been treated under two headings as horizontal and vertical mathematization (Van den Heuvel-Panhuizen, 1998; Treffers, 1987). Horizontal mathematization defines real-life problems presented to students using mathematical expressions to solve them mathematically (Gravemeijer & Doorman, 1999). Vertical mathematization uses mathematical situations in mathematical language abstraction and integrates this new knowledge into previously acquired mathematical knowledge. In other words, vertical mathematization is the process of working with symbols and reaching general or individual formulas by presenting the relationships between concepts (Altun, 2006; Zulkardi, 2002). According to Alacacı et al. (2016), the steps of RME based teaching are as follows: teacher presents and distributesthe problem to students, students read and understand the problem, they work on the problem as groups, they share and discuss the solutions under the guidance of the teacher, the teacher asks summative questions by and they discuss the mathematical basis of the results/outcomes.

The positive effects of the RME approach on student performance are frequently reported in previous studies (Demir, 2017; Gravemeijer et al., 1990; Ödemiş, 2019). In addition, Kaplan et al. (2015) conducted a meta-analysis study on the effect of RME-supported educational practices on student achievement. They stated that RME-based educational practices revealed a moderate, positive and statistically significant change in student achievement. In a study on the development of mathematical skills, Noviani et al. (2017)

suggested that the students' spatial skills who participated in the activities based on the RME approach developed more effectively than the students who received education with the traditional approach. In the literature, many researchers stated that the RME approach increases student achievement and facilitates the concept teaching process by helping students comprehend mathematical subjects more effectively (Laurens et al., 2017). For example, Uça and Saracaloğlu (2017) identified that students could effectively establish part and whole relations between weight units, make intuitive explanations about fractions and make sense of decimal fractions based on integer fractions as a result of educational activities on fractions with RME. In addition, it is emphasized that teaching with the RME approach increases the permanence of learning (Cihan, 2017; Kan, 2019) and is effective in developing high-level cognitive skills (Altun, 2001; Cansız, 2016).

Based on the international examination results, reform initiatives have been launched in the Turkish education system to change the required skills of students, the teaching and learning process, and the roles and responsibilities of teachers. Today, accessing, organizing, sharing and interpreting information, as well as applying and understanding mathematics in daily life are becoming more important than mere memorization or direct transfer of information (MEB, 2009). On the other hand, mathematics is a cumulative and interconnected subject consisting of topics that build on each other; therefore, difficulties may arise in learning some subjects if the prerequisites are not fully understood (Kuzu, 2017). In this regard, it is assumed that working on the subjects that form the basis of mathematics, such as addition, subtraction, multiplication and division, contributes to literature. In fact, it can be argued that these four basic operations (addition, subtraction, division, multiplication) occupy a very important place in mathematical calculations and that mathematics is based on these basic operations. Considering all these needs, this study investigated the effects of RME approach on fourth grade elementary students' performance and problem solving skills in problems involving four basic arithmetic operations. Answers to the following research questions were sought.

1. Does teaching with RME approach have an effect on the achievement of primary school fourth grade students in regards to problem posing and problem solving?

2. How did teaching with RME approach change the primary school fourth grade students' problem posing and problem solving skills in regards to four basic operations?

Method

Research Model

Both qualitative and quantitative research approaches were used in this study, which was designed using a mixed methods approach. Mixed methods research (Creswell & Plano Clark, 2015), which can compensate for the weaknesses of both quantitative and qualitative research, provides a broader perspective by using two different perspectives obtained from closed and open data (Creswell, 2009). According to Cresswell (2009), using qualitative and quantitative approaches together provides a better understanding of research problems. In this study, a parallel design was used in which the quantitative and qualitative approaches were equally weighted and collected simultaneously. In parallel design, qualitative and quantitative data that are equally important are collected simultaneously and used together to answer the research question (Creswell & Plano Clark, 2011). In this study, the case study model was used in the qualitative part, which examines the RME approach to problem setting and problem solving skills of fourth grade elementary students in relation to four-operation problems. The case study model is defined as an in-depth description and investigation of a bounded system (Merriam & Tisdell, 2015). On the other hand, the quasi-experimental pretest-posttest design with experimental and control groups, one of the quantitative research designs, was used in the quantitative part, which investigates the effect of the RME approach on fourth grade elementary students' performance on problems with four basic operations. Table 1 shows the symbolic representation of the quasi-experimental design.

Table 1.

Symbolic Representation of the Quasi-experimental Design

Groups	Pretests	Experimental Pprocess	Posttests
Experimental Group	AATTFO	Teaching Based on the RME Approach	AATTFO
Control Group	AATTFO	Traditional Teaching (Lecture, question-answer, discussion)	AATTFO

AATTFO: Academic Achievement Test towards Four Basic Mathematical Operations

Study Group

The study group consisted of a total of 70 fourth grade students studying in the Central Anatolian region of Turkey during the 2019-2020 school year. Of these students, 35 were randomly included in the experimental group and 35 in the control group. The purposive sampling method was used to determine the study group, which is a non-probabilistic method (Cohen et al., 2000). Since the study aimed to determine the effectiveness of the method used in the research, it was not necessary to select a sample representative of the population (Büyüköztürk, Kılıç-Çakmak, Akgün, Karadeniz ve Demirel, 2017).The study group was composed of a total of 70 fourth grade students studying in the Central Anatolian Region of Turkey in the 2019-2020 academic year. Out of these students, 35 were included in the experimental group and 35 in the control group through random method. Purposive sampling method, which is a non-probabilistic sampling method, was used to determine the study group (Cohen et al., 2000). Since the study aimed to determine the effectiveness of the method used in the research, it was not necessary to select a sample representing the universe (Büyüköztürk, Kılıç-Çakmak, Akgün, Karadeniz, & Demirel, 2008). 2017). The ethical permission of this study was obtained from the T.R. Ministry of National Education, Innovation and Educational Technologies General Directorate (date 25.04.2018, number 81576613/605.01/8278421).

Data Collection Tool

20-item "Academic Achievement Test towards Four Basic Mathematical Operations" (AATTFO) developed by Aytekin-Uskun et al. (2020) was used in this study to investigate the effect of the RME approach on primary school fourth grade students' academic achievement on problem posing and problem-solving concerning four basic mathematical operations (addition, subtraction, multiplication, division). The tool has .87 reliability. The mean item difficulty index of the test, which consists of 12 open-ended and 8 multiple-choice items, is .46, the mean discrimination index is .61, and the mean point biserial correlation coefficient is .54. When the items in the test are examined, it is identified that there are two open-ended items for the statement "poses a problem" and one open-ended and two multiple-choice items for the statement "solves problems" for each acquisition. The items in the test were created in line with four themes that are related to real-life and have contexts in themselves. Among the basic mathematical operations, addition is related to the bicycle theme, subtraction to the library theme, multiplication to the greengrocer theme, and division to the global warming theme.

Data Collection and Analysis

During the data collection process, the classes conducted in the control group were carried out with the traditional teaching method (lecture, question-answer, discussion), taking into account the acquisitions in the curriculum. The classes continued as usual under the supervision of the classroom teachers in line with student needs and levels. The classes conducted in the experimental group were carried out taking into account the five basic features of RME emphasized by Gravemeijer (1994) and six basic principles expressed by Van den Heuvel-Panhuizen & Wijers (2005). Activities were carried out in line with the principles of "realistic" and "real life problems" and the teaching process was organized with real life problems. Secondly, considering the "activity principle" and "use of materials", attention was paid to the use of materials during the activity by using models, schemes and symbols that were meaningful in students' lives. Thirdly, students were given the opportunity to produce and use new things in line with the "raising the level of the learning process" and "using their own constructions and productions" principles and students were given the opportunity to reflect their talents with the activities carried out in the classroom. Regarding the fourth principle, "interconnectedness principle" and "intertwined learning strands", attention was paid to presenting the subjects in a patterned structure like interconnected threads, instead of dealing with them separately. Fifth, according to the principle and characteristic of "interaction", the strategies used and/or emerged in both the activity and the problem-solving processes were discussed between the students and the teacher. Finally, considering the "guidance principle", the ground was prepared for the students to develop their own strategies and come up with new ideas under the guidance of the teacher.

First, the experimental and control group students were given AATTFO as a pretest in one class hour. Then, the control group students were taught the four basic operations with the traditional teaching method, while the experimental group students were taught with the RME approach. In this process, for the control group; two-course hours were allocated for each of the topics of addition and subtraction and multiplication and division operations were taught during three course hours each, followed by a general review during four class hours. After lecture and general review, AATTFO was administered again as a posttest in one class hour. The process completed in 16 course hours: 14 hours of teaching and general review and two hours of pretest and posttest. The 7-week experimental process included four weeks teaching (two hours a week), a week of review (four hours a week) and pre and posttests (one hour each). However, since the curriculum in this study was considered in the presentation of the topics and the purpose of the study was to investigate how students pose and solve problems with the four basic operations, the acquisitions were not given sequentially in this study. Still, the other acquisitions related to the four operations specified in the program were also taught. In this context, the process, which started in the second week of November, continued for 17 weeks and was completed in the last week of February (including one week of semester break during fall and two weeks of regular semester break). Table 2 presents the detailed representation of the experimental process.

Table 2.

Detailed Presentation of the Experimental Process

Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Topic	PreT	Ad	Fb	Da	Su	Da	Da	Mu	Da	Da	Di	Sb	Sb	Da	Da	Gr	PosT

PreT: Pretest; PosT: Posttest; Fb: Fall break; Sb: Semester break; Da: Different learning goals of basic mathematical operations; Gr: General Review; Ad: Addition; Su: Subtraction; Mu: Multiplication; Di: Division.

During data analysis, first of all, the data obtained from the 12 open-ended items in the AATTFO were scored independently by two mathematics educators, taking into account the scoring key in the study of Aytekin-Uskun et al. (2020). In addition, the answers given by the participants to the open-ended questions were analyzed by content analysis to describe the change in students' problem posing and problem-solving skills in regards to the four operations.

The data obtained this way were transferred to a statistical program used in social sciences and the agreement between the scores given by two independent raters was calculated with the weighted kappa (Moskal & Leydens, 2000). Weighted kappa, which is a type of kappa statistics, is a method used to calculate the agreement between two raters in multi-rating rubrics (Şencan, 2005). Kappa statistic provides a value between -1 and +1 and a value of at least .60 is recommended. Values between 60 and 80 indicate substantial inter-rater agreement, while values above .80 indicate a perfect agreement (Fleiss, 1971; Wood, 2007). In this context, the obtained inter-rater agreement was perfect for open-ended items 6, 11 and 16, and substantial for the other open-ended items (Table 3).

Table 3.

Inter-rater Agreement Values

	1	2	4	6	9	10	11	12	15	16	19	20
κ	.75	.64	.65	.86	.65	.61	.94	.64	.75	.82	.64	.63

After calculating inter-rater agreement, the raters met again on the items where there was no agreement and reached consensus on those items. Thus, 100% agreement was achieved. Next, the homogeneity of variances and normal distribution were tested for the experimental and control group data for the 20 test items. Table 4 shows the descriptive statistical results of the distribution.

Table 4.

Descriptive Statistics Results of the Distribution

Group	Test	Mode	Median	$\bar{X}$	Sd	Skewness	Kurtosis	Min	Max	Kolm. Smir.
Experi-	Pre	39	39	45.26	22.75	.57	-.74	13	95	.18
mental	Post	37	57	61.29	20.67	.23	-1.31	30	97	.14
Control	Pre	39	57	54.69	17.53	-.34	-.24	17	89	.14
	Post	44	53	58.63	16.17	.30	-.97	31	89	.15

Table 4 shows that the descriptive statistics values for the distribution such as arithmetic mean, mode and median were close to each other. The skewness coefficients of the distribution were .57 for the experimental group pretest; .23 for the experimental group posttest; -.34 for the control group pretest and.30 for the control group posttest. The kurtosis coefficients were -.74 for the experimental group pretest; -1.31 for the experimental group posttest; -.24 for the control group pretest and -.97 for the control group posttest, and it was determined that these values did not differ significantly from the range of -1 to +1 (Morgan et al., 2004). The histogram, box plots and Q-Q graphs showed that the data showed normal distribution. On the other hand, according to the Kolmogorov-Smirnov test result, p-value was $p<.05$ and it was determined that the normality condition was not met. However, since it is necessary to evaluate the Kolmogorov-Smirnov test results and descriptive and graphical methods together when examining the normality of the distribution (Abbott, 2011; Gnanadesikan, 1997; McKillup, 2012; Stevens, 2009), it was concluded that the distribution of data for both groups was normal in this study.

It can be argued that the sample sizes in the groups did not affect the parametric test assumption, since the score distributions in the study met the parametric test assumptions and the score distribution showed a normal distribution. In this context, the paired sample t-test was utilized to examine the difference between the experimental and control group students' pretest and the posttest score. The difference between the experimental and control group students' pretest scores was examined with the independent sample t-test. Analysis of covariance (ANCOVA) was used to determine whether there was a statistically significant difference between the posttest achievement scores of the students in the experimental and control groups. By using ANCOVA, the effect of pretest scores on posttest scores was aimed to be eliminated. First of all, whether the ANCOVA assumptions were met was checked and the normality of the distribution, the linearity of the relationship between the pretest and posttest scores, the equality of the within-group regression curves and the homogeneity of the variances were examined. In this study, the significance level was accepted as .05 and the effect size was calculated to test the significance of the changes after the implementation. Significant differences between the mean scores do not always guarantee the difference that exists in practice. Therefore, effect size statistics should be used in the interpretation of the results of tests based on the comparison of mean scores. Regardless of its sign, the Cohen d value used as the effect size is interpreted as small, medium and large effect sizes for .2, .5 and .8 respectively; The η^2 value is interpreted as small effect size in the range of $.01<\eta^2<.06$, medium in the range of $.06<\eta^2<.14$ and large in the range of $.14<\eta^2$ (Cohen, 1988).

Findings

The Effect of RME Approach on Achievement in Regards to Problem Posing and Problem-solving

Before starting the teaching process, whether there was a statistically significant difference between the achievements of the students in the experimental and control groups was investigated in regards to problems with four operations. Analysis of students' achievement levels o based on their scores provided the following: $0 \le$ score ≤ 20: Very low, $20 <$ score ≤ 40: Low, $40 <$ score ≤ 60: Moderate, $60 <$ score ≤ 80: High, $80 <$ score ≤ 100: Very high. Table 5 presents the obtained findings.

Table 5.

Independent Sample t-Test Results Regarding the Difference Between Experimental and Control Group Students' Pretest Scores

Achievement Test	Group	n	$\bar{X}$	Sd	t	df
Pretest	Experimental	35	45.26	22.76	-1.94	68
	Control	35	54.69	17.53		

* p<.05

According to Table 5, there was no statistically significant difference between the control and experimental groups' pretest scores and both student groups were moderately successful ($\bar{X}_{\text{Experimental}=45.26}$; $\bar{X}_{\text{control}=54.69}$; t=-1.94; p>.05). Experimental group students were taught four operations (addition, subtraction, multiplication and division) with a teaching method based on the RME approach while the control group students were taught the same topics with the traditional teaching method and then AATTFO was re-administered as a posttest. The paired sample t-test was performed to determine whether there was a statistically significant difference between the pretest-posttest achievement scores of the experimental and control group students and the results are presented in Table 6.

Table 6.

Paired Sample t-Test Results for the Difference between Pretest-Posttest Achievement Scores of Experimental and Control Group Students

Achievement Test	Test	n	$\bar{X}$	Sd	t	df	d
Experimental Group	Pretest	35	45,26	22,75	-7,982	34	-1,90*
	Posttest	35	61,29	20,67			
Control Group	Pretest	35	54,69	17,53	-1,443	34	-0,34
	Posttest	35	58,63	16,17			

* p<.05

According to Table 6, there was no statistically significant difference between the pretest and posttest scores of the control group students regarding their problem-solving achievement for the four operations ($\bar{X}_{\text{pretest}} = 54.69$; $\bar{X}_{\text{posttest}} = 58.63$; $p > .05$) while there was a significant difference in favor of the experimental group students' post-test scores ($\bar{X}_{\text{pretest}} = 45.26$; $\bar{X}_{\text{posttest}} = 61.29$; $p < .05$). The teaching method based on the RME approach was observed to have positive effects on student achievement and increased moderate achievement level to high-achievement level. The effect size was calculated as -1.90 and it was very large. The independent sample t-test was used to examine whether the difference between the pretest and posttest scores of the students was statistically significant between the groups and the results are presented in Table 7.

Table 7.

Independent Sample T-Test Results Regarding the Difference between Experimental and Control Group Students' Pretest-PostTest Scores

Differences in PostTest-Pretest Scores		n	$\bar{X}$	Sd	t	df	d
Achievement Test	Experimental	35	16,02	11,88	3.56	68	.85*
	Control	35	3,94	16,16			

* p<.05

According to Table 7, the difference between experimental and control group students' pretest and posttest scores regarding their problem-solving achievement for four operations was statistically significant in favor of the experimental group students ($\bar{X}_{\text{Experimental}} = 16.02$; $\bar{X}_{\text{control}} = 3.94$; $p < .05$). When the effect sizes were examined, it was seen that the effect was at a large level (d=.85). ANCOVA was used to determine whether the difference between the posttest scores of the groups was statistically significant. First, it was checked whether the assumptions of ANCOVA were violated. As a result of the tests carried out in this context, it was determined that the groups had a normal distribution; that there was a linear relationship between pretest and posttest achievement scores (r=.70) and homogeneity of variances was met (F=2.55; p>.05). In addition, the difference between the slopes of the regression lines between the groups was examined with the "Group x Pretest" joint effect test and it was concluded that the difference was not statistically significant (F=3.23; p>.05). According to these results, it was determined that the difference between the posttest scores of the groups can be examined with ANCOVA and the data obtained as a result of the analyses are presented in Table 8, Table 9 and Table 10.

Table 8.

Groups' Actual PostTest Scores and Posttest Scores Adjusted according to Pretest Scores

chievement Test	Posttest			Adjusted Posttest	
	n	$\bar{X}$	Standard Error	$\bar{X}$	Standard Error
Experimental Group	35	61,29	2,14	64,94	3,58
Control Group	35	58,63	2,17	56,27	3,58

Table 9.

ANCOVA Results for Posttest Scores Adjusted for Groups' Pretest Scores

Variances	Sum of Squares	Sd	Mean Square	F	p	η^2
Pretest (Regression)	12727.89	1	12727.89	79.71	.00	.36
Groups (Posttest)	6698.27	1	6698.27	41.95	.00	
Error	10697.43	67	159.66			
Total Adjusted	275189.00	70				

According to the ANCOVA result, there was a significant difference between the posttest scores that were adjusted according to the academic achievement pretest scores (F= 41,953, p<.05). The effect size value for the difference (η^2) was calculated as .36 and it was seen that it had a large effect. Bonferroni comparison test results presented in Table 10 also demonstrated that the significant difference between groups' adjusted posttest scores was in favor of the experimental group (p<.05).

Table 10.

Bonferroni test results for groups' adjusted posttest scores

Groups	n	Difference between Means	Standard	p	Direction of the Difference
Experimental	35	9,007	3,103	,005	Experimental>Control

The Effect of RME Approach on Problem Posing and Problem-solving Skills

A statistically significant and positive increase was observed in experimental group students' academic achievement as a result of the activities conducted with the four basic operations prepared with the RME approach. This section aimed to describe the change in student achievement qualitatively and reflect the development of students' problem posing and problem-solving skills. The data obtained within the scope of the study describing the problem posing and problem-solving skills of the students were collected under the themes presented in Table 11.

Table 11.

Themes for Problem Posing and Problem-solving Skills

Problem Posing	Problem Solving
• Findings About Understanding the Root Cause of the Question	• Findings About Selecting the Appropriate Operation
• Findings Regarding Incorrect Data Use	• Findings Regarding Incorrect Data Use
• Findings Regarding Lack of Expression	

Findings Regarding the Development of Problem Posing Skills

It was observed that the students were insufficient to follow the instructions while solving the problem-posing questions in the Achievement Test, to pose problems using an understandable language and to accurately use the data presented to them before they were taught with the RME approach.

Findings about Understanding the Root Cause of the Question. As part of the Achievement Test, instructions were provided to the students to guide them in problem posing questions for the four operations. For example, in the bicycle theme, the students were presented with Tables showing the prices of the parts that will make up the bicycle and they were asked to create a question for addition using this data. When the data were analyzed, it was understood that the students' ability to follow the instructions presented to them while writing the questions improved visibly after the RME training. For example; while Uras wrote a problem before the RME training involving addition which required subtraction instead, he was able to use the data presented to him more effectively to pose a problem involving subtraction in accordance with the instructions after the training:

Table 12.

Problems Uras Posed for Subtraction

Before RME	After RME
Naci Abay Primary School has 712 boys and 20 more girls than the number of boys. How many students are in this school?	*Hello, I am Ahmet Bey, the principal of the school. This year, 40 girls from the 4th grades left our school. Hence, how many 4th grade girls are left in our school?*

In a problem posing question involving division on the theme of global warming, Esra preferred to express her opinion about pandas instead of posing a problem in accordance with the guidelines before the RME training, however, she tried to construct a problem statement after the training:

Table 13.

The Problems Posed by Esra Regarding Division

Before RME	After RME
I saw a panda today, it was so cute and I loved it.	*Pandas are endangered. There are 1280 pandas. There are currently 120 pandas left. 120 pandas divided into 4 islands?*

Similarly, while Toprak gave information about his own bike by ignoring the instructions given in the question before the RME training, he tried to form a problem statement about addition by following the instructions after the training:

Table 14.

The Problems Posed by Toprak Regarding Addition

Before RME	After RME
My bike has got a flat tire.	*My bike is ice blue, it has got a light and a bell. Their costs; ice blue is 138 TL, bell is 30 TL, light is 45 TL.*

In the problem posing question, Selinay made a mistake by writing a problem involving addition in the library theme which required subtraction instead before the RME training, but she was able to correct her mistake and write a problem containing subtraction in line with the instruction and outcome:

Table 15.

The Problems Posed by Selinay Regarding Subtraction

Before RME	After RME
In Naci Abay Primary School, the number of female students is 187 and the number of male students is 123 in the first grade. According to this, what is the sum of these?	*The number of boys and girls in the 4th grade of Naci Abay Primary School is provided above. The number of male and female students in the 2nd grades is also given. What is the difference in the number of female students in the 2nd and 4th grades?*

In regards to the problem posing question on the theme of global warming, Ayaz wrote a problem involving addition before the RME training and after the training he demonstrated improvement by posing a problem that included both subtraction and division in accordance with the directive:

Table 16.

The Problems Posed by Ayaz Regarding Division

Before RME	After RME
The temperature is 138 degrees on 5 week days. It is 10 degrees higher on the weekend. How many degrees is it on the weekend?	*It was 138 degrees in total for 5 days on weekdays. How many degrees was the temperature in one day?*

While Melike posed a problem requiring subtraction in the library theme by writing a problem involving addition before the RME training, she gave a full and correct answer by writing a question involving subtraction after the training:

Table 17.

The Problems Posed by Melike Regarding Subtraction

Before RME	After RME
If there are 109 stories, 198 fairy tales, 237 novels, 115 poems and 250 test books in a library, how many books are there in total?	*A teacher throws away 16 of 250 test books and distributes 35 of them. How many books are left?*

Findings Regarding Incorrect Data Use*.* The Achievement Test provided clues and data (number, table, operation, etc.) together with the problem posing statements and the students were asked to form problem statements in line with this information. It was observed that while the students ignored the data presented to them or used random data before the training given with the RME approach, they were able to form question sentences by using the data more effectively after the training. For example, it was found that Esra prepared a question by using her own data instead of selecting suitable data from the Table before the RME training, but she was able to pose and solve the problem correctly and in full using the provided data after the training:

Figure 1.

The Problems Posed by Esra Regarding Addition

Before RME

Now, you pose a problem that involves addition to customize your bike and calculate the result. Don't forget; you can make use of the accessories and colors in the catalog while writing your question.

Solution:

- Basic Bike Parts Catalogue
  Steering Wheel: 60
  Wheel: 60
  Bicycle Frame: 285
  Bicycle Seat/Saddle: 30
- Accessory Catalogue
  Bell:20
  Light:50
  Basket:60
  Flower: 20
  20+50+60+20=150
- Color Catalogue
  Rainbow: 300
  Ice blue: 100
  300+100=400

After RME

Now, you pose a problem that involves addition to customize your bike and calculate the result. Don't forget; you can make use of the accessories and colors in the catalog while writing your question.

Solution:

My dad bought me a bike as gift for my report card; it has been a year now so my bike is old. I got new parts. I got a steering wheel for 80 TL, wheels for 125 TL; a frame for 225 TL and a bike seat for 40 TL. How much did it cost? 225+125+80+40=465

In the following example, while Toprak preferred to use random data instead of selecting data from the Table, he created a problem by using the data presented to him after the training:

Figure 2.

The Problems Posed by Toprak Regarding Addition

Before RME

By using the visual provided above, pose a problem that involves addition and solve the problem:

Solution:

The distance between Aksaray and Ordu is 300 km; the distance between Ordu and Gaziantep is 570. The distance between Ordu and Niğde is 240. We can add these three.

300+570+240=100

After RME

By using the visual provided above, pose a problem that involves addition and solve the problem:

Solution:

My father first left Aksaray then drove 400 km. He arrived in Ordu. He left the stuff and left Ordu. He got some things from Ordu. He arrived in Niğde, he drove 250 km. He left the stuff in Niğde and got new materials from Niğde. He then arrived in Gaziantep, it was 580 km. He left the things and then he got back to Aksaray.

580+400+250=1230

In another example, Merve ignored the vegetable and fruit price table presented to her while posing a problem and created a question with random numbers. After the RME training, she was able to select the necessary data for problem-solving and improved her skills in reading Tables:

Table 18.

The Problems Posed by Merve Regarding Multiplication

Before RME	After RME
When I went to the grocery store today, I bought a banana for 11 TL, a salad for 4 TL and an orange for 12 TL. How much will I pay at the cash register?	*When my mother went to the grocery store, she bought 3 kg of bananas and 15 kg of oranges. So, how much did she pay?*

Examination of Şüheda's answer about posing a problem on the bicycle theme showed that she tried to pose a problem about the total number of crates instead of using the data about the distances presented to her. But she posed a problem based only on the unit of length after the RME training:

Table 19.

The Problems Posed by Şüheda Regarding Addition

Before RME	After RME
How much is it for 50 crates from Aksaray to Ordu?	*Ms. Ayşe lives in Aksaray. Ayşe's sister lives in Niğde. How many kilometers does it take Ms. Ayşe to go to Niğde?*

Before the RME training, Talha wrote a question about the total price of the books, although there was no information about the prices of the books in the data presented to him, but after the training, he formed a correct and meaningful problem statement in accordance with the data:

Table 20.

The Problems Posed by Talha Regarding Subtraction

Before RME	After RME
In our school, teachers bought 109 storybooks and 250 test books for the library. According to this, how many liras did the teachers pay?	*What is the difference in number between the storybooks and novels in Naci Abay Primary School library?*

Findings Regarding Lack of Expression. Problem statements should be constructed in a manner to express a single judgment in a clear and understandable manner without creating any confusion and to bring everyone to the same truth. It was found that students formed more understandable problem statements with the help of the RME education. For example, while Songül could not accurately integrate information about the price for the bike bell, light and basket in the question statement before the training, she posed a problem using a clear and understandable language after the RME training:

Table 21.

The Problems Posed by Talha Regarding Subtraction

Before RME	After RME
They bought me a bike. Bell 300, lamp 450, basket 50. How much was it paid?	*My sister Sule got a basket and a bell for her bell as accessories. She had her bike painted in rainbow colors. How much will she pay?*

While it was impossible to solve the problem based on the problem statement that Enes wrote before the RME training, it was observed that he posed a problem that was clear and had a solution after the training:

Table 22.

The Problems Posed by Enes Regarding Subtraction

Before RME	After RME
Our teacher wants to take the students to the library. There are 237 novels, 198 storybooks and 115 poetry books in the library. There are 250 testbooks as well. What is the difference among these books?	*There are 237 novels in Atatürk Primary School Library. 50 of the novels were checked out. How many books are in the library now?*

Similarly, while Miraç was unsuccessful in forming a question sentence before the RME training, he posed a clear and understandable problem after the training:

Table 23.

The Problems Posed by Miraç Regarding Subtraction

Before RME	After RME
Teacher Ali wants to match the storybooks and test books in the library, but finds the difference between these two types of books.	*There are 237 novels and 198 storybooks in a library. There are as many poetry books as the difference between these books. How many poetry books are there?*

In another example, İlyas wrote a sentence with no root cause, with a lack of expresion and elements before the RME training, but he formed a correct question sentence after the RME training using a clear and understandable language:

Table 24.

The Problems Posed by İlyas Regarding Addition

Before RME	After RME
Steering wheel, seat, basket, wheel.	*I use the following when designing my own bike; Steering wheel 80 TL, wheels 120 TL, seat (saddle) 40 TL, bell 30 TL, basket 50 TL, ice blue paint 138 TL. How much would the bike I want cost?*

Findings Regarding the Development of Problem-solving Skills

Students' solutions to open-ended questions in the Achievement Test were examined and the change in students' problem-solving skills was described in this section. A noticeable improvement was observed in students' ability to select the appropriate operation and use the data in the appropriate manner after the RME training.

Findings about Selecting the Appropriate Operation. In the Achievement Test, students were not only asked to pose a problem, but also to solve the problems they posed. Before the training, some students were able to find a suitable solution to the problems they posed, while others showed limited success. After the RME training, it was identified that the students improved especially in understanding the problem and choosing the appropriate operation for the problem. This development is presented below with the some examples. For example, since Aycan could not fully understand the question before the RME training, she added 95 to 125 in the problem she posed, making an operation that was not called for and was not able to find a complete solution. However she was able to provide a complete and correct answer to the problem she posed after the training:

Figure 3.

Aycan's Problems Solutions Regarding Addition

Before RME

Now, you pose a problem that involves addition to customize your bike and calculate the result. Don't forget; you can make use of the accessories and colors in the catalog while writing your question.
Solution:
In the bike shop, bell is 30 TL, light is 45 TL and basket is 50 TL. How much should I pay?
50+45=95
95+30=125
125+95=220

After RME

Now, you pose a problem that involves addition to customize your bike and calculate the result. Don't forget; you can make use of the accessories and colors in the catalog while writing your question.
Solution:
How much did I pay for my bike when I designed it from the accessory catalogue?
50+45+30=125

In another example, Mücahit demonstrated limited ability in selecting the correct operation and he performing addition where he had to use multiplication to find the total cost. After the RME training, he demonstrated that he could select the appropriate operation for the question by performing the necessary multiplication operations:

Figure 4.

Mücahit's Problems Solutions Regarding Multiplication

Before RME

Your mother will make orange jam and pickled cucumbers. She wanted you to buy 14 kg of oranges and 13 kg of cucumbers. How much do you need to pay the greengrocer?
Solution:
250+14+13=277

After RME

Your mother will make orange jam and pickled cucumbers. She wanted you to buy 14 kg of oranges and 13 kg of cucumbers. How much do you need to pay the greengrocer?
Solution:
14x12=168
13x6=68
168+68=236 TL

In a similar figure, Selçuk overlooked the need to perform multiplication before the RME training and could not select the appropriate operation for the problem, but he was able to perform the operations required by the problem text even if he made a mistake in the operation after the training:

Figure 5.

Selçuk's Problems Solutions Regarding Multiplication

Before RME

Your mother will make orange jam and pickled cucumbers. She wanted you to buy 14 kg of oranges and 13 kg of cucumbers. How much do you need to pay the greengrocer?
Solution:
12+14=26
6+13=29
26+19=45

After RME

Your mother will make orange jam and pickled cucumbers. She wanted you to buy 14 kg of oranges and 13 kg of cucumbers. How much do you need to pay the greengrocer?
Solution:
14x12=168
13x6=78
168+78=236
250-236=14

Esra, on the other hand, made a mistake by selecting multiplication and addition operations when she should have solved the problem with subtraction in the library theme. After RME training, she reached the complete and correct solution by performing addition and subtraction:

Figure 6.

Esra's Problem Solutions Regarding Addition and Subtraction

Before RME

In our school, a group of students wanted to organize a campaign to raise awareness about the need for traveling libraries/bookmobiles. The number of books that students targeted to collect was 958. They already had 102 books. They were able to gather 364 books with the campaign, how many books are needed to reach the target?

Solution:

32+5=37

958+120+364=1442

1442x37=5214

After RME

In our school, a group of students wanted to organize a campaign to raise awareness about the need for traveling libraries/bookmobiles. The number of books that students targeted to collect was 958. They already had 102 books. They were able to gather 364 books with the campaign, how many books are needed to reach the target?

Solution:

364+102=466

958-466=492 books left

In the following example, Sefer could not select the right operations, but after the RME training, he developed the ability to select the appropriate operations, even though he performed an extra operation:

Figure 7.

Sefer's Problem Solutions Regarding Subtraction

Before RME

Your mother will make orange jam and pickled cucumbers. She wanted you to buy 14 kg of oranges and 13 kg of cucumbers. How much do you need to pay the greengrocer?

Solution:

14+6=624

12+13=1326

After RME

Your mother will make orange jam and pickled cucumbers. She wanted you to buy 14 kg of oranges and 13 kg of cucumbers. How much do you need to pay the greengrocer?

Solution:

14x12=168 oranges

13x6=78

168+78=246

250-246=004

While İpek was unable to select the appropriate operation for the text before the RME training, she understood the text correctly after the training and reached the complete and correct result by choosing division in solving the questions:

Figure 8.

İpek's Problem Solutions Regarding Division

Before RME

Due to climate change, the melting rates of the ice masses in the poles have increased 7 times compared to the 1900s. Today, approximately 455 kilograms of ice mass melt in 1 minute. How many kilograms of ice mass melted in the same amount of time in 1900s?
Solution:
455x77=3185

After RME

Due to climate change, the melting rates of the ice masses in the poles have increased 7 times compared to the 1900s. Today, approximately 455 kilograms of ice mass melt in 1 minute. How many kilograms of ice mass melted in the same amount of time in 1900s?
Solution:
455:7=65

Findings Regarding Incorrect Data Use. Examination of the data obtained within the scope of this study showed that the students experienced difficulties in selecting the appropriate data while solving problems as they had while posing problems. For example, before the RME training, İlayda could not comprehend the basic parts because she had limited ability to read and interpret tables, and therefore answered the question incorrectly. After the training, she selected the appropriate data from the table and solved the question in a correct manner:

Figure 9.

İlayda's Problem Solutions Regarding Addition

Before RME

Osman bought only the basic parts while he was designing his own bike. How much did Osman pay for his bike?
Solution:
371+239+225+138+120+50+80+40+20+45=1338

After RME

Osman bought only the basic parts while he was designing his own bike. How much did Osman pay for his bike?
Solution:
225+120+80+40=465

Merve used the wrong data and could not find the correct answer because she could not understand the problem statement before the RME training, but she solved the question using the appropriate data after the training and got a full score.

Figure 10.

Merve's Problem Solutions Regarding Addition and Subtraction

Before RME

In our school, a group of students wanted to organize a campaign to raise awareness about the need for traveling libraries/bookmobiles. The number of books that students targeted to collect was 958. They already had 102 books. They were able to gather 364 books with the campaign, how many books are needed to reach the target?

Solution:

958+102=1060

1060-364=0696

After RME

In our school, a group of students wanted to organize a campaign to raise awareness about the need for traveling libraries/bookmobiles. The number of books that students targeted to collect was 958. They already had 102 books. They were able to gather 364 books with the campaign, how many books are needed to reach the target?

Solution:

364+102=466

958-466=492 books left

In another example, while Ömer did not use the data presented in the catalog before the RME training, he obtained the right result by selecting the appropriate data after the training:

Figure 11.

Ömer's Problem Solutions Regarding Addition

Before RME

Osman bought only the basic parts while he was designing his own bike. How much did Osman pay for his bike?

Solution:

371+45=416

416+225=641

After RME

Osman bought only the basic parts while he was designing his own bike. How much did Osman pay for his bike?

Solution:

225+120+80+40=465

In the following example, Toprak could not understand the table presented to him and could not choose the appropriate data for the problem, but he was able to perform the operation with the correct data after the RME training and reached the correct result:

Figure 12.

Toprak's Problem Solutions Regarding Addition

Before RME

Osman bought only the basic parts while he was designing his own bike. How much did Osman pay for his bike?

Solution:

120+225=345

138+239=376

After RME

Osman bought only the basic parts while he was designing his own bike. How much did Osman pay for his bike?

Solution:

225+120+80+40=465

Conclusion, Discussion and Suggestions

Examination of the studies in the field of mathematics education shows that the number of studies investigating the effect of the RME approach on the teaching process of the four operations is limited. This study explored the effect of the educational activities based on the RME approach on fourth grade students' problem posing and problem-solving skills concerning four basic mathematical operations.

This study found that using the RME approach in teaching four operations at primary school fourth grade level produced more effective results on student achievement compared to the traditional teaching method. It was identified that the RME approach significantly increased students' academic achievement in problem posing and problem-solving skills in addition, subtraction, multiplication and division. Other studies in the literature also reported that teaching within the framework of the RME approach produced more effective results on student achievement than teaching via only the traditional method (Webb et al., 2011; Okuyucu & Bilgin, 2019; Ünal & İpek, 2009). The opportunity provided to students to develop their own strategies and make new discoveries during the RME process allows them to be more creative in the problem-solving process (Olkun & Toluk-Uçar, 2007; Wubbels et al., 1997; Kalaw, 2012). The results of the RME based approach training which was implemented within the scope of this study demonstrated that the students understood the problem statements more clearly, they were able to select the necessary data in a more effective manner while solving and posing problems and they were able to create more meaningful problem statements while posing problems. In this context, the reasons why the educational activities prepared with the RME approach have more positive effects on student achievement compared to traditional approaches may be related to the fact that the trainings prepared with the RME approach present problems in relation to real life and provide the students with opportunities to easily associate various concepts with each other and to develop various strategies in the problem-solving process.

As noted in the literature, educational processes prepared with RME not only positively affect academic achievement, but also support students in developing mathematical skills. For example, the study conducted by Noviani et al. (2017) reported that the RME based training has a positive effect on the development of students' spatial skills. Within the scope of this study, comparison of the answers provided by the students to the open-ended questions showed that the students participating in the study improved their ability to select the appropriate data while posing and solving questions, they perceived the question root in a more accurate manner and they succeeded in creating more meaningful and purposeful expressions while posing problems. In other words, the students participating in the study not only increased their academic achievement in mathematics, but also improved their mathematical literacy, problem solving and problem posing skills with the RME training provided in this study. With the 2023 Education Vision, the Ministry of National Education announced that the national exams in Turkey will focus on measuring skills as reasoning, critical thinking and interpretation rather than directly measuring knowledge in regards to school subjects (MEB, 2018b) by signaling the transition from a content-based education to a skill-based education. Hence, the need to implement innovative approaches such as RME, which can be organized by focusing on skill-based education instead of the traditional methods currently used in our schools, is evident to achieve this goal and to increase achievement in national and international exams which consist of skill-based questions.

Providing opportunities for students to use their mathematical knowledge and skills in a more effective manner to solve problems encountered in daily life is another positive effect of the RME approach on student development. Within the scope of this study, students who were taught with the RME approach not only tried to find solutions to mathematical questions related to daily life, but also posed problems for potential problems they may encounter in life. For example, they associated daily life with - problem-solving and problem-solving skills related to four basic operations by writing questions in which the cost of a bicycle can be calculated according to their needs or the total distance traveled during a journey. MoNE defines mathematical competence as "the development and application of mathematical thinking style to solve a series of problems encountered in daily life" (MEB, 2018b) and lists the specific goals of mathematics education as “understanding and using mathematical concepts in daily life” (MEB, 2018a) and " making sense of the relations between people and objects and the relations of objects with each other by using the meaning and language of mathematics" (MEB, 2018a). Considering these statements, it is evident that efforts should be made to popularize the use of educational approaches such as RME that offer the opportunity to transfer mathematical knowledge and skills to daily life. As a result, the effective use of the RME approach will provide opportunities for associating mathematical knowledge with daily life skills and will help increase student achievement in national and international exams.

Examination of participating students’ problem-solving and problem posing skills at fourth-grade level points to rather significant limitations; especially in fourth grade students’ problem posing skills. As a matter of fact, acquisitions regarding problem-solving and problem posing are also included in the lower grades, which makes these limitations even more remarkable. The pretest data of the students participating in the study demonstrated that they experienced difficulty in writing meaningful question statements, they mostly left the problem posing questions unanswered and they even preferred to perform operations with the presented data instead of writing a problem statement in problem posing questions. In the light of these data, it is identified that the education offered at previous grade levels for the problem posing process was not sufficiently comprehended by the students. In a similar manner, it is observed that students had limited knowledge about the basic features of a problem. Comparison of student answers in the pretest and posttest clearly demonstrates the development in their problem posing skills. Students' failures in the pretest are believed to be related to the very limited education they received for problem posing, especially in the lower grades. Kuzu et al. (2019) cautioned that when more than one educational outcome or skill is taught within the same acquisition, it becomes more difficult to express the educational concept in a clear manner. Considering that the skills related to posing problems are presented as a sub-acquisition within the problem-solving outcome until the fourth grade, it becomes clear why the training was limited for the participating students until now. Examining primary school students' problem posing and problem-solving skills in detail in further studies will contribute to describe the effect of providing these skills in a single outcome on students' skill acquisition.

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