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A GENERALIZATION OF REDUCED RINGS

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Abstract

Let R be a ring with identity. We introduce a class of rings which is a generalization of reduced rings. A ring R is called *central rigid* if for any a, $b \in R$, $a^2b = 0$ implies ab belongs to the center of R. Since every reduced ring is central rigid, we study sufficient conditions for central rigid rings to be reduced. We prove that some results of reduced rings can be extended to central rigid rings for this general setting, in particular, it is shown that every reduced ring is central rigid, every central rigid ring is central reversible, central semicommutative, 2-primal, abelian and so directly finite.

Keywords: Reduced rings, Central rigid rings, Central reversible rings, Central semicommutative rings, Abelian rings.

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1. Introduction

Throughout this paper all rings are associative with identity unless otherwise stated. A ring is *reduced* if it has no nonzero nilpotent elements. Recently the reduced ring concept was extended to R-modules by Lee and Zhou in [13], that is, an R-module M is called *reduced* if, for any $m \in M$ and any $a \in R$, $ma = 0$ implies $mR \cap Ma = 0$. According to Cohn [9] a ring R is called *reversible* if for any a, $b \in R$, $ab = 0$ implies $ba = 0$. A ring R is called *central reversible* if for any a, $b \in R$, $ab = 0$ implies ba belongs to the center of R. A ring R is called *semicommutative* if for any $a, b \in R$, $ab = 0$ implies $aRb = 0$, while the ring R is said to be *central semicommutative* [3] if for any $a, b \in R$, $ab = 0$ implies arb is a central element of R for each $r \in R$. A ring R is called *right* (*left*) *principally quasi-Baer* [8] if the right (left) annihilator of a principal right ideal of R is generated by an idempotent. Finally, a ring R is called *right* (*left*) *principally projective* if the right (left) annihilator of an element of R is generated by an idempotent [7]. For a positive integer n, $\mathbb{Z}_n$ denotes the ring of integers modulo n. We write $R[x]$ and $R[x, x^{-1}]$ for the polynomial ring and the Laurent polynomial ring, respectively.

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In this paper we introduce and study a class of rings, called central rigid rings, which is a generalization of reduced rings. We prove that some results of reduced rings can be extended to central rigid rings for this general setting. We supply some examples to show that all central rigid rings need not be reduced. We show that the class of central rigid rings lies strictly between classes of reduced rings and central reversible rings. Among others we prove that if R is a right principally projective ring or a semiprime ring, then R is reduced if and only if R is central rigid if and only if R is reversible if and only if R is central reversible if and only if R is semicommutative if and only if R is central semicommutative if and only if R is abelian. We also prove that a ring R is central rigid if and only if the Dorroh extension of R is central rigid. Moreover, it is proven that if R is a right principally projective ring, then R is central rigid if and only if $R[x]/(x^n)$ is central Armendariz, where $n \geq 2$ is a natural number and (x^n) is the ideal generated by x^n. Finally, it is shown that the polynomial ring $R[x]$ is central rigid if and only if the Laurent polynomial ring $R[x, x^{-1}]$ is central rigid.

2. Central rigid rings

Let α be a homomorphism of a ring R. A ring R is called α-rigid if $a\alpha(a) = 0$ implies $a = 0$ for any $a \in R$ (see [11]). Regarding a generalization of α-rigid rings as well as a reduced module, recall that a module M is called *α-rigid* [1] if $ma\alpha(a) = 0$ implies $ma = 0$ for any $m \in M$ and $a \in R$. Hence M is *rigid* if, for any $m \in M$ and $a \in R$, $ma^2 = 0$ implies $ma = 0$. It is easy to show that if M is a reduced module, then it is rigid. For rings, R is said to be *rigid* if for any a, $b \in R$, $a^2b = 0$ implies $ab = 0$. Then we have the following.

2.1. Proposition. *Let R be a ring. Then the following are equivalent.*

(1) *R is a reduced ring.*
(2) *R_R is a reduced module.*
(3) *R_R is a rigid module.*
(4) *R is a rigid ring.*

Proof. Clear by the definitions. □

We now define central rigid rings as a generalization of reduced rings.

2.2. Definition. A ring R is called *central rigid* if for any a, $b \in R$, $a^2b = 0$ implies ab is central.

It is clear that commutative rings and reduced rings are central rigid. For the central case Proposition 2.1 is not true in general as the following example shows.

2.3. Example. Consider the ring $R = \mathbb{Z}_3 \oplus \mathbb{Z}_3$, where multiplication is defined by $(a, b) * (c, d) = (ac, ad + bc)$ and addition is componentwise. The ring R is commutative and has an identity $(1, 0)$. Since R is commutative, it is central rigid. But $(0, 1)$ is a nonzero nilpotent element in R and so R is not reduced.

Recall that a ring R is *semiprime* if $aRa = 0$ implies $a = 0$ for $a \in R$. Our next endeavor is to find conditions under which a central rigid ring is reduced.

2.4. Proposition. *If R is a reduced ring, then R is central rigid. The converse holds if R satisfies any of the following conditions.*

(1) *R is a semiprime ring.*
(2) *R is a right (left) principally projective ring.*
(3) *R is a right (left) principally quasi-Baer ring.*

Proof. The first statement is clear. Conversely,

(1) Let R be a semiprime ring and $x \in R$ with $x^2 = 0$. By hypothesis, x is central. We have $xRx = 0$. Since R is semiprime, we have $x = 0$. Thus R is reduced.

(2) Let $x \in R$ with $x^2 = 0$. By hypothesis, x is central. Since R is a right principally projective ring, there exists an idempotent $e \in R$ such that $x \in r_R(x) = eR$. It follows that $x = ex = xe = 0$. Thus R is reduced.

(3) Similar to the proof of (2). □

2.5. Corollary. *If R is a central rigid ring, then the following conditions are equivalent.*

(1) *R is a right principally projective ring.*
(2) *R is a left principally projective ring.*
(3) *R is a right principally quasi-Baer ring.*
(4) *R is a left principally quasi-Baer ring.*

Proof. This follows from Proposition 2.4 since in either case R is reduced. □

Next we prove that central rigid rings are closed under finite direct sums.

2.6. Proposition. *Let $\{R_i\}_{i\in I}$ be a class of rings for a finite index set I. Then R_i is central rigid for all $i \in I$ if and only if $\bigoplus_{i\in I} R_i$ is central rigid.*

Proof. Clear from the definitions. □

The following result is a direct consequence of Proposition 2.6.

2.7. Corollary. *Let R be a ring. Then eR and $(1-e)R$ are central rigid for some idempotent e in R if and only if R is central rigid.* □

The next example shows that for a ring R and an ideal I, if R/I is central rigid, then R need not be central rigid.

2.8. Example. Let $R = \begin{bmatrix} F & F \\ 0 & F \end{bmatrix}$, where F is any field. Notice

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}^2 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

but for $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \in R$ we have,

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \neq \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

So R is not central rigid.

Consider the ideal $I = \begin{bmatrix} F & F \\ 0 & 0 \end{bmatrix}$ of R. Hence R/I is central rigid because of $R/I \cong F$.

2.9. Lemma. *Let R be a ring. If R/I is a central rigid ring with a reduced ideal I, then R is central rigid.*

Proof. If $a, b \in R$ with $a^2b = 0$, then $(a+I)^2(b+I) = 0 + I$. Since R/I is a central rigid ring, $arb - rab \in I$ for all $r \in R$, and so $aba \in I$. On the other hand $(aba)^2 = 0$ and the reducibility of I implies that $aba = 0$. Then $abra \in I$ for all $r \in R$. Hence $abra = 0$, due to $(abra)^2 = 0$ for all $r \in R$. Thus $(abr - rab)^2 = 0$ and by hypothesis $abr = rab$, for all $r \in R$. This completes the proof. □

Note that the homomorphic image of a central rigid ring need not be central rigid. Consider the following example.

2.10. Example. Let D be a division ring, $R = D[x, y]$ and $I = \langle y^2 \rangle$ where $xy \neq yx$. Since R is a domain, R is central rigid. On the other hand, $(yxy + I)^2(x + I) = I$ but $(yxy + I)(x + I)$ does not commute with $y + I$. Hence R/I is not central rigid.

It is well known that a ring is a domain if and only if it is prime and reduced. In addition to this fact, we have the following proposition when we deal with the central case.

2.11. Lemma. *Let R be a ring. Then R is a prime and central rigid if and only if it is a domain.*

Proof. Let $a, b \in R$ with $ab = 0$. Then $(ba)^2 = 0$ and so ba is central. Thus $(arb)^2 = 0$ for all $r \in R$. We have arb is central for all $r \in R$. Hence we get $(arb)R(arb) = R(arb)^2 = 0$. Since R is prime, $a = 0$ or $b = 0$. The rest is clear. $\square$

Recall that a ring R is called *weakly semicommutative* [14], if for any $a, b \in R$, $ab = 0$ implies arb is a nilpotent element for each $r \in R$. It is well known that every reduced ring is semicommutative. For weakly semicommutative rings we have the following.

2.12. Lemma. *If R is central rigid, then R is weakly semicommutative.*

Proof. Let $ab = 0$ with $a, b \in R$. Since R is central rigid, we have ba is central. It follows that $(arb)^2 = 0$. That is R is weakly semicommutative. $\square$

The following example shows that there is a weakly semicommutative ring which is not central rigid.

2.13. Example. Let S be a division ring and consider the ring

$$R = \left\{ \begin{bmatrix} a & b & c \\ 0 & a & d \\ 0 & 0 & a \end{bmatrix} \mid a, b, c, d \in S \right\}$$

Then R is weakly semicommutative. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, then $A^2 = 0$. If $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, then $BA = 0$, but $AB = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \neq 0$. So, R is not central rigid.

2.14. Lemma. *Let R be a central rigid ring. Then R is abelian.*

Proof. Let $e^2 = e \in R$. For any $r \in R$, $(re - ere)^2 = 0$ and so $re - ere$ is central. Commuting $re - ere$ by e we have $ere = re$. Similarly for any $r \in R$, $(er - ere)^2 = 0$ implies $ere = er$. Thus R is abelian. $\square$

The converse of Lemma 2.14 is not true in general, that is, every abelian ring need not be central rigid, as the following example shows.

2.15. Example. Consider the ring

$$R = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a \equiv d \ (mod\ 2),\ b \equiv c \equiv 0 \ (mod\ 2) \right\}$$

Since $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are the only idempotents of R, R is abelian. On the other hand, $\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ but $\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$ is not central. Hence R is not central rigid.

In the sequel, we give relations between reduced, central rigid, reversible, central reversible, semicommutative, central semicommutative and abelian rings by using right principally projective rings and semiprime rings.

2.16. Theorem. *Let R be a right principally projective ring. Then the following are equivalent.*

(1) *R is reduced.*
(2) *R is central rigid.*
(3) *R is reversible.*
(4) *R is central reversible.*
(5) *R is semicommutative.*
(6) *R is central semicommutative.*
(7) *R is abelian.*

Proof. Note first that xR is a projective right ideal for any $x \in R$. The isomorphism $xR \cong R/r_R(x)$ implies $r_R(x)$ is a direct summand of R so that there exists an idempotent $e^2 = e \in R$ such that $r_R(x) = eR$. Also if R is a right principally projective ring, then every idempotent is central.

(3) $\Longrightarrow$ (4), (5) $\Longrightarrow$ (6), (6) $\Longrightarrow$ (7) and (7) $\Longrightarrow$ (1) Clear.

(1) $\Longleftrightarrow$ (2) Proposition 2.4.

(2) $\Longrightarrow$ (3) Let $x, y \in R$ with $xy = 0$. Then $y \in r_R(x) = eR$ for some $e^2 = e \in R$. So $y = ey$ and $xe = 0$. On the other hand $(yx)^2 = 0$ and R is central rigid, we have yx is central. So $yx = eyx = yxe = 0$. Thus R is reversible.

(4) $\Longrightarrow$ (5) Let $x, y \in R$ with $xy = 0$. Then $y \in r_R(x) = eR$ for some $e^2 = e \in R$. So $y = ey$ and $xe = 0$. Hence $xry = xr(ey) = xery = 0$ for all $r \in R$ and so (5) holds. □

2.17. Theorem. *Let R be a semiprime ring. Then the following are equivalent.*

(1) *R is reduced.*
(2) *R is central rigid.*
(3) *R is reversible.*
(4) *R is central reversible.*
(5) *R is semicommutative.*
(6) *R is central semicommutative.*

Proof. (1) $\Longleftrightarrow$ (2) Proposition 2.4.

(2) $\Longrightarrow$ (4) Let $a, b \in R$ with $ab = 0$. Then $(ba)^2 = baba = 0$ and so ba is central.

(4) $\Longrightarrow$ (2) Suppose now R is a central reversible and semiprime ring. Let $a, b \in R$ with $a^2b = 0$. Since R is central reversible, aba is central. Hence we have $(aba)R(aba) = 0$ and by assumption $aRab$ is in the center of R. Thus $(abrab)R(abrab)$=0 for all $r \in R$. Since R is semiprime, we have $abRab = 0$ and so ab is central.

(2) $\Longrightarrow$ (6) Let $a, b \in R$ with $ab = 0$. Then ba is central. $(arb)^2 = 0$ for all $r \in R$. Since R is central rigid, arb is central for all $r \in R$. Hence R is central semicommutative.

(6) $\Longrightarrow$ (2) Let $a, b \in R$ with $ab = 0$. Then $(ba)^2 = 0$ and so ba is central. $(arb)^2 = 0$ for all $r \in R$. Thus we have $(arb)R(arb) = 0$ for all $r \in R$. Since R is semiprime, R is semicommutative.

(3) $\Longrightarrow$ (4) Clear.

(4) $\Longrightarrow$ (3) Assume that R is a central reversible ring and $a, b \in R$ with $ab = 0$. Then ba is central. Since R is a semiprime ring, $baRba = 0$ implies $ba = 0$.

(5) $\Longrightarrow$ (6) Clear.

(6) $\Longrightarrow$ (5) Let a,$b \in R$ with $ab = 0$. Then for any $r \in R$, arb is a central element and so a^2rb, arb^2 are central. For any $r \in R$, $b(arb)a = ba(arb) = b(a^2rb) = a^2rb^2 = a(arb)b = ab(arb) = 0$. Hence $baRba = 0$. By hypothesis $ba = 0$, so $aRb = 0$. □

Let $P(R)$ denote the prime radical and $N(R)$ the set of all nilpotent elements of the ring R. The ring R is called 2-*primal* if $P(R) = N(R)$ (See namely [10] and [12]). In [17, Theorem 1.5] it is proved that every semicommutative ring is 2-primal. In this direction we obtain the following result.

2.18. Theorem. *If R is a central rigid ring, then it is 2-primal. The converse holds for semiprime rings.*

Proof. Let R be a central rigid ring. We always have $P(R) \subseteq N(R)$, since $P(R)$ is a nil ideal of R.

For the converse inclusion, let $a \in N(R)$ with $a^n = 0$ for some positive integer n. Assume that $a \notin Q$ for a prime ideal Q. Since R is central rigid, a is central. For any $r_{n-1}, r_{n-2}, \dots, r_2, r_1 \in R$, we have $ar_{n-1}ar_{n-2}a \cdots ar_2ar_1a = r_{n-1}r_{n-2} \cdots r_2r_1a^n = 0$. For all prime ideals P, we have $aR(ar_{n-2}a \cdots ar_2ar_1a) \subseteq P$. Since $a \notin Q$, $ar_{n-2}a \cdots ar_2ar_1a \in P$ for all prime ideals P and $r_{n-2}, \dots, r_2, r_1 \in R$. Hence $aR(ar_{n-3}a \cdots ar_2ar_1a) \subseteq P$ for all prime ideals P and $r_{n-3}, \dots, r_2, r_1 \in R$. By a similar reasoning, $aR(ar_{n-4}a \cdots ar_2ar_1a) \subseteq P$, $ar_{n-4}a \cdots ar_2ar_1a \in P$ for all prime ideals P and for all $r_{n-4}, \dots, r_2, r_1 \in R$. By doing a downward induction, we may reach $aRa \subseteq P$ for all prime ideals P. Hence $a \in P$ for all prime ideals P. This is the required contradiction. Thus if a is nilpotent, then $a \in P(R)$ and so $N(R) \subseteq P(R)$.

Conversely, let R be a semiprime and 2-primal ring. Then $P(R) = 0$ and so $N(R) = 0$. Hence R is reduced and so central rigid. This completes the proof. □

2.19. Corollary. *Let R be a central rigid ring. Then the ring $R/P(R)$ is central rigid.* □

A module M has the *summand intersection property* if the intersection of two direct summands is again a direct summand of M. A ring R is said to have the *summand intersection property* if the right R-module R has the summand intersection property. A module M has the *summand sum property* if the sum of two direct summands is a direct summand of M and a ring R is said to have the *summand sum property* if the right R-module R has the summand sum property.

2.20. Proposition. *Let R be a central rigid ring. Then we have*

(1) *R has the summand intersection property.*
(2) *R has the summand sum property.*

Proof. (1) Let e and f be idempotents of R. By Lemma 2.14, e and f are central, we have $eR \cap fR = efR = feR$ and $(ef)^2 = ef$. This completes the proof.

(2) Let eR and fR be right ideals of R with $e^2 = e$, $f^2 = f \in R$. Then $e + f - ef$ is an idempotent of R. Since R is abelian, it is easy to check that $eR + fR = (e + f - ef)R$. So $eR + fR$ is a direct summand of R. □

The *Dorroh extension* $D(R, \mathbb{Z}) = \{(r, n) \mid r \in R,\ n \in \mathbb{Z}\}$ of a ring R is a ring with operations $(r_1, n_1) + (r_2, n_2) = (r_1 + r_2, n_1 + n_2)$ and $(r_1, n_1)(r_2, n_2) = (r_1r_2 + n_1r_2 + n_2r_1, n_1n_2)$. Obviously R is isomorphic to the ideal $\{(r, 0) \mid r \in R\}$ of $D(R, \mathbb{Z})$. Then we have the following.

2.21. Proposition. *A ring R is central rigid if and only if the Dorroh extension $D(R, \mathbb{Z})$ of R is central rigid.*

Proof. Let R be a central rigid ring and $(r, n), (s, m) \in D(R, \mathbb{Z})$ with $(r, n)^2(s, m) = 0$. Since $n^2m = 0$, it follows that $n = 0$ or $m = 0$. Assume that $n = 0$, so $r^2s + mr^2 = 0$. Thus $r(s + m1_R)$ is central. Then $(r, n)(s, m)(u, t) = (u, t)(r, n)(s, m)$ for any $(u, t) \in D(R, \mathbb{Z})$. Therefore $D(R, \mathbb{Z})$ is central rigid.

The converse is clear. □

Let R be a ring and M an (R, R)-bimodule. Recall that the *trivial extension* of R by M is defined to be ring $T(R, M) = R \oplus M$ with the usual addition and the multiplication $(r_1, m_1)(r_2, m_2) = (r_1r_2, r_1m_2 + m_1r_2)$. This ring is isomorphic to the ring $\left\{ \begin{bmatrix} r & m \\ 0 & r \end{bmatrix} \mid r \in R,\ m \in M \right\}$ with the usual matrix operations and isomorphic to $R[x]/(x^2)$, where (x^2) is the ideal generated by x^2. The trivial extension of R by M need not be a central rigid ring, as the following example shows.

2.22. Example. Let $\mathbb{H}$ be the division ring of quaternions over the real numbers. Then $\mathbb{H}$ is a reduced ring but not commutative. Consider the nilpotent element $\begin{bmatrix} 0 & j \\ 0 & 0 \end{bmatrix}$ of $T(\mathbb{H}, \mathbb{H})$. Since $\begin{bmatrix} 0 & j \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} \neq \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 0 & j \\ 0 & 0 \end{bmatrix}$, $T(\mathbb{H}, \mathbb{H})$ is not central rigid.

Let R be a ring and $f(x) = \sum_{i=0}^{n} a_ix^i$, $g(x) = \sum_{j=0}^{m} b_jx^j \in R[x]$. Rege and Chhawchharia [16] introduce the notion of an Armendariz ring, that is, a ring R is called *Armendariz* if $f(x)g(x) = 0$ implies $a_ib_j = 0$ for all i and j. The name of the ring was given due to Armendariz who proved that reduced rings satisfied this condition [6]. The interest of this notion lies in its natural and useful role in understanding the relation between the annihilators of the ring R and the annihilators of the polynomial ring $R[x]$. So far, Armendariz rings have been generalized in different ways. A ring R is called *weak Armendariz* [15], if whenever$f(x)g(x) = 0$, then a_ib_j is a nilpotent element of R for each i and j, while a ring R is called *nil-Armendariz* [5], if whenever $f(x)g(x)$ has nilpotent coefficients, then a_ib_j is nilpotent for $0 \leq i \leq n$, $0 \leq j \leq s$. Clearly every nil-Armendariz ring is weak Armendariz. According to Harmanci et al. [2], a ring R is called *central Armendariz*, if $f(x)g(x) = 0$ implies that a_ib_j is a central element of R for all i and j. In [4, Theorem 5], Anderson and Camillo proved that for a ring R and $n \geq 2$ a natural number, $R[x]/(x^n)$ is Armendariz if and only if R is reduced. For central rigid rings, we obtain the following result.

2.23. Theorem. *Let R be a right principally projective ring and $n \geq 2$ a natural number. Then R is central rigid if and only if $R[x]/(x^n)$ is central Armendariz.*

Proof. Suppose R is a central rigid ring. By Proposition 2.4, R is a reduced ring. From [4, Theorem 5], $R[x]/(x^n)$ is Armendariz and so central Armendariz.

Conversely, assume that $R[x]/(x^n)$ is central Armendariz. By hypothesis and [2, Theorem 2.5], $R[x]/(x^n)$ is Armendariz. It follows from [4, Theorem 5] that R is reduced and so central rigid. $\square$

We end the paper with some observations.

2.24. Theorem. *If R is a central rigid ring, then R is nil-Armendariz.*

Proof. If R is central rigid, then it is 2-primal by Theorem 2.18 and so $N(R)$ is an ideal of R. [5, Proposition 2.1] states that in a ring in which the set of all nilpotent elements forms an ideal, then the ring is nil-Armendariz. $\square$

2.25. Corollary. *If R is a central rigid ring, then $R[x]/(x^n)$ is nil-Armendariz, where $n \geq 2$ is a natural number and (x^n) is the ideal generated by x^n.*

Proof. If R is central rigid, then it is nil-Armendariz by Theorem 2.24. From [5, Proposition 4.1], $R[x]/(x^n)$ is nil-Armendariz. $\square$

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